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Surplus production models in a Bayesian framework applied to Greenland halibut in SA2+Div 3KLMNO

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Abstract

A number of formulations of surplus production models in a Bayesian framework were explored using data for Greenland halibut in SA2+Div. 3KLMNO. The purpose of this modelling was to explore models that do not include ageing. The reason for doing this was because of issues with ageing and lack of consistency in age compositions both within and among surveys. The best model formulation of those tested included the following survey data: Canadian fall 2J3K, spring 3LNO, and the EU 3M series split, i.e. 0-700m 1995-2015 and 700-1400m from 2004-2015. All model runs showed a similar pattern with B_{ratio} estimated to have decreased substantially and rapidly in the early 1990s with the large catches in that period. Since that time B_{ratio} remained at a lower level, slightly above B_{lim} (30% B_{MSY}), with some variation in the level relative to B_{lim} in the recent years. The consistency in the estimated biomass trajectory over time across the various model formulations indicates that at least some of the operating models in the management strategy evaluation of Greenland halibut should be consistent with this view of the population trajectory.

Introduction

A number of formulations of surplus production models in a Bayesian framework were explored using data for Greenland halibut in SA2+Div. 3KLMNO. The purpose of this modelling was to explore models that do not include ageing. The reason for doing this was because of issues with ageing and lack of consistency in age compositions both within and among surveys.

Methods

The Schaefer (1954) form of a surplus production model used here is:

$P_t=[P_{t-1}+ r\bullet P_{t-1} (1 - P_{t-1})- C_{t-1}/K]\bullet\eta_t$

where P_{t-1} and C_{t-1} denote exploitable biomass (as a proportion of carrying capacity) and catch, respectively, for year t-1 (Meyer and Millar, 1999a, 1999b). Carrying capacity, K, is the level of stock biomass at equilibrium prior to commencement of a fishery, r is the intrinsic rate of population growth, and η_t is a random variable describing stochasticity in the population dynamics (process error). The model utilizes biomass proportional to an estimate of K in order to aid mixing of the Markov Chain Monte Carlo (MCMC) samples and to help minimize autocorrelation between each state and K (Meyer and Millar, 1999a, 1999b).

An observation equation is used to relate the unobserved biomass, P_t, to the research vessel survey indices:

$$I_t=q\bullet P_t \bullet\varepsilon_t$$

where q is the catchability parameter, P_t is an estimate of the biomass proportional to K at time t, and ε_t is observation error.

All models used landings – 1960-2015. Several different combinations of survey data were used. See table 1 for full set of data. Results for selected parameters with comments are found in Table 2.

Run 1 (All surveys K 200,10, r 0.2,0.1)

The starting point included all of the survey data (option 1)

	O1
Fall 2J3K	1996-2015
Spring 3LNO	1996-2014
EU 3M 0-700	1995-2015
EU 3M 0-1400	
EU 3M 700-1400	2004-2015
EU 3L	2006-2015
EU 3NO	1997-2015
Fall 3LNO	1996-2015

Priors were as follows (mean, precision):

r ~ dlnorm(-1.72,4.49)

K~dlnorm(5.297,400.48)

prior distribution of q's same for all surveys

pq.f2J3K~dgamma(1,1)

q.f2J3K<-1/pq.f2J3K

Prior for process error, sigma

sigma ~ dunif(0,10)

isigma2 <- pow(sigma, -2)

Prior for observation errors, tau same for all surveys

a0<-1

b0<-1

tau.f2J3K~dgamma(a0,b0)

itau2.f2J3K <- 1/tau.f2J3K

The prior for K in this run was too informative and the posterior was very similar to the prior. The estimate of r was unreasonably high for a long lived, slow growing, late maturing species (0.75). Further, the process error was large relative to all observation errors.

Run 2 (All surveys K 400,100, r 0.2, 0.1). A second run was conducted with a less informative prior on K.

K~dlnorm(5.961,16.498)

In this case the parameter estimate for r was still unreasonably large (0.62) and the process error large relative to all observation errors.

Run 3 (All surveys K 400,40, r 0.2, 0.1). Since K and r are not independent, a more informative prior on K with a larger mean than run 1 was attempted to examine the impact on r.

K~dlnorm(5.986,100.5)

Results were similar to previous runs.

Run 4 (All surveys K 400,100, r 0.12,0.01) . An informative prior on r was the next run.

r ~ dlnorm(-2.124,144.5)

K~dlnorm(5.961,16.498)

This resulted in a reasonable r for a species like Greenland halibut (0.13), but a large increase in process error and an increase in observation error.

The size of the process error was an issue in all these runs. So the next series of runs attempted to decrease the process error.

Run 5 (Surveys last accepted assessment, K 400,40, r 0.12,0.1). The starting point for these runs was the surveys used in the last accepted assessment. With the following priors on r and K:

r ~ dlnorm(-1.72,4.49)

K~dlnorm(5.986,100.5)

This resulted in a decrease in r to 0.39 (still large for Greenland halibut but more biologically plausible) and a smaller process error than other runs, although process error was still large relative to the observation error.

Run 6 (Surveys last accepted assessment, K 500,100, r 0.12,0.1). To gauge the impact on r of a less informative K with a higher mean the following prior was used:

K~dlnorm(6.195,25.508)

Results of this model were very similar to the previous run.

Run 7 (Surveys last accepted assessment plus Engels 2J3K K 500, 100, r0.12 0.1 Engels). To see if the model would benefit from the addition of older data the Engel time series from 1978 to 1989 was added. This resulted in a small decrease in r to 0.35 and some improvement in the magnitude of process error relative to observation error for some surveys. But improvements were not substantial.

Run 8 (Surveys last accepted assessment EU split k 500, 100, r0.12 0.1. The next model formulation included the Canadian fall 2J3K, spring 3LNO, and the EU 3M series split, i.e. 0-700m 1995-2015 and 700-1400m from 2004-2015.

r ~ dlnorm(-2.38,1.896)

K~dlnorm(6.195,25.508)

This formulation improved the size of the process error relative to the observation error for most surveys (but not for fall 2J3K).

Run 9 (Surveys last accepted assessment EU split k 400, 40, r0.12 0.1). This same data set was used in a formulation with a more informative prior on K.

K~dlnorm(5.986,100.5)

This run had the smallest process error.

The size of the process error is an issue for all runs. A graphical comparison of process error over time is presented in Figure 1. It is clear that run 4 with an informative prior on r that resulted in a seemingly reasonable value of r, had very large process error compared to other runs. While run 1 was amongst the smallest process error, its process error was larger than the 2 runs with the EU split. The smallest process error was for run 9 and this run is explored further.

Model fit Run 9

Priors and posteriors for r and K are given in Figure 2. All posteriors (only r and K shown) were updated from their priors.

Figure 3 shows the observed and predicted survey along with standardized residuals. Model fit to the surveys seems reasonably good with little or no pattern in the residuals.

Process error (Figure 4) shows no trend over time.

Convergence diagnostics for Run 9 are shown in Appendix 1.

Results Run 9

B_{ratio} (Figure 5) is estimated to have decreased substantially and rapidly in the early 1990s with the large catches in that period. Since that time B_{ratio} has remained at a lower level, slightly above B_{lim} (30% B_{MSY}). All model runs showed this same pattern, with some variation in the level relative to B_{lim} in the recent years. Run 9 estimates a B_{ratio} of 0.41 in 2015. The most pessimistic run has estimates of B_{ratio} of near 0.3 in the most recent years, while the most optimistic estimates B_{ratio} at about 0.58. Runs also differed in the size of the credible intervals in the period prior to the surveys, with credible intervals for Run 9 being amongst the smallest.

F_{ratio} in Run 9 (Figure 6) was estimated to generally be above 1 since 1990 but has been decreasing since about 2010. The trend in F_{ratio} was similar across runs but the period of time above F_{MSY} varied.

The consistency in the estimated biomass trajectory over time across the various model formulations indicates that at least some of the operating models in the management strategy evaluation of Greenland halibut should be consistent with this view of the population trajectory.

References

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Table 1. Data used in the Bayesian Surplus Production models. Values are in thousands of tons.

Year	Landings	f2J3K	sp3LNO	f3LNO	EU3M7	EU3M14	EU3L	EU3NO	F2J3K	EU3M7all	EU3M714
1960	0.9										
1961	0.7										
1962	0.6										
1963	2										
1964	4										
1965	10										
1966	19										
1967	27										
1968	32										
1969	37										
1970	37										
1971	25										
1972	30										
1973	29										
1974	28										
1975	28.814										
1976	24.611										
1977	32.048										
1978	39.07								184.2717		
1979	34.104								133.3		
1980	32.867								145.187		
1981	30.754								154.6271		
1982	26.278								175.1035		
1983	27.861								176.1205		
1984	26.711								194.0843		
1985	20.347								141.4081		
1986	17.976								185.3908		
1987	32.442								127.2533		
1988	19.215								103.7177		
1989	20.034								111.2168		
1990	47.454										
1991	65.008										
1992	63.193										
1993	62.455										
1994	51.029										
1995	15.272				10.875					10.875	
1996	18.84	185.1071	15.71855	26.71187	11.594					11.594	
1997	19.858	212.6417	25.23187	29.56447	16.098			6.859332		16.098	
1998	19.946	204.3071	47.8396	42.66205	24.229			11.30522		24.229	
1999	24.226	262.7791	28.78056	28.75192	21.207			11.24637		21.207	
2000	34.177	198.1868	31.32762	25.25606	16.959			9.331294		16.959	
2001	38.232	194.4974	15.3435	21.59101	13.872			7.721428		13.872	
2002	34.062	120.59	7.43686	15.01681	12.1			2.379667		12.1	
2003	35.151	131.2216	15.12497	16.99365	6.214			4.700956		6.214	
2004	25.486	149.6441	11.78614	19.23214		28.561		3.437284		12.292	16.382
2005	23.255	173.6577	17.20721	28.63054		20.46		3.070872		11.698	8.762
2006	23.531	216.0995	14.4352	25.46323		23.475	8.795172	2.719831		11.706	11.767
2007	22.747	236.8296	31.21188	24.45516		30.731	9.602631	3.285677		13.04	17.691
2008	21.18		21.07216	29.51543		38.444	14.49439	7.271533		11.995	27.617
2009	23.156	162.6193	6.990234	16.27724		36.047	12.03035	12.92692		7.775	28.27
2010	26.174	166.2302	17.15382	17.42467		27.094	13.46615	12.46162		6.656	20.438
2011	24.96	214.7178	10.816	22.64046		32.021	8.476713	6.483437		6.713	25.308
2012	22.98	191.1353	19.56726	17.62161		23.504	8.475973	6.829932		4.29	19.214
2013	19.98	245.104	7.513936	26.70615		23.391	10.01795	4.959137		2.799	20.592
2014	21.43	271.0874	6.837792			29.288	13.74271	5.48212		5.167	24.121
2015	15.27	181.2614		8.945409		58.17913	13.46174	8.519134		6.577032	51.60209

Table 2. Parameter estimates from different formulations of surplus production models. Sigma is process error. Tau referred to in comments is observation error. See text for details.

RUN 1 All surveys, K 200,10, r 0.2,0.1

R	K	MSY	FMSY	sigma	comments
0.75	200.5	38.02	0.37	0.21	K not updated, r very large, sigma large relative to all tau

RUN 2 All surveys, K 400,100, r 0.2,0.1

R	K	MSY	FMSY	sigma	comments
0.62	298.4	47.04	0.31	0.26	r very large, sigma large relative to all tau

RUN 3 All surveys, K 400,40, r 0.2,0.1

R	K	MSY	FMSY	sigma	comments
0.54	388.3	52.27	0.27	0.27	r very large, sigma large relative to all tau

RUN 4 All surveys, K 400,100, r 0.12,0.01

R	K	MSY	FMSY	sigma	comments
0.13	460.3	14.64	0.06	0.40	r reasonable for turbot, sigma large relative to all tau and to other formulations, tau larger

RUN 5 Surveys last accepted assessment, K 400,40, r 0.12,0.1

R	K	MSY	FMSY	sigma	comments
0.39	379.7	36.39	0.19	0.18	r somewhat large, sigma large relative to most tau

RUN 6 Surveys last accepted assessment, K 500,100, r 0.12,0.1

R	K	MSY	FMSY	sigma	comments
0.37	410.6	37.75	0.19	0.18	r somewhat large, sigma large relative to most tau

RUN 7 Surveys last accepted assessment plus Engels 2J3K K 500, 100, r0.12 0.1 Engels

R	K	MSY	FMSY	sigma	comments
0.35	418.8	36.84	0.18	0.17	r somewhat large, sigma large but improved for some tau

RUN 8 Surveys last accepted assessment EU split k 500, 100, r0.12 0.1

R	K	MSY	FMSY	sigma	comments
0.38	407.7	37.05	0.19	0.17	r somewhat large, sigma large relative to 2J3K

RUN 9 Surveys last accepted assessment EU split k 400, 40, r0.12 0.1

R	K	MSY	FMSY	sigma	comments
0.38	378.3	35.52	0.19	0.15	r somewhat large, sigma smallest, sigma large relative to 2J3K

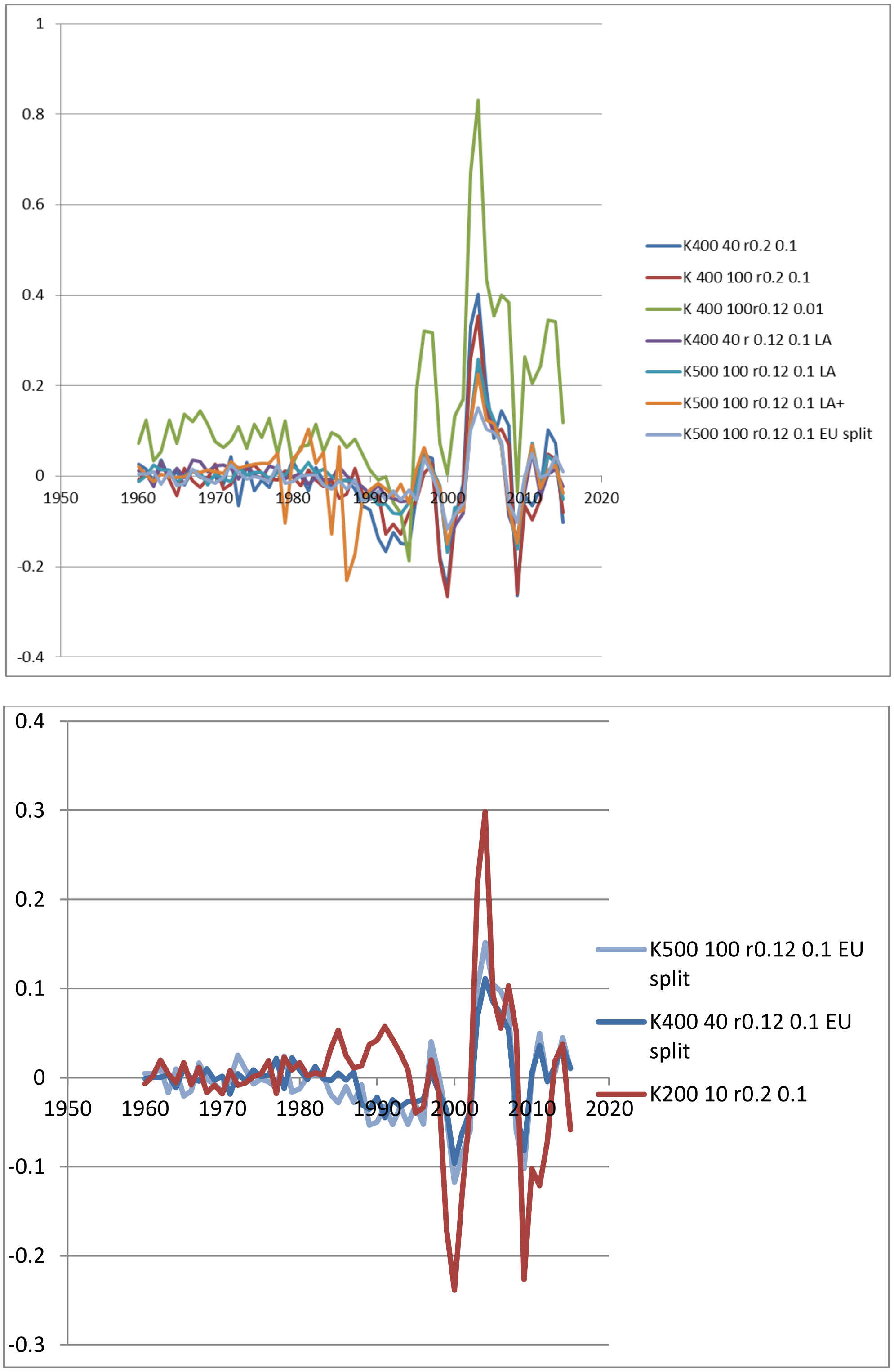

Fig.1. Process error from various model runs. See text for description of model runs.

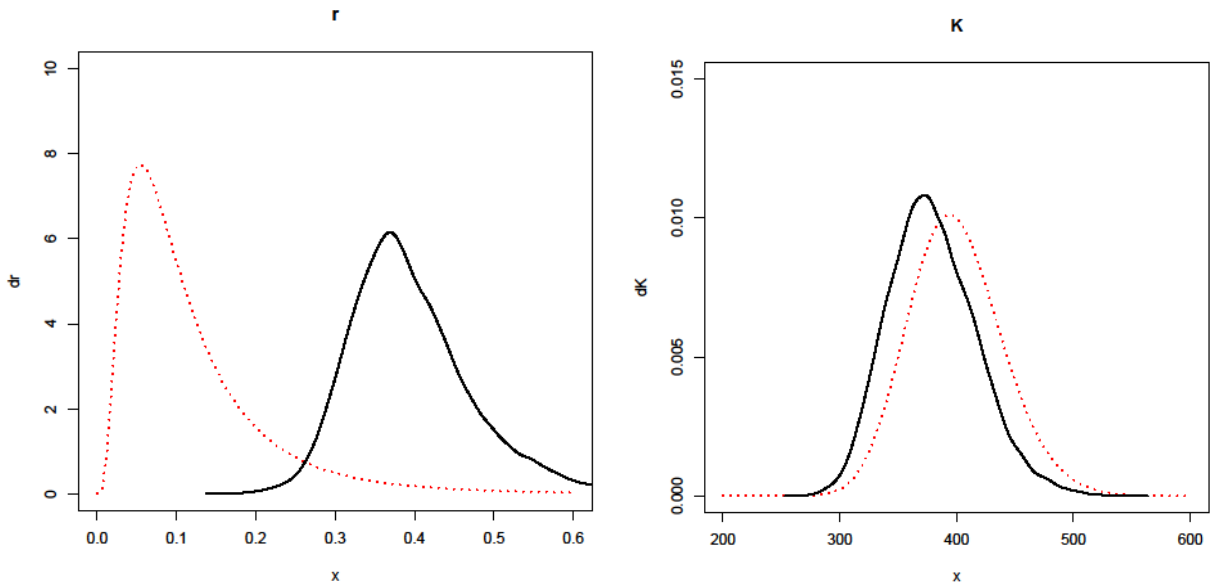

Fig. 2. Priors (red dotted) and posteriors (black solid) for r and K from run 9.

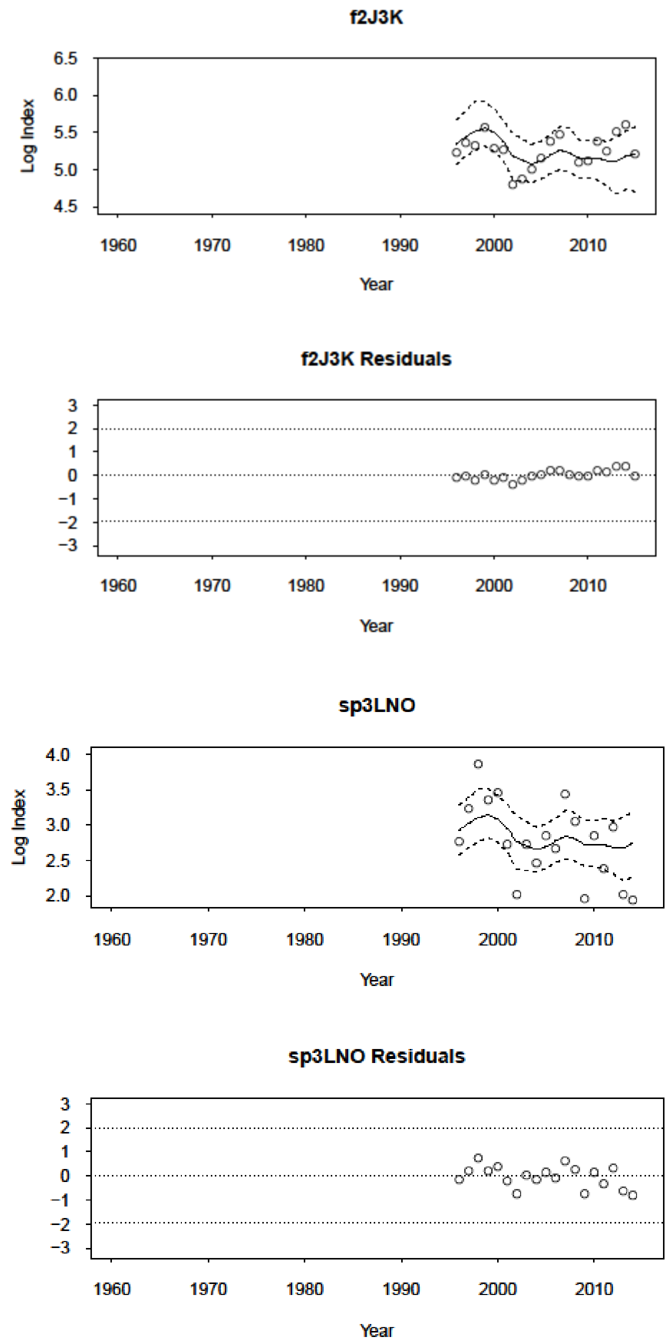

Fig. 3a. Observed and predicted for each survey along with standardized residuals ((x-mean(x))/sd(x)), for Canadian fall 2J3K (f2J3K) and Canadian spring 3LNO (sp3LNO) surveys.

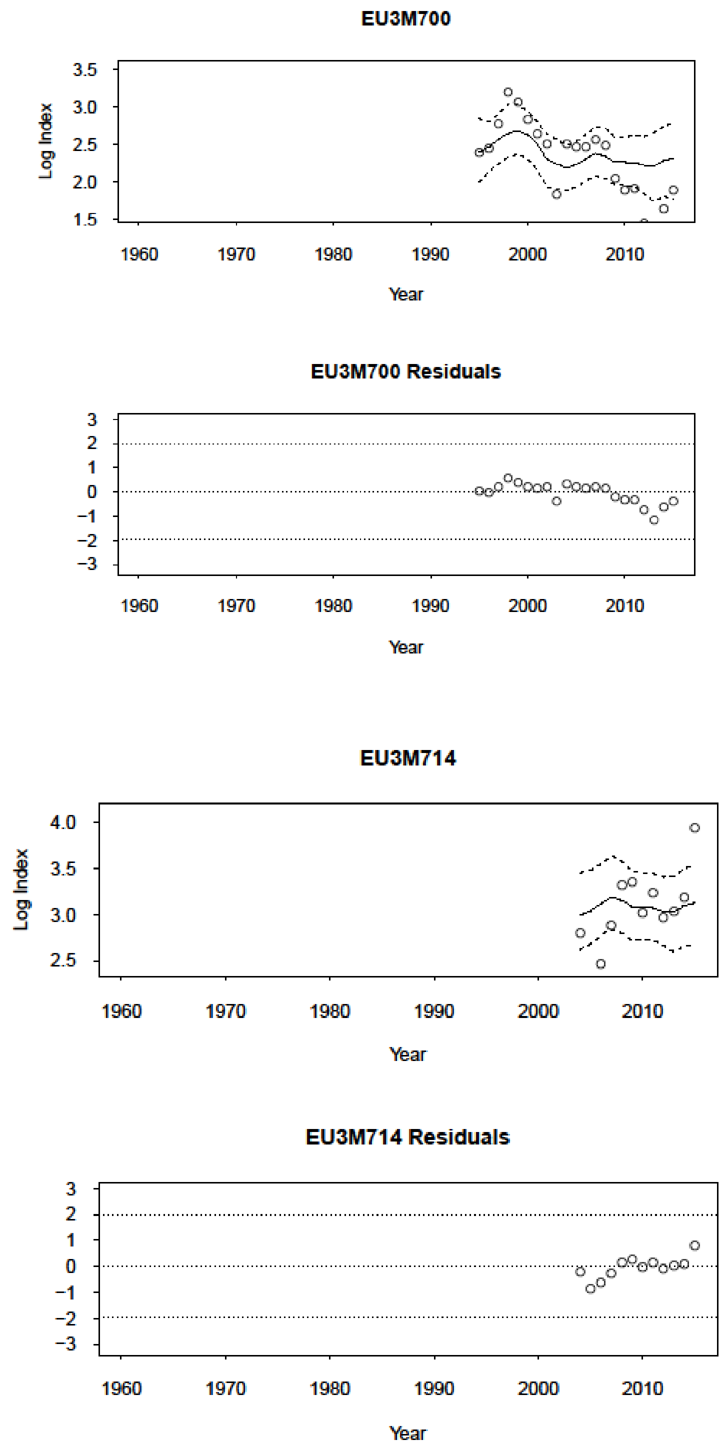

Fig.3b. Observed and predicted for each survey along with standardized residuals ((x-mean(x))/sd(x)), EU 3M survey from 0-700m (EU3M700) and EU 3M survey from 700-1400m (EU3M714).

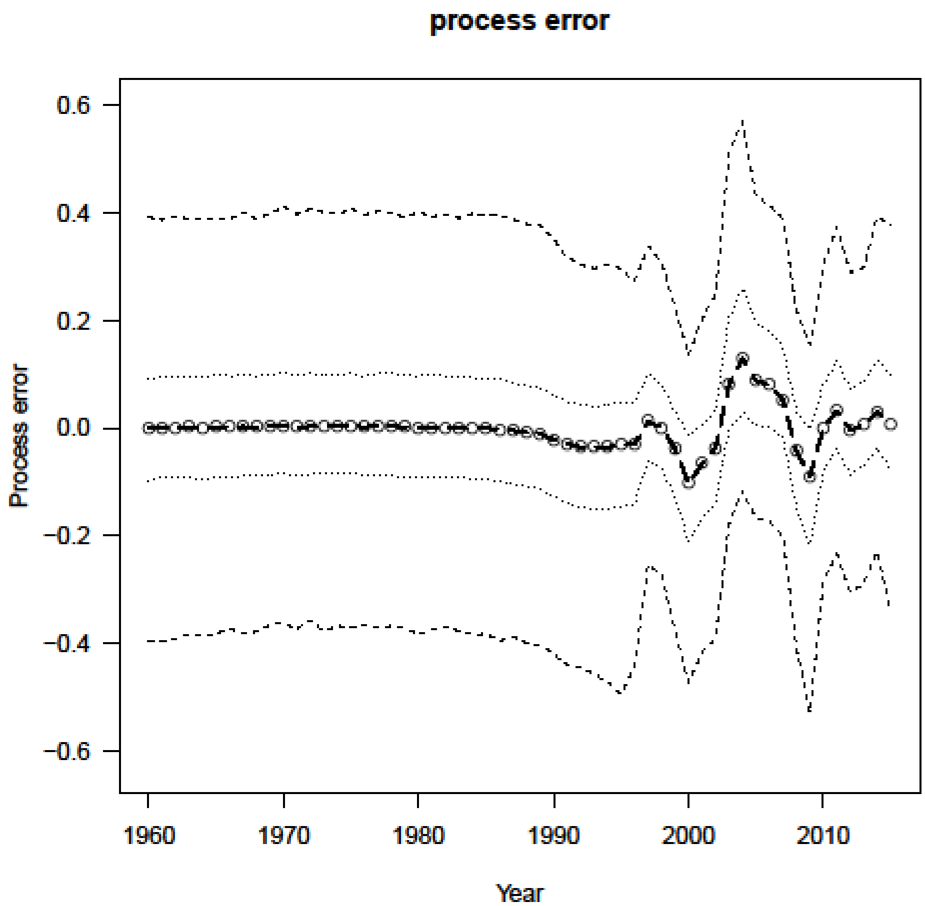

Fig. 4. Process error with 50% and 90% credible intervals for run 9.

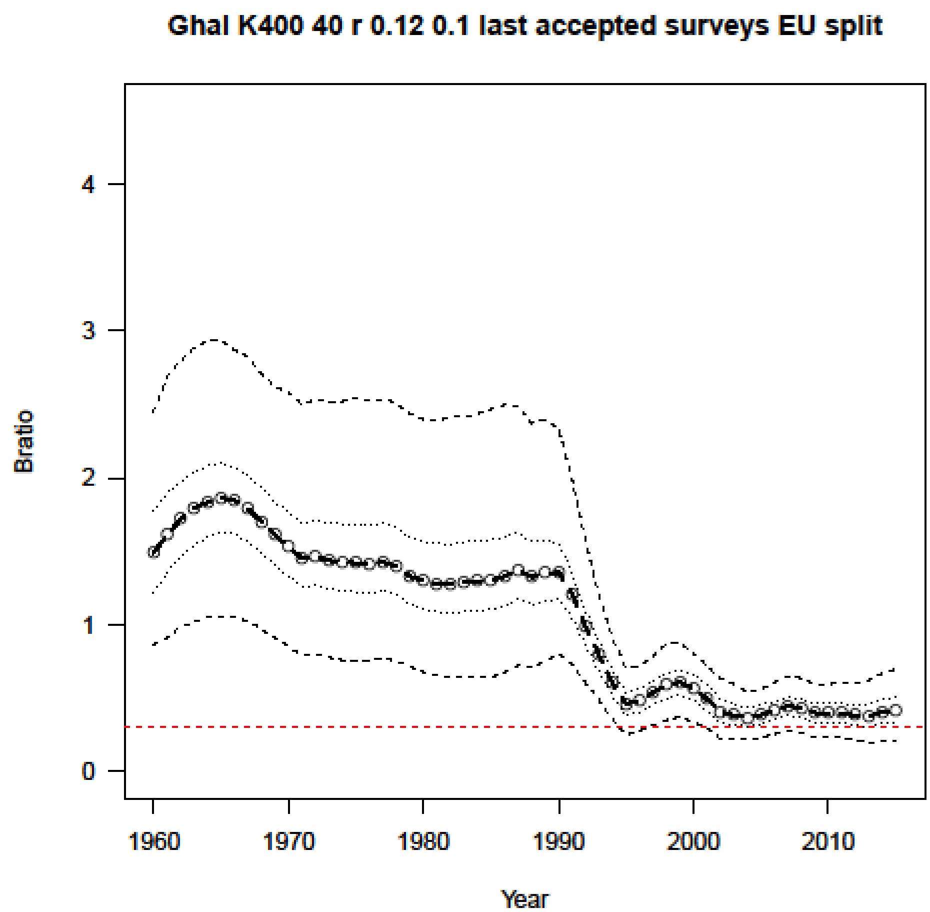

Fig. 5. Ratio of Biomass to B_{MSY} along with 50% and 90% credible intervals for run 9. The red dotted line is B_{lim} which is 30% B_{MSY}.

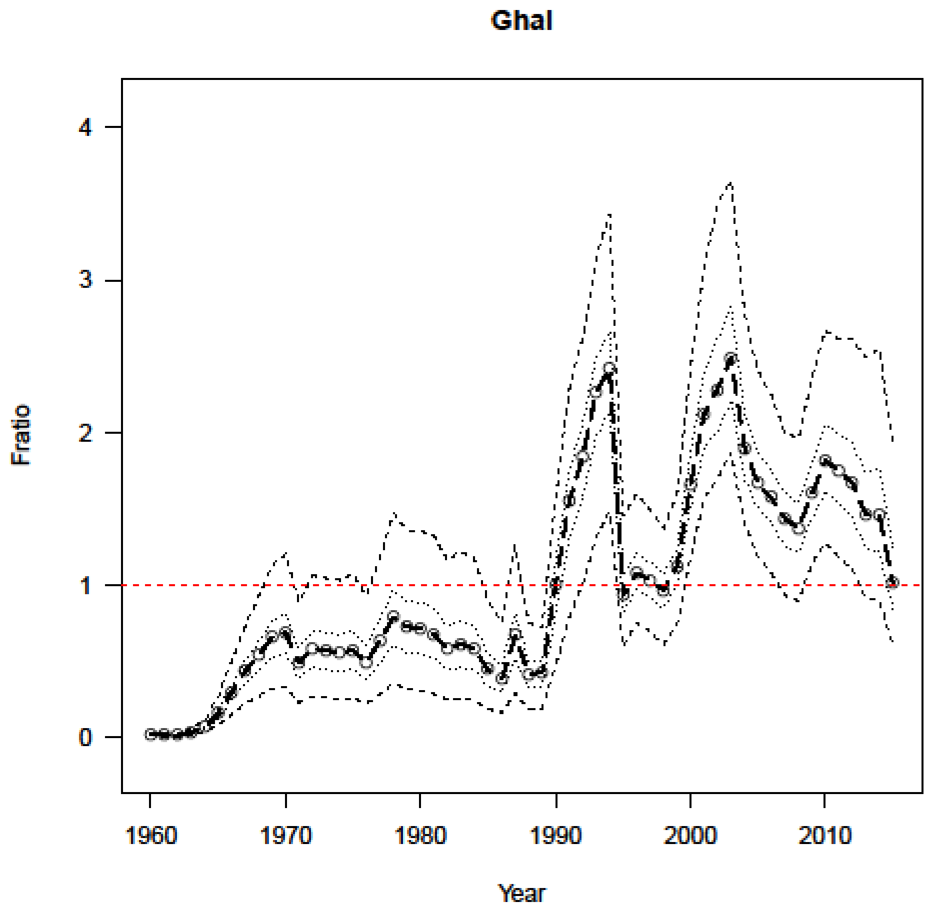

Fig. 6. Ratio of fishing mortality to F_{MSY} along with 50% and 90% credible intervals for run 9. The red dotted line is F_{lim} which is F_{MSY}.

Appendix 1

Convergence diagnostics Run 9

Convergence diagnostics r

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3936	0.0799	0.0011	0.0011	0.0012	-0.1350	0.2693	0.382	0.5801	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3972	0.0786	0.0011	0.0014	0.0013	-0.0270	0.2717	0.3857	0.5802	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3962	0.0785	0.0011	0.0012	0.0012	-0.0489	0.2738	0.3839	0.5844	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000414

Multivariate Potential Scale Reduction Factor = 1.000733

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000747 1.00246

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 1.1980373

p-value 0.2309025

Chain: turbotchain2

x

Z-Score -0.6146555

p-value 0.5387822

Chain: turbotchain3

x

Z-Score -1.4836602

p-value 0.1378991

Diagnostics K

SUMMARY STATISTICS:

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Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
380.3658	37.6772	0.5616	0.5609	0.5500	0.0940	312.5	377.2	461	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
379.0429	37.9241	0.5653	0.6593	0.6113	0.1047	315.0	376.3	460.9	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
379.2188	36.9817	0.5512	0.6301	0.6286	-0.0507	314.1	376.9	455.2	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000146

Multivariate Potential Scale Reduction Factor = 1.000331

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000423 1.001415

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 0.004322175

p-value 0.996551414

Chain: turbotchain2

x

Z-Score 0.4548116

p-value 0.6492448

Chain: turbotchain3

x

Z-Score -0.3964448

p-value 0.6917769

Convergence diagnostics process error

SUMMARY STATISTICS:

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1648	0.0840	0.00125	0.0019	0.0017	-0.0324	0.0143	0.1591	0.3484	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1655	0.0821	0.0012	0.0018	0.0017	-0.0211	0.0151	0.161	0.3432	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1665	0.0841	0.0012	0.0015	0.0015	-0.1404	0.0200	0.1591	0.3532	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

0.999897

Multivariate Potential Scale Reduction Factor = 0.9999566

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000021 1.000342

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 0.2011824

p-value 0.8405560

Chain: turbotchain2

x

Z-Score -1.1152361

p-value 0.2647493

Chain: turbotchain3

x

Z-Score -0.2767956

p-value 0.7819371

Diagnostics q f2J3K

SUMMARY STATISTICS:

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Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
2.3842	0.5080	0.0075	0.0082	0.0083	-0.0834	1.6314	2.3065	3.5962	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
2.3965	0.4983	0.0074	0.0092	0.0086	0.0154	1.617	2.33	3.5411	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
2.3914	0.4970	0.0074	0.0070	0.0076	-0.1227	1.6094	2.329	3.5530	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

0.9998208

Multivariate Potential Scale Reduction Factor = 0.9998423

Corrected Scale Reduction Factors

Estimate 0.975

x 0.9998375 0.9999533

GEWEKE CONVERGENCE DIAGNOSTIC:

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 0.3465736

p-value 0.7289117

Chain: turbotchain2

x

Z-Score -1.049050

p-value 0.294155

Chain: turbotchain3

x

Z-Score -0.4327403

p-value 0.6652034

Convergence diagnostics q sp3LNO

SUMMARY STATISTICS:

==================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.2115	0.0475	0.0007	0.0006	0.0006	-0.0967	0.1386	0.2045	0.3244	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.2129	0.0473	0.0007	0.0008	0.0008	0.0052	0.1399	0.2066	0.3207	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.2125	0.0462	0.0006	0.0006	0.0007	-0.1862	0.1371	0.2061	0.3180	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

0.9998128

Multivariate Potential Scale Reduction Factor = 0.9998303

Corrected Scale Reduction Factors

Estimate 0.975

x 0.9998747 0.9999689

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

X

Z-Score 0.3278664

p-value 0.7430127

Chain: turbotchain2

x

Z-Score -0.3900970

p-value 0.6964648

Chain: turbotchain3

x

Z-Score 0.6889215

p-value 0.4908727

Convergence diagnostics q sp3LNO

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1329	0.0281	0.0004	0.0005	0.0004	0.0111	0.0897	0.1288	0.1981	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1337	0.0282	0.0004	0.0004	0.0004	-0.0570	0.0895	0.1297	0.1981	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1335	0.0276	0.0004	0.0004	0.0004	-0.1071	0.0902	0.1302	0.1973	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000524

Multivariate Potential Scale Reduction Factor = 1.000897

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000649 1.002655

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 1.98783367

p-value 0.04683009

Chain: turbotchain2

x

Z-Score -0.7251590

p-value 0.4683545

Chain: turbotchain3

x

Z-Score -0.8416623

p-value 0.3999770

Convergence diagnostics q EU3M714

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: turbotchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample

0.3037	0.0797	0.0011	0.0012	0.0012	-0.0495	0.1837	0.2899	0.4881	1	4500	4500

Chain: turbotchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3036	0.0780	0.0011	0.0012	0.0011	0.0274	0.1858	0.2913	0.4905	1	4500	4500

Chain: turbotchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3047	0.0800	0.0011	0.0010	0.0012	-0.0934	0.1839	0.2929	0.4962	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000301

Multivariate Potential Scale Reduction Factor = 1.000563

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000726 1.002135

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: turbotchain1

x

Z-Score 0.05289689

p-value 0.95781407

Chain: turbotchain2

x

Z-Score 0.1137123

p-value 0.9094658

Chain: turbotchain3

x

Z-Score -0.3094060

p-value 0.7570127