Serial No. 1139
(D.c.2)

ANNUAL MEETING-JUNE, 1963.

Document No. 68.

Length-Weight Relationship of Redfish Collected from U.S. Landings in 1951

by Bradford E. Brown
Bureau of Commercial Fisheries
Woods Hole, Mass.

Introduction

Fishery investigations often require information on the relationship between length and weight of fish. Some studies, for example, require estimates of the number of fish caught, and these estimates can be obtained from length samples of the catch using weight at length data. This report contains the results of analysis of available data on length and weight of redfish caught in the Northwest Atlantic, collected by the United States Fish and Wildlife Service. Geographical and temporal variations are discussed, and equations for estimating weight at length are presented.

The Data

Lengths in milimeters and weight in grams were recorded for redfish sampled from the commercial landings. Redfish are landed round, but have been stored on ice for varying periods up to two weeks. A total of 2000 fish, in 22 samples, were obtained from landings caught during the months of April through August 1951, in 12 sampling areas. In addition 300 more fish were taken in 6 samples from landings from mixed subareas. These sampling areas include fishing grounds in ICNAF Subareas

4 and 5. A two digit code was assigned to sampling units within these Subareas, (fig. 1). The first digit of the code indicates an area corresponding to an ICNAF Division. Division 4T is considered as a separate sampling area, 30.

Analysis

An equation of the form - $W = cl^b$ was assumed. Lengths and weights were transformed by natural logarithims:

$y = \log_e W$, $x = \log_e l$.

Regressions were fitted by the least square method to the equation

$$y = a+bx,.$$

where -

$$a = \log_e c.$$

Notation for regression and covariance analyses throughout this report follows that of Snedecore (1956).

Comparisons between sexes

Regressions were first computed for each sex within each sample, (Table 1). The differences between regressions of males and females were tested by covariance analyses. The F value obtained by pooling all 28 analysis (Table 2) indicated that there was, on the average, no significant difference between the regressions. Five of the 28 individual comparisons were significantly different, but the pooled analysis showed that these differences were not consistent. Therefore, further analyses in this study were made with measurements of males and females combined within each sample.

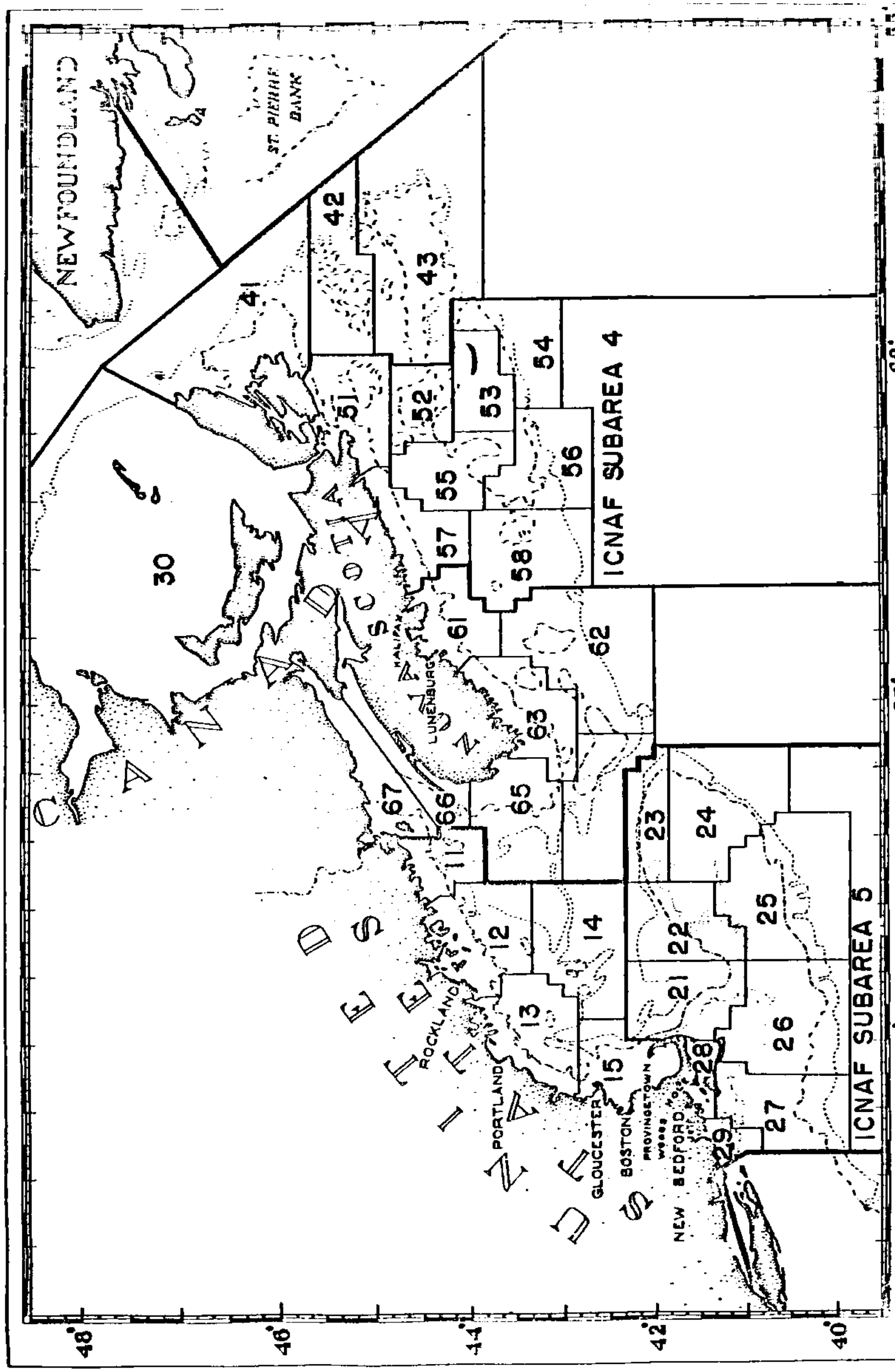

Fig. I. Sampling areas for Redfish length-weight study.

Table 1.--Regression statistics for samples of redfish by area, month and sex.

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
30	July		24	0.11523	0.32270	1.05549	0.1518	0.0069	2.800	-9.930
			26	0.08539	0.21239	0.65947	0.1312	0.0057	2.487	-8.101
		Total	50	0.36475	0.94052	2.71613	0.2909	0.0061	2.578	-8.639
	Aug		25	0.04467	0.09947	0.43175	0.2102	0.0091	2.227	-6.665
			25	0.05289	0.14992	0.56126	0.1363	0.0059	2.835	-10.212
		Total	50	0.15936	0.45689	1.69019	0.3802	0.0079	2.867	-10.422
41	June		27	0.28391	0.80920	2.49962	0.1932	0.0077	2.850	-10.282
			23	0.27197	0.88985	3.14925	0.2378	0.0113	3.272	-12.747
		Total	50	0.76450	2.27220	7.22513	0.4718	0.0098	2.972	-10.988
42	April		47	0.98630	3.05170	9.77028	0.3280	0.0073	3.094	-11.568
			53	0.87077	2.49728	7.54828	0.3863	0.0076	2.868	-10.220
		Total	100	2.46744	7.50708	23.59967	0.7597	0.0078	3.042	-11.259
42 + 43	May		71	0.51276	1.43585	4.38748	0.3667	0.0053	2.800	-9.962
			79	0.82608	2.24454	6.70163	0.6030	0.0078	2.717	-9.436
		Total	150	2.08301	5.99023	18.25818	1.0317	0.0070	2.876	-10.373

Table 1. -- Continued

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
42 + 43	July		54	0.53926	1.49829	4.42039	0.2575	0.0050	2.778	-9.822
			96	1.51474	4.25208	12.92551	0.9894	0.0105	2.807	-9.930
		Total	150	2.38788	6.88092	21.16943	1.3414	0.0091	2.881	-10.380
43	Aug		61	0.44415	1.14423	3.20282	0.2551	0.0043	2.576	-8.652
			89	0.91122	2.72397	8.74197	0.5991	0.0069	2.989	-11.012
		Total	150	2.03406	5.88745	17.95642	0.9156	0.0062	2.894	-10.461
51	June		7	0.03926	0.11020	0.34127	0.0319	0.0064	2.807	-9.959
			43	0.59723	1.60063	4.46735	0.1775	0.0043	2.680	-9.239
		Total	50	0.72675	1.95692	5.47993	0.2105	0.0044	2.693	-9.312
	Aug		25	0.22757	0.62303	1.79704	0.0913	0.0040	2.738	-9.641
			25	0.33868	0.88557	2.39962	0.0841	0.0037	2.615	-8.855
		Total	50	0.81978	2.34947	6.98615	0.2526	0.0053	2.866	-10.324
58	Aug		13	0.06397	0.17929	0.53153	0.0290	0.0026	2.803	-9.945
			37	0.39971	1.17958	3.68097	0.1106	0.0032	3.027	-11.223
		Total	50	0.54007	1.57463	4.75157	0.1606	0.0033	2.916	-10.592

Table 1. -- Continued

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
62	April		11	0.04716	0.11662	0.33595	0.0476	0.0053	2.473	-8.207
			39	0.23698	0.73070	2.44644	0.1935	0.0052	3.083	-11.492
		Total	50	0.36545	1.17424	4.09470	0.3217	0.0067	3.213	-12.240
63	May		67	0.52769	1.49988	4.94759	0.6844	0.0105	2.842	-10.244
			83	1.09058	3.38931	11.07156	0.5382	0.0066	3.108	-11.709
		Total	150	1.95032	5.88641	19.01575	1.2495	0.0084	3.018	-11.209
	June		63	1.05229	3.15135	9.78461	0.3471	0.0057	2.995	-11.086
			87	1.23654	3.38153	15.11810	5.8707	0.0691	2.735	-9.602
		Total	150	2.70966	7.92694	29.52191	6.3321	0.0428	2.925	-10.683
	July		38	0.13437	0.40981	1.51819	0.2683	0.0074	3.050	-11.373
			62	1.08171	3.19145	9.73251	0.3165	0.0053	2.950	-10.835
		Total	100	1.37918	4.06433	12.56595	0.0589	0.0060	2.947	-10.813
	Aug		18	0.28873	0.85915	2.66644	0.1099	0.0069	2.976	-11.004
			32	0.43103	1.20313	3.59970	0.1817	0.0061	2.791	-10.007
		Total	50	0.89064	2.54128	7.54912	0.2981	0.0062	2.853	-10.347

Table 1.-- Continued

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
14	April		45	0.37834	1.19043	3.94307	0.1974	0.0046	3.146	-11.887
			55	0.88841	2.91183	9.91516	0.3714	0.0070	3.278	-12.600
		Total	100	1.41400	4.61426	15.63428	0.5767	0.0059	3.263	-12.527
	May		77	0.75061	2.29163	7.42410	0.4277	0.0057	3.053	-11.409
			123	2.33334	7.87085	27.32233	0.7723	0.0064	3.373	-13.183
		Total	200	4.04510	13.41010	45.71014	1.2538	0.0063	3.315	-12.857
	June		36	0.22650	0.64049	2.10001	0.2889	0.0085	2.828	-10.145
			64	0.67230	2.09985	6.90146	0.3428	0.0055	3.123	-11.785
		Total	100	1.21777	3.73599	12.10504	0.6434	0.0066	3.068	-11.473
	July		39	0.37099	1.16214	3.95193	0.3115	0.0084	3.132	-11.831
			61	1.08276	3.26459	10.38724	0.5443	0.0092	3.015	-11.203
		Total	100	1.51419	4.58112	14.73584	0.8759	0.0089	3.025	-11.252
21	May		25	0.07618	0.23795	0.79446	0.0512	0.0022	3.123	-11.811
			25	0.25889	0.82442	2.68815	0.0628	0.0027	3.184	-12.167
		Total	50	0.57736	1.80443	5.75635	0.1170	0.0024	3.125	-11.825

Table 1. -- Continued

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
	June		29	9.34754	1.07932	3.47733	0.1254	0.0046	3.106	-11.700
			21	0.35405	1.11313	3.58475	0.0851	0.0045	3.144	-11.950
		Total	50	0.89661	2.75110	8.66153	0.2202	0.0046	3.068	-11.504
22	Aug		24	0.59193	1.71435	5.22678	0.2617	0.0119	2.896	-10.492
			26	0.24429	0.72714	2.22960	0.0652	0.0027	2.976	-10.986
		Total	50	1.02368	2.92856	8.72264	0.3446	0.0072	2.861	-10.312
41+43+51	June		16	0.03964	0.12026	0.43403	0.0692	0.0049	3.034	-11.305
			34	0.09395	0.24011	0.69318	0.0795	0.0025	2.556	-8.5860
		Total	50	0.38318	1.04057	2.98045	0.1546	0.0032	2.716	-9.511
14+21	July		22	0.21929	0.66551	2.13745	0.1177	0.0059	3.035	-11.302
			28	0.56795	1.72772	5.42021	0.1644	0.0063	3.042	-11.346
		Total	50	0.90088	2.73430	8.57924	0.2802	0.0058	3.035	-11.306
12+14	Aug		21	0.26140	0.76772	2.39729	0.1425	0.0075	2.937	-10.741
			29	0.64259	1.80846	5.37695	0.2873	0.0106	2.814	-10.027
		Total	50	1.09438	3.16837	9.61548	0.4426	0.0092	2.895	-10.495

Table 1.-- Continued

Sampling Area	Month	Sex	Number	Σx^2 [1]	Σxy [2]	Σy^2 [3]	SS [4]	MS [5]	b	a
13-14	Aug		21	0.19371	0.52152	1.51258	0.1085	0.0057	2.692	-9.311
			29	0.22307	0.65753	2.16164	0.2234	0.0083	2.948	-10.806
		Total	50	0.58562	1.59613	4.70338	0.3530	0.0074	2.726	-9.516
14-22	Aug		21	0.08340	0.20988	0.66099	0.1329	0.0070	2.516	-8.462
			29	0.44167	1.28352	3.83257	0.1025	0.0038	2.900	-10.906
		Total	50	0.69638	2.04501	6.26959	0.2641	0.0055	2.937	-10.772
12-13-14	June		23	0.14520	0.45618	1.55808	0.1249	0.0060	3.142	-11.872
			27	0.45506	1.43713	4.65598	0.1174	0.0047	3.158	-11.986
		Total	50	0.60814	1.91025	6.25157	0.2512	0.0052	3.141	-11.882

1/ $\Sigma x^2 = \Sigma X^2 - (\Sigma X)^2/N$

2/ $\Sigma xy = \Sigma XY - (\Sigma X)(\Sigma Y)/N$

3/ $\Sigma y^2 = \Sigma Y^2 - (\Sigma Y)^2/N$

4/ $SS = \Sigma y^2 - (\Sigma xy)/\Sigma x^2$

5/ $MS = SS/(N-2)$

Table 2. --Pooled analysis of covariance between sexes.

Source of variation	DF	SS	MS	F
Total	2244	19.8526	0.0088	
Common	2216	19.8219	.0089	
Within	2188	19.5048	0.0089	
Between regression coefficients	28	0.3171	0.0113	1.27
Between adjusted means	28	0.0307	0.0011	0.012

Comparisons among month and areas

Differences among areas-within-months (Table 3) and among months-within-areas (Table 4), were studied by both separate and pooled covariance analyses. The differences in adjusted means among areas-within-months were highly significant in the pooled analyses, and in all but one of the individual months. This could well indicate that redfish within each area comprise independent stock units. It most certainly means that the condition of the fish were different among the various areas within the same months.

The adjusted means were significantly different for sampling area 30 and for Subarea 4 in the pooled analyses of covariance among months-within-areas, although two of the three individual sampling areas did not show such differences. However, in the pooled analyses for Subarea 5 and Subareas 4 and 5 combined, the regression coefficients were significantly different. In this case, the further test of adjusted means was superfluous.

Gonadal condition undoubtedly changed throughout the months studied herein, which include the period of extrusion of the young. As only mature fish would experience these changes, it was possible that differences in the proportion of mature fish from sample to sample could have been responsible for the differences in slopes.

Table 3.--Redfish length-weight regressions: Analysis of covariance among Areas within months.

ICNAF Subarea	4	4	4	4	4
Month	April	May	June	July	August
N	150	300	250	250	300
M.S. for total regression	0.0074	0.0078	0.0289	0.0083	0.0065
M.S. for common regression	0.0074	0.0077	0.0287	0.0078	0.0055
M.S. within samples	0.0074	0.0077	0.0287	0.0078	0.0056
M.S. among regression coefficients	0.0099	0.0205	0.0185	0.0037	0.0006
M.S. among adjusted means	0.0090	0.0469*	0.0697	0.1425**	0.1083**

Area	4 pooled,	5	5	5 pooled,	4& 5 pooled,
Month	All months	May	June	All months	All months
M	1250	250	150	400	1650
M.S. for total regression	0.0118	0.0058	0.0060	0.0060	0.0104
M.S. for common regression	0.0114	0.0057	0.0059	0.0057	0.0100
M.S. within samples	0.0114	0.0058	0.0059	0.0057	0.0100
M.S. among regression coefficients	0.0090	0.0181	0.0001	0.0090	0.0090
M.S. among adjusted means	0.0828**	0.0741**	0.0327*	0.0534**	0.0770**

* 5% level of significance

** 1% level of significance

Table 4. --Redfish length-weight regressions: Analysis of covariance among months within Areas.

Area	30	42 & 43	51	63	Pooled sub-area 4
N	100	550	100	450	1100
M.S. for total regression	0.0083	0.0081	0.0050	0.0192	0.0124
M.S. for common regression	0.0070	0.0075	0.0049	0.0191	0.0120
M.S. within samples	0.0089	0.0075	0.0048	0.0192	0.0120
M.S. among regression coefficients	0.0070	0.0152	0.0116	0.0063	0.0108
M.S. among adjusted means	0.1332*	0.1273**	0.0151	0.0390	0.0719**

Area	14	21 & 22	Pooled sub-area 5	Pooled sub-areas 4 & 5
M	500	150	650	1750
M.S. for total regression	0.0077	0.0049	0.0071	0.0104
M.S. for common regression	0.0077	0.0049	0.0070	0.0100
M.S. within samples	0.0074	0.0047	0.0068	0.0101
M.S. among regression coefficients	0.0536**	0.0165	0.0387**	0.0225**
M.S. among adjusted means	0.0143	0.0098	0.0125	0.0472

* 5% level of significance

** 1% level of significance

To examine the above hypothesis, the samples were divided into mature and immature fish, and covariance analyses made separately on these two groups to test differences among months within an area. From data on size at maturity provided by George Kelly, (U. S. Fish and Wildlife Service, Woods Hole, Massachusetts - unpublished data), the author decided to use the following approximate dividing lines between mature and immature fish: Areas 14, 51, and 63, 25 mm.; Areas 21 and 22, 26 mm.; and Areas 42 and 43, 29 mm. In the pooled covariance analyses (Table 5) the immature fish did not show significant differences among months. However, slope differences were still significant for the mature fish. Therefore, the slope differences observed in the original analyses did not appear to be explainable by differing proportions of mature and immature fish in the samples. The reason for the apparent change in relative rate of growth among months is not known.

The assumption that there was no inter-action between months and areas was made in order to test separately differences among months-within-areas and among areas-within-months. Inspection of Table 6 leads to the conclusion that such interactions are probable. The b values tended to decrease from April to August for each area, and increase from North to South for each month. The values were generally lower for samples from Subarea 4 than for Subarea 5 for corresponding months. However, when b values for samples taken in June and July in Subarea 5 are compared with those from Subarea 4 taken in April and May,

Table 5. --1951 Redfish length-weight regressions.
Covariance analysis.

	Pooled Small+ Fish Only	Pooled Large+ Fish Only
N	488	1255
Total A	-10.952	-12.012
Common A	-11.064	-12.015
Total B	2.966	3.167
Common B	2.986	3.167
M.S. for total Regression	0.0186	0.0095
M.S. for common Regression	0.0185	0.0093
M.S. within samples	0.0189	0.0092
M.S. among Regression Coefficients	0.0030	0.0148**
M.S. among adjusted means	0.0221	0.0327

* 5% level of significance

** 1% level of significance

\+ See text for definition of "Large" and "Small".

Table 6.--Regression statistics for redfish by month and area.

Month		AREA 30	41	42+43	51	58	62	63	Subarea 4 pooled common regressions	14	21+22	Subarea 5 pooled common regressions	Subarea 4+5 pooled common regressions
April	N			100	-	-	50		150	100		100	
	b			3.042	-	-	3.213		3.064	3.263		3.263	
	a			-11.259	-	-	-12.240		-11.393	-12.527		-12.527	
May	N			150				150	300	200	50	250	
	b			2.876				3.018	2.945	3.315	3.068	3.291	
	a			-10.373				-11.086	-10.786	-12.857	-11.825	-12.731	
June	N		50		50			150	250	100	50	150	
	b		2.972		2.693			2.925	2.344	3.068	3.068	3.068	
	a		-10.988		-9.312			-10.683	-10.509	-11.473	-11.504	-11.483	
July	N	50		150				100	250	100		100	
	b	2.578		2.881				2.947	2.906	3.025		3.025	
	a	-8.639		-10.380				-10.813	-10.544	-11.252		-11.252	
August	N	50		150	50	50		50	300		50	50	

Table 6. -- Continued

A R E A

Month		30	41	42+43	51	58	62	63	Subarea 4 pooled common regressions	14	21+22	Subarea 5 pooled common regressions	Subarea 4-5 pooled common regressions
	b	2.867		2.894	2.866	2.916		2.853	2.883		2.861	2.861	
	a	-10.422		-10.461	-10.324	-10.592		-10.347	-10.422		-10.312	-10.312	
Areas within months, pooled common regressions	N	100	50	550	100	50	50	450	1.250	500	150	650	1.900
	b	2.665	2.972	2.927	2.784	2.916	3.213	2.946	2.930	3.173	2.996	3.132	3.003
	a	-9.186	-10.988	10.644	-9.851	-10.592	-12.240	-10.813	-10.687	-12.054	-11.890	-12.025	-11.165

C 4

they are approximately equal. Further studies are necessary to investigate the possibility of interaction. Differences in latitude between Subareas 4 and 5 suggest the possibility that changes in environmental factors, which may affect condition of fish, occur at different times in the Subareas. Such changes could produce an interaction between months and areas.

Estimators of length-weight conversion

The data examined in this study indicated that in most cases a separate estimating equation would be desirable for each month and area. As data was lacking in many cells, such a procedure would not be practical. In addition, further analysis of within cell variation is desired. In combining regression equations where significant differences exist, the common equation is the least biased estimator. The parameters of the estimating equation $\log_e W = a+b \log_e l$ necessary for converting length to weight are given in Table 6 for each area and month, as well as the areas and months combined.

The error mean square of the pooled analysis of covariance among areas-within-months for Subareas 4 + 5 (0.086) was greater than that for months-within areas (0.070). This indicated that differences between areas were greater than those between months. When Subareas 4 and 5 were combined, the pooled within mean square for month-within-areas was considerably larger than that for Subarea 5 separately, and only sightly less than the <u>within</u> mean square for Subarea 4, (Table 4). The b values for Subarea 5 are also generally higher than those for Subarea 4, and those from sampling area 30 were lower still. These observations indicated that a single equation may be used for the Subarea 5, for sampling area 30, (Division T of Subarea 4) and Division V, W, and X of Subarea 4, without too much loss in precision, depending, of course, on the purposes to which the data are put.

<u>Sample size</u>

The formula:

$$(\bar{y} + t_{.05} S\bar{y}) - (y - t_{.05} S\bar{y}) = \frac{Sy.x \; t_{.05}}{\sqrt{N}}$$

where t was chosen at (N-2) degrees of freedom was used to estimate requirements of sample size for future length-weight sampling. The pooled estimate of the <u>within</u> mean square (0.01) for area 21 and area 22 combined (Tables 2 and 3) was used as an estimate of $Sy.x^2$. Table 7 gives the widths of the confidence intervals for various values of N. These data indicate that there is little to be gained by increasing size above 150 fish, and that 100 fish per sample would be sufficient for most purposes.

Table 7.--Confidence intervals for various sample sizes.

Number of fish	Confidence interval in grams
25	24
50	17
75	14
100	12
150	10
250	8
500	7

Literature Cited

Snedecore, George W.

1956. Statistical Methods, 5th cd., Iowa State College Press, Ames Iowa XIII, 534 pp.