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A contribution to the discussion of the effects
of error on the action of catch quotas and effort quotas

by

J. G. Pope and D. J. Garrod
Fisheries Laboratory, Lowestoft, Suffolk.

1 INTRODUCTION

Many of the commercially important fish stocks of the ICNAF area have become subject to catch quotas. It has been further proposed by the USA that in addition to these catch quotas there should also be an overall restriction on fishing effort in sub-areas 5 and 6.

The aim of each of these management schemes is to attain the long-term (Maximum Sustainable Yield), MSY / or, in the case of one herring stock, to maintain a specific stock level. In other circumstances the management objective might be to stabilize the fishing mortality at the level of the maximum economic yield; to allow a depleted stock to recover or to stabilize a new fishery at some safe level pending further scientific investigation. The management scheme appropriate to any of these aims, except possibly the last, should be based on the best available scientific assessment of the resource. Unfortunately, all assessments of fishery resources are subject to some degree of error. The relative importance of the error is a function of the inherent variability of the species and the amount of research and sampling devoted to it but, because the errors exist, all schemes of management, in the short term at least, will fail to some extent to achieve their precise objectives. It is therefore

important to try to give some idea of the likely impact of assessment errors on the schemes of management proposed for and in action on the fishery resources of sub-areas 5 and 6 of the ICNAF region.

Assessments of many of the stocks of fish in these areas are as yet incomplete and the variability of the parameters contributing to catch, fishing effort and population size is virtually unknown. Necessarily, therefore, this paper cannot examine the effects of errors of many of the actual management schemes proposed and already in action but it does attempt to provide a general framework to investigate the effects of assessment errors. This is in the hope that it will assist the acknowledged experts for each species to gather some idea of the likely level of uncertainty associated with a particular management scheme. In addition to the errors that result from inaccurate assessment of stocks, effort quotas will, by their nature, be the subject of systematic and random errors in their application due to the variable performance of fishing vessels on a year-to-year, or trip-to-trip, basis.

2 SOURCES OF INACCURACY

Errors in assessments may be classified into two types. The first type are biases in sampling techniques and parameters, which tend to imply objectives of management that are incorrect in themselves. The second type are random errors in sampling and estimates of parameters. These, while less often resulting in the choice of incorrect objectives, tend to make it impossible to achieve precisely a stated objective in a given year.

2.1 Causes and effect of biases

Biases in sampling schemes and estimates of stock parameters are caused mostly by inadequate sampling but they can also arise as a result of once-only assessment of particular parameters. Thus, for example, the

natural mortality of a fish stock is often estimated by forming a regression of the yearly estimates of the total mortality acting on the stock against the yearly estimates of fishing effort deployed on the species. An extrapolation of the regression line to the point that corresponds to zero fishing effort gives a value of total mortality that is an estimate of the natural mortality from the available data. In practice, however, this may well be too high or too low. Since an error in the estimate of the natural mortality of a stock will lead to an error in the estimated form of the yield curve it may well result in schemes of management which are designed to achieve an objective which is in fact incorrect. This can be illustrated very simply. Halliday (1972) developed three possible yield curves for the Eastern Scotian Shelf cod stock complex. These three yield curves were calculated under the alternative assumptions that M , the natural mortality, was: 0.1, 0.2 and 0.3, the corresponding fishing mortalities associated with MSY are 0.3, 0.4 and 0.6 respectively. If it was assumed that M was 0.2 then, if the aim of management was to achieve the Maximum Sustainable Yield (MSY), the management objective would be to achieve a fishing mortality (F) of about 0.4. If in fact the true value of M was 0.1 management action to generate $F = 0.4$ would produce an actual $F = 0.5$ which, because when $M = 0.1$, $F_{MSY} = 0.3$, would be 67% higher than the desired objective. If, alternatively, the true value of M was 0.3 then the level of fishing which would produce an apparent fishing mortality of 0.4 relative to the assumption that M was 0.2 would in fact produce a true fishing mortality of approximately 0.3; this true fishing mortality would be about 50% too small to achieve the MSY, which for an M of 0.3 occurs at a fishing mortality of approximately 0.6 (a bias in M produces an approximately equal but opposite bias in fishing mortality as calculated by Virtual Population Analysis; see Agger et al. 1971). Thus erroneous

assumptions about natural mortality can generate errors in the objectives of fisheries management.

Similar errors might well be introduced by biases in sampling schemes. The non-reporting of catch of species, assigning the catch to the wrong area and the non-measuring and ageing of fish or the measuring and ageing of fish in some biased fashion might all lead to assessments of fisheries which suggested management objectives which differed substantially from the true optimum management scheme. The effects of biases in sampling schemes cannot be overcome without drastic improvement in the data base of assessment work and they underline the plea for more and better sampling effort made by the ICNAF Statistician.

2.2 Causes of random errors and their effects on management objectives

Random errors in sampling methods and estimates of stock parameters are created by the process of sampling populations and catches for characteristics of length and age. This inevitably leads to random errors in estimates of these characteristics. Gulland (1955) examines the errors inherent in estimating age distribution from samples of catches. In addition to errors introduced as a result of sampling, some parameters may vary about their mean value from year to year in an apparently random fashion. An example of the random variation of a parameter may often be observed in the catchability coefficient (q) which relates fishing effort (f) to fishing mortality (F), ($F = qf$) in a particular stock. In these circumstances a given level of fishing effort would produce a fishing mortality that varied from year to year. Therefore a level of fishing effort designed to be compatible with the maximum sustainable yield might in fact produce a series of fishing mortalities which varied to a greater or lesser extent about the optimum level. While it has been

shown by Pope (1972) and by Garrod (1973) that fluctuations about an optimum do not inevitably lead to a smaller average yield, large fluctuations might be embarrassing in that they could lead to periodic shortages and could conceivably do permanent damage to stocks by impairing their ability to produce adequate numbers of recruits. This might conceivably be the case for a stock with a steeply parabolic stock-recruitment relationship as for example the curve developed by Herrington (1948) for the Georges Bank haddock stock.

Thus, random variations are caused in estimates of parameters and estimates of stock by both the lack of precision of sampling regimes and by the natural variability of some parameters.

3 ERRORS IN CATCH QUOTAS AND EFFORT QUOTAS

Biases are by their nature difficult to establish since they are often the result of incomplete data. To some extent they may be studied by considering alternative possibilities and choosing courses of action that minimize any risk.

Random errors are more amenable to analysis but the error components depend on the frequency of adjustment of the management regime. Catch quotas, as currently envisaged, necessitate annual adjustment with reference to an estimate of the existing stock level. Effort quotas may be adjusted annually, in which case they will be influenced by errors in estimates of the reference stock. Alternatively, effort quotas may be set to approximate MSY over a longer period.

Basically errors in catch quotas result from errors in estimates of the population size at the beginning of the year in question. Incorrect estimates of weight at age and selection at age may also cause random errors but these effects are usually smaller than the effect of errors in population estimates.

If C_a is the weight of catch of fish aged (a) from a stock with P_a fish of that age, whose selection to the fishery is defined by S_a , and whose weight is W_a , then

$$C_a = P_a \frac{S_a F}{M + S_a F} \left(1 - \exp \left\{ -(S_a F + M) \right\} \right) W_a. \quad 3.1$$

This may be simplified if S_a can be regarded as the proportion of the population (P_a), available to capture as opposed to the proportion of the (fully recruited fish) fishing mortality (F) that acts on the age group.

Then

$$C_a = A_a \frac{F}{Z} \left(1 - \exp(-Z) \right) \quad 3.2$$

where $A_a = P_a S_a W_a$ may be called the exploitable biomass of fish aged a and A , the sum of all the A_a 's, may be called the total exploitable biomass. The catch quota (Q) that will cause a certain fishing mortality (F) on the stock is given approximately by

$$Q = A \frac{F}{Z} \left(1 - \exp(-Z) \right). \quad 3.3$$

In practice the A_a can be separated into three components. If r is the age of first capture, fish for which $a < r$ are young unexploited fish for which $A_a = 0$. Fish for which $a = r$ are the recruits of the year, and the value of A_r cannot be determined from the results of previous years' catch and effort data. Fish for which $a > r$ are fish exploited in previous years.

A_r may be estimated in some cases by young fish surveys. In other cases, it may not be known and for the purpose of setting catch quotas the average value of A_r may have to be used or the value of A_r predicted by a stock-recruitment relationship. Clearly this can lead to large errors in the catch quota if A_r is a large proportion of A and if the year-to-year variation in recruitment to the stock is large.

Where $a > r$ the A_a 's may be estimated in two main ways. The first of these is to use estimates of the catch at age in the previous year together

with the estimates of the fishing mortality (obtained from a knowledge of fishing effort), selectivity and natural mortality. The second method is to use the estimates of relative yearly biomass obtained from groundfish surveys to estimate the absolute abundance in the current year. Using the first method random errors occur in the total available biomass (A) as a result of errors in the estimates of the numbers caught at each age in the previous year and the selectivity in the previous year. The error in selectivity is often small compared to errors in the other estimates and will be ignored for the purposes of this investigation in the interests of simplicity. If it can be assumed that the numbers caught at age have a fairly constant coefficient of variation (this is often the objective of sampling schemes) then the errors in the estimate of fishing mortality and catch will induce errors in $\sum_{r} A_{ra}$ (the biomass of fish aged greater than the age of first capture, i.e. $A - A_r$) such that,

$$\text{Var}(A - A_r) = (A - A_r)^2 \left(\frac{\text{Var}(F)}{F^2} \right) + \frac{(A - A_r)^2}{\theta} \left(\frac{\text{Var}(C)}{C^2} \right) \quad 3.4$$

where $\frac{\text{Var}(C)}{C^2}$ is the average value of this ratio for all ages. The derivation of this formula is shown in the mathematical annex. θ in the formula is a factor depending on the growth and mortality of the stock and the variability of its recruitment, which is also explained in the annex.

Since A_r and $A - A_r$ are statistically independent,

$$\text{Var}(A) = (A - A_r)^2 \left(\frac{\text{Var}(F)}{F^2} \right) + \frac{(A - A_r)^2}{\theta} \left(\frac{\text{Var}(C)}{C^2} \right) + \text{Var } A_r. \quad 3.5$$

Since from 3.3,

$$\text{Var}(Q) = \text{Var}(A) \left(\frac{F}{Z} (1 - \exp(-Z)) \right)^2 \quad 3.6$$

which may be simplified to

$$\text{Var}(Q) = \text{Var}(A) \left(1 - \exp(-F)\right)^2 \cdot \exp(-M), \quad 3.7$$

using the basic assumption of cohort analysis (Pope 1972).

The error in the catch quota causes an error in the value of \hat{F} generated on the actual populations. This is given approximately by

$$\text{Var}(\hat{F}) = \frac{\text{Var}(Q)}{A^2} \cdot \exp(2\hat{F} + M) \quad 3.8$$

or using formulae 3.7 and 3.5

$$\text{Var}(\hat{F}) = (1 - e^{-F})^2 \frac{e^{2F}}{A^2} \left\{ (A - A_r)^2 \frac{\text{Var}(F)}{F^2} + \frac{(A - A_r)^2}{\theta} \frac{\text{Var}(C)}{C^2} + \text{Var}(A_r) \right\}. \quad 3.9$$

Let $\frac{A - A_r}{A} = u$ $\frac{A_r}{A} = v$.

Then approximately

$$\frac{\text{Var}(\hat{F})}{\hat{F}^2} = e^{2\hat{F}} \left\{ u^2 \left(\frac{\text{Var}(F)}{F^2} + \frac{\text{Var}(C)}{C^2 \theta} \right) + v^2 \frac{\text{Var}(A_r)}{A_r^2} \right\}. \quad 3.10$$

Thus if \hat{F} were the level of fishing mortality associated with MSY, 3.10 gives the error associated with catch quota. By way of comparison the variance of fishing mortality (\hat{F}), that would be achieved by an effort quota, is given by:

$$\frac{\text{Var}(\hat{F})}{\hat{F}^2} = \frac{\text{Var}(q)}{q^2}, \quad 3.11$$

where q was the catchability associated with the effort measure adopted in the quota. If this measure of effort was the same as that which was used to estimate the level of fishing mortality in the previous year in the estimation of the catch quota (presumably the best available measure) then:

$$\frac{\text{Var}(q)}{q^2} = \frac{\text{Var}(F)}{F^2}. \quad 3.12$$

In this case it is likely that an effort quota would be more accurate than a catch quota. If however the effort quota were based on some measure of

effort which related less well to the fishing mortality of the stock in question, either through choice of unit or its generalization over a number of stocks, then the catch quota might well be the more accurate. For example, for a cod stock the best estimate of fishing effort might be Spanish trawler hours fishing specifically for cod, which might relate quite well to the fishing mortality, while overall days on ground for all species might hardly relate to the fishing mortality on cod at all.

Equations 3.10 and 3.11 relate to the case where the object of stock management is to generate some specific level of \hat{F} (e.g. MSY). When the objective of management is the maintenance of some specific level of stock biomass B in the following year then if a catch quota is used a variance will be induced in B so that where \hat{F} is the fishing mortality of the current year,

$$\frac{\text{Var}(B)}{B^2} = e^{2\hat{F}} \left\{ u^2 \frac{\text{Var}(A_S)}{A_S^2} + v^2 \left(\frac{\text{Var}(F)}{F^2} + \frac{\text{Var}(C)}{\theta C^2} \right) \right\}. \quad 3.13$$

When an effort quota is used

$$\frac{\text{Var}(B)}{B^2} = u^2 \frac{\text{Var}(A_S)}{A_S^2} + v^2 \left(\frac{\text{Var}(F)}{F^2} + \frac{\text{Var}(C)}{\theta C^2} \right) + \frac{\text{Var}(q)}{q^2} \hat{F}^2. \quad 3.14$$

This tends to mean that in an annually adjusted regime, such as is necessary to maintain a specified stock size, effort quotas have a less variable effect than catch quotas, but the greater precision of effort quotas is wholly dependent on there being an adequate relationship between fishing effort and fishing mortality.

Equation 3.4 is the expression for the variance of $A - A_T$ when it is estimated from the previous year's catch and effort data. If the estimate of $A - A_T$ was based on groundfish surveys the variance of this estimate should be substituted for 3.4. The likely precision of estimates of available biomass

are given by Grosslein (1971) and by Jones and Pope (1972). Methods of estimating the various variances are shown in the examples of Appendix B.

4 THE VARIABILITY OF FISHING VESSEL PERFORMANCE IN RELATION TO CATCH AND EFFORT QUOTAS

If effort quotas are set either with or without catch quotas then the need to allocate the quota between countries will require a knowledge of the relative performance of the fleets from the various countries. Similarly, each country would have to assess the relative performance of the individual vessels that it intended to allow to use the national quota.

The relative performance of a particular vessel is a quantity that varies to some extent from year to year and therefore the fishing effort developed by a particular vessel would in practice be greater or smaller than the actual effort allocated to it. Because of this, the variability of vessel performance would be a problem to any country trying to meet a fishing effort quota. In order that the magnitude of such variations might be appreciated an analysis was made of all British fishing vessels fishing at Iceland and in the north-east Arctic in the years 1969-71. The analysis consists of an examination of the variation of catch per effort of individual vessels from the tonnage group mean for each year and an examination of the average variation of each vessel's catch per effort between years.

Catch per day of individual vessels will be influenced by the stock abundance, skipper-trawler combination and season of fishing. Within years the stock abundance is assumed constant, i.e. the average catch per unit effort taken over the whole year should be the same for equivalent skipper-vessel combinations. In the British statistics skippers are not recorded and as a

first approximation the tonnage of the trawler is taken as a characteristic that might be related to the skipper-trawler combination (see Gulland 1956). Seasonality has not been analysed as such but it may be reflected to some extent in the analyses of the number of days spent in the area during the year; trawlers which only occasionally visit an area are more likely to do so at the most favourable seasons.

Tables 1A and B summarize the relative performance of trawlers in different tonnage categories fishing at Iceland and in the north-east Arctic, performance being expressed as the deviation from the mean performance of all vessels in each area. The catch per unit effort of trawlers 0-499 GRT is below the overall mean, owing to their smaller size and greater specialization in their fishery objectives. There is a considerable increase in catch per unit effort through the 500-900 GRT range: this does not extend to the 900+ GRT class and over the whole range the variation attributable to tonnage only accounts for about half of the total variation. The standard deviation of performance of all vessels fishing Iceland is 25-30%; the dispersion is greater than in the north-east Arctic because of the wider range of vessel operations there. Within each tonnage grouping the between-trawler deviation is stable at $\pm 20\%$.

The alternative analysis at Table 1C summarizes the within-ship variation over the three years (i.e. the deviation of a trawler performance in one year from the average relative performance over the three years) in relation to the amount of time spent on the grounds. There is a weak trend of above-average performance with shorter periods, but this could easily be an artefact of the statistics or a reflection of improvement due to fishing tactics which select for season. The period groupings show standard deviation

of individual vessel performance decreasing from 20 to 10% as the period increases. The similarity between this figure and the between-trawler performance indicates that the identity of the vessel has very little connection with the annual fishing performance and each year's fishing might be considered as though it were carried out by a different boat.

The trend in variation with fishing time has a bearing on the within-country allocation of an effort quota. At an extreme, if one vessel used all the quota during a year (S.D. 11%) over the years 95% of the annual catches would be at most $\pm 23\%$ of what was predicted on the basis of average stock and average performance. If the same effort quota were divided amongst a number of vessels, each with a shorter time allocation, the variation of individual performance would increase, but the accuracy of meeting the overall national catch and effort quotas would improve.

5 VARIATION IN THE CATCHABILITY COEFFICIENT

The formulation of potential errors set out in Section 3 defines the importance of variations in the catchability coefficient in relation to regulation of fishing effort. These variations spring from variation in the fishing performance of vessels and biological variation in the availability of the stock. Section 4 examined variation in the fishing performance of UK vessels as it might affect the allocation of catch and effort quotas. This section examines variation in the catchability coefficient itself.

Virtual population estimates of fishing mortality (F) on fully recruited age groups have been taken from ICES and ICNAF publications and each mortality allocated to component fishing fleets according to the ratio of the fleet catch to the total international catch. The estimated fishing mortality per fleet has then been related to the recorded fishing activity of that fleet

to provide an estimate of the catchability coefficient ($F/f = q$), where both F and f have been measured independently. Strictly, the partitioned values of fishing mortality are not instantaneous coefficients but they may be used as such in considering the effect of a national fleet in relation to an annual quota.

Table 2 sets out for various fisheries and fleets the mean value of q , its standard deviation and coefficient of variation. Plots of individual year's points are illustrated in Figure 1. These show:

For the Arcto-Norwegian cod the yearly values of q for both UK and USSR vessels is remarkably consistent, though Figure 1A shows some differences between the fleets over time. For UK vessels the variation with time is similar in both major fisheries on this stock and neither relate very well to the trend in tonnage of UK vessels (Figure 1D), which for these vessels is usually taken as an indicator of fishing power. This lack of correlation with tonnage indicates additional sources of variation in q .

Estimates of q for trawlers of the Federal Republic of Germany (FRG) vary more widely than estimates for the UK and USSR because of a smaller amount of fishing, with timing and fishery objectives which are rather more variable. The estimated q per Norwegian fisherman at Lofoten has shown a steady increase over time (Figure 1C), but it is not clear how far this might be caused by increased fishing power per man or biological change (however, see below).

For Greenland (1A-1D), all estimates of q show a coefficient of variation close to 50%, but this contains variation due to the increase in q in recent years (Figure 2A) which has been reported to ICNAF previously (Schumacher 1970). Hitherto, the increase in q has been attributed to

improved efficiency of trawler fleets concentrating activity at the most advantageous season for fishing. It is therefore of considerable interest that estimates of q for Portuguese dory vessels show the same trend (Figure 2B). Dory effort measurements exclude increases in fishing power of the mother ship, and the stability of q for these same dory vessels fishing at Newfoundland indicates that the trend in q at Greenland is not caused by a trend in fishing power. The identical trend in all sets of data could result from systematic overestimate of the total fishing mortality, but the four-fold increase in q during the decade is too great to be entirely accounted for in this way, and one can only conclude that there has been a real change in the biological availability of the cod stock at Greenland in recent years.

For Labrador (2G-3L), the trawler fleets of West Germany and USSR (Figure 2C) are relatively recent entrants to this fishery compared to Spain and Portugal (Figure 2D), and have other objectives besides cod. As a result, the coefficient of variation of the catchability for cod of the West German and USSR fleets is considerably higher than that of Spain and Portugal. There is no clear trend in q with time for all fleets and, as noted above, it is interesting that there is no trend in q with respect to Portuguese dories which could parallel the increase at West Greenland referred to above.

The possibility of biological variation in q is taken further in Figure 3 for those stocks which show evidence of trend in q with time. The estimates of catchability coefficient are plotted against estimated stock size. Trend lines have been fitted by eye only because too little is known to predict any form of relationship between the variables, but it is evident that the relationship is inverse, catchability increasing as stock decreases.

In Figure 3A/B the data for Arcto-Norwegian cod cover a period of decrease and recovery of stock size so that the relationship cannot be an entirely spurious effect of increased fishing power coinciding with a time series of decreasing stock size. Likewise, for West Greenland cod the correlation between q for West German trawlers and q for Portuguese dories shown in Figure 3C would suggest that fishing power changes are not the source of trend in q with respect to West German trawlers shown in Figure 3D.

There is therefore evidence that the catchability of demersal resources is inversely proportional to stock size. This has been suspected to occur in species with strong shoaling characteristics, e.g. 'pelagic' species, and models have been described to show this must occur if behaviour causes fish to tend to an optimum density per unit area. The effect could be expected to be less pronounced in demersal resources, but if a smaller stock occupies a smaller geographical area then a given level of fishing effort must generate a higher fishing effort per unit area and catchability will appear to increase.

Table 3 compares the variability of q resulting from different possible measures of fishing effort used by fleets of the USSR and Spain fishing at Labrador. The results indicate that hour fished gives the most precise indication of the fishing mortality generated by the effort. It is interesting to note that the coefficient of variation of q associated with hours fishing is broadly similar to the 20% level that was associated with British vessels fishing at the north-east Arctic and at Iceland.

It is known that for some fisheries there are marked changes in catchability with season. Table 4 shows the results of a series of analyses of variance for various fleets and cod stocks. Each analysis shows a very significant change in catchability between quarters. A knowledge of seasonal

change in catchability could of course be used to reduce the coefficient of variation of catchability, but only at the expense of more complicated definitions of fishing effort and therefore of any resulting effort quota. Even if this adjustment were made there would still be significant changes in catchability on a year-to-year basis as is indicated by the table which records significant differences between years for most of the analyses.

Thus in brief it may be concluded that:

- (1) Estimates of the catchability coefficient may contain a bias related to the size of the stock in question.
- (2) For fleets known to fish specifically for cod and catching a large proportion of the total catch, the coefficient of variation of the catchability coefficient is of the order 15-20% (95% confidence limits \pm 40%). The coefficient of variation increases to the order 50% (95% confidence limits \pm 100%) with respect to fleets taking smaller 'samples' on a more opportunistic basis. This scale of variation combines the effects of variation in fishing performance of a given effort unit with variation in biological availability. The variation is therefore slightly greater than that recorded in Section 4 which, in effect, describes the variation that can be attributed to variation in vessel performance alone.

6 SUMMARY

This paper considers the effect of some of the errors in catch quotas and effort quotas and derives approximate formulae to indicate the expected levels of inaccuracy of catch quotas and effort quotas relative to the inaccuracies of the parameters that are used in the calculation of such regulations for specific stocks. Additionally, the problem of variation in catchability is examined in depth, both variations in vessel performance and variation in the

biology of stocks being considered, since both of these causes are relevant to the accuracy with which an effort quota system could be established. From this investigation and from the formulae for the errors in catch and effort quotas it is apparent that while catch quotas are by their nature more subject to error than effort quotas based on the best measures of effort available they are nevertheless likely in many cases to give more accurate results than effort quotas based on effort measurements chosen for their ease of enforcement (days on ground etc.).

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Table 1 Variations in vessel performance

	Tonnage category				
	0-499	500-699	700-899	900+	All vessels
<u>Between vessels</u>					
A. Iceland					
Group mean relative to overall mean	0.74	1.06	1.16	1.19	1.00
S.D. within years					
1969	0.18	0.20	0.32	0.62	0.34
1970	0.21	0.23	0.19	0.22	0.29
1971	0.19	0.14	0.21	0.16	0.23
Average S.D.	0.19	0.19	0.24	0.33	0.29
B. North-east Arctic					
Group mean relative to overall mean	0.66	0.99	1.04	1.00	1.00
S.D. within years					
1969	0.18	0.16	0.20	0.10	0.23
1970	0.14	0.23	0.20	0.23	0.24
1971	0.18	0.22	0.22	0.19	0.23
Average S.D.	0.16	0.21	0.21	0.18	0.23
<u>Days on grounds</u>					
	0-50	51-100	101-150	151-200	
C. <u>Within vessels</u> (Iceland only)					
Group mean relative to overall mean	1.08	1.00	0.97	0.96	
S.D.	0.19	0.22	0.16	0.11	

Table 2 Estimates of the catchability coefficient in various cod fisheries

Stock	Country	Gear	Effort unit	Period	q	S.D.	Coeff. of variation
Arcto-Norwegian							
I + IIB	UK	Steam O.T.	10 ⁶ hours	1950-68	0.53	0.11	20.7
		Motor O.T.	10 ⁶ hours	1951-68	0.53	0.14	26.5
	USSR	O.T.	10 ⁶ hours	1950-68	0.49	0.14	27.8
		O.T.	10 ⁶ days (F)	1955-61	9.26	7.11	76.8
IIA	UK	Steam O.T.	10 ⁶ hours	1950-68	0.40	0.11	27.3
		Motor O.T.	10 ⁶ hours	1951-68	0.42	0.15	36.0
	Norway	Men	10 ⁶ Lofoten	1950-68	9.77	6.10	62.4*
		O.T.	10 ⁶ days (F)	1954-68	2.86	1.02	35.7
Iceland	UK	Steam O.T.	10 ⁶ hours	1960-68	0.23	0.051	22.11
		Motor O.T.	10 ⁶ hours	1960-68	0.16	0.025	15.54
Greenland A-D	F.R. Germany	501-900	10 ⁶ days	1956-68	28.66	18.14	63.27*
		901+	10 ⁶ days (F)	1960-68	40.47	21.14	52.24*
	Portugal	501-900 O.T.	10 ⁶ hours	1956-68	2.05	1.03	50.30
		501-900 dory	10 ⁶ hours	1956-68	0.089	0.045	49.94
Labrador 2G-3L	F.R. Germany	901+ dory	10 ⁶ hours	1956-68	0.088	0.038	42.83
		501+ O.T.	10 ⁶ days (F)	1961-68	15.74	9.46	60.13
	USSR	901-1800 O.T.	10 ⁶ days (G)	1961-68	5.77	3.01	52.27
		1800+ O.T.	10 ⁶ days (F)	1961-68	6.38	4.23	66.27
Spain	901-1800 O.T.	10 ⁶ days (G)	1961-68	20.26	6.38	31.51	
		10 ⁶ hours	1961-68	1.95	0.44	22.31	
	151-500 P.T.	10 ⁶ days (G)	1961-68	14.02	2.01	14.36	
		10 ⁶ days (F)	1961-68	16.13	2.32	14.38	
Portugal	901-1800 O.T.	10 ⁶ hours	1961-68	1.21	0.16	13.39	
		10 ⁶ days (G)	1961-68	9.49	3.59	37.77	
	< 500 dory	10 ⁶ days (F)	1961-68	11.29	4.13	36.61	
		10 ⁶ hours	1961-68	1.14	0.37	32.15	
501-900 dory	10 ⁶ hours	1961-68	1.42	0.23	16.36		
	10 ⁶ days	1961-68	0.046	0.0065	14.20		
901-1800 dory	10 ⁶ hours	1961-68	0.036	0.0068	18.75		
	10 ⁶ days	1961-68	0.039	0.0166	42.10		

(F) = days fished; (G) = days on ground.

* Includes trend with time.

Table 3 Coefficient of variation of different units of effort, as measures of the same fishing mortality at Labrador

Country	Gear	Days on grounds	Days fished	Hours fished
USSR	901-1800 O.T.	52.27	66.27	
	1800+ O.T.	31.51		22.31
Spain	901-1800 O.T.	14.36	14.38	13.39
	151-500 P.T.	37.77	36.61	32.15

Table 4 Analyses of quarterly and annual variation of catchability coefficient in selected fisheries

Fishery	Country	Gear	Unit	Mean square F ratio	
				Between quarters	Between years
Arcto-Norwegian I + IIB	UK	Steam O.T.	10 ⁶ hours	33.54***	4.51***
		Motor O.T.	10 ⁶ hours	24.88***	8.68***
Greenland A-D	F.R. Germany	501-900 O.T.	10 ⁶ days (F)	11.49***	24.49***
Labrador 2G-3L	Spain	901-1800 O.T.	10 ⁶ days (F)	13.52***	2.21
		151-500 P.T.	10 ⁶ days (F)	6.12**	4.94**
		901-1800 O.T.	10 ⁶ hours	27.28***	2.60*

(F) = days fished.

*** P = < 0.001.

** P = < 0.01.

* P = < 0.05.

ANNEX A

The derivation of equation 3.4.

From 3.2,

$$C_a = A_a \frac{F}{Z} (1 - \exp(-Z)).$$

Using the cohort analysis approximation this becomes,

$$C_a = A_a (1 - e^{-F}) e^{-M/2}. \tag{A.1}$$

If ΔC_a , ΔA_a and ΔF represent small differences in C_a , A_a and F respectively, then from Taylor's theorem

$$\Delta C_a \approx (1 - e^{-F}) e^{-M/2} \Delta A_a + A_a e^{-F} e^{-M/2} \Delta F. \tag{A.2}$$

Dividing by A.1 gives

$$\frac{\Delta C_a}{C_a} \approx \frac{\Delta A_a}{A_a} + \frac{e^{-F}}{(1 - e^{-F})} \Delta F. \tag{A.3}$$

For small F this is approximately equivalent to

$$\Delta A_a \approx A_a \left\{ \frac{\Delta C_a}{C_a} - \frac{\Delta F}{F} \right\}. \tag{A.4}$$

Since

$$\text{Covariance } (A_i, A_j) = \text{all } \Sigma \text{ values } \Delta A_i \Delta A_j \tag{A.5}$$

and since

$$\text{Var}(A - A_r) = \sum_{i>r} \sum_{j>r} \text{Cov}(A_i, A_j) \tag{A.6}$$

it follows that

$$\text{Var}(A - A_r) = A^2 \frac{\text{Var}(F)}{F^2} + \sum_{a>r} A_a^2 \frac{\text{Var}(C_a)}{C_a^2}. \tag{A.7}$$

If we assume $\frac{\text{Var}(C_a)}{C_a^2}$ has a similar value for each age (a frequent objective

of sampling schemes) then if we call the average value $\frac{\text{Var}(C)}{C^2}$ it follows that

$$\frac{\text{Var}(A - A_r)}{(A - A_r)^2} \approx \frac{\text{Var}(F)}{F^2} + \frac{\text{Var}(C)}{\theta C^2} . \quad \text{A.8}$$

The form of 3.4 where

$$\theta = \frac{(A - A_r)^2}{\sum_{a > r} A_a^2} . \quad \text{A.9}$$

ANNEX B

Examples of the use of the equations of Section 3.

Example I

Scotian Shelf cod

Halliday's (1972) description of this stock gives cohort analysis estimates of the fishing mortality on each age. If, as an example, it had been intended to achieve a fishing mortality of 0.45 in 1970 (the appropriate level to achieve the MSY) using a catch quota, then this would have to be based on the catch at age data of 1969. Normally for this stock new recruits form a negligible proportion of the exploitable biomass. On average, using 3.10, $v = 0.006$ and $u = 0.994$.

In 1969 $\theta = 5.056$, which was calculated using equation A.9.

$\frac{\text{Var}(C)}{C^2}$ can be estimated from the within-year coefficient of variation (28%) in fully recruited fishing mortality which gives estimates $100 \sqrt{\frac{\text{Var}(C)}{C^2}}$ (see Pope 1972). Halliday considers there was little variation in F over the period analysed and thus the between-year coefficient of variation in the average fully recruited fishing mortality gives an estimator for $100 \sqrt{\frac{\text{Var}(F)}{F^2}}$.

$$\text{Thus } \frac{\text{Var}(F)}{F^2} = 0.012$$

$$\frac{\text{Var}(C)}{C^2} = 0.084.$$

Inserting these various values in equation 3.10 leads to an estimate of the coefficient of variation (of the fishing mortality achieved by the catch quota) of 27%.

Thus the fishing mortality in fact achieved (\hat{F}) would most probably be in the range

$$0.22 < \hat{F} < 0.67.$$

Alternatively, had an effort quota been applied this would have a coefficient of variation of about 11% if the same effort measure was used and hence in this case the fishing mortality achieved would most probably be in the range

$$0.35 < \hat{F} < 0.55.$$

However, for the measures of effort more likely to be adopted for effort regulations the coefficient of variation is likely to be far greater, since it is unlikely that these measures would bear any great relationship to cod fishing effort.

If it was the object of management to get a catch on effort quota on this stock (stock composition as at 1969) so that the biomass of fish (B) available for capture aged 4 and over was 144 metric tons 10^{-3} , this would be achieved by a catch quota of 49 metric tons 10^{-3} or an effort quota which produced a fishing mortality of 0.45.

From equations 3.12, 3.13 the coefficient of variation of B would then be about 27% when a catch quota was used and about 18% when an effort quota was used.

Example II

Georges Bank herring

Considerable doubts have been expressed about the level of recruitment to this stock in recent years and it is probable that the estimates of variance given here are on the conservative side.

Formally, 3 year-old recruits do not form a large proportion of the total stock but there is some doubt as to whether the catch of 3 year-old fish adequately predicts the catch of 4 year-olds in the following year. If it is assumed that it does, then

$$\theta = 5.15.$$

$\frac{\text{Var}(C)}{C^2}$ was estimated as 0.193, i.e. coefficient of variation = 43%,

$\frac{\text{Var}(F)}{F^2}$ was estimated as 0.011, i.e. coefficient of variation = 10%.

These lead to a variance ratio of 35% for $\hat{F} = 0.48$ when a catch quota is applied.

Therefore, the fishing mortality that would be achieved by the catch quota would be in the range

$$0.14 < \hat{F} < 0.82.$$

For an effort quota the variance ratio would be 25%. If alternatively days fished with learning was used as the basis of an effort quota then

$$0.24 < \hat{F} < 0.72.$$

If it were aimed to set a catch or effort quota on this stock (stock composition as at 1971) so that the biomass available to capture (B) in the next year was 277 metric tons $\times 10^3$ excluding 3 year-old recruits, with a catch quota the variance ratio of B would be about 35%, while with an effort quota it would be 25%.

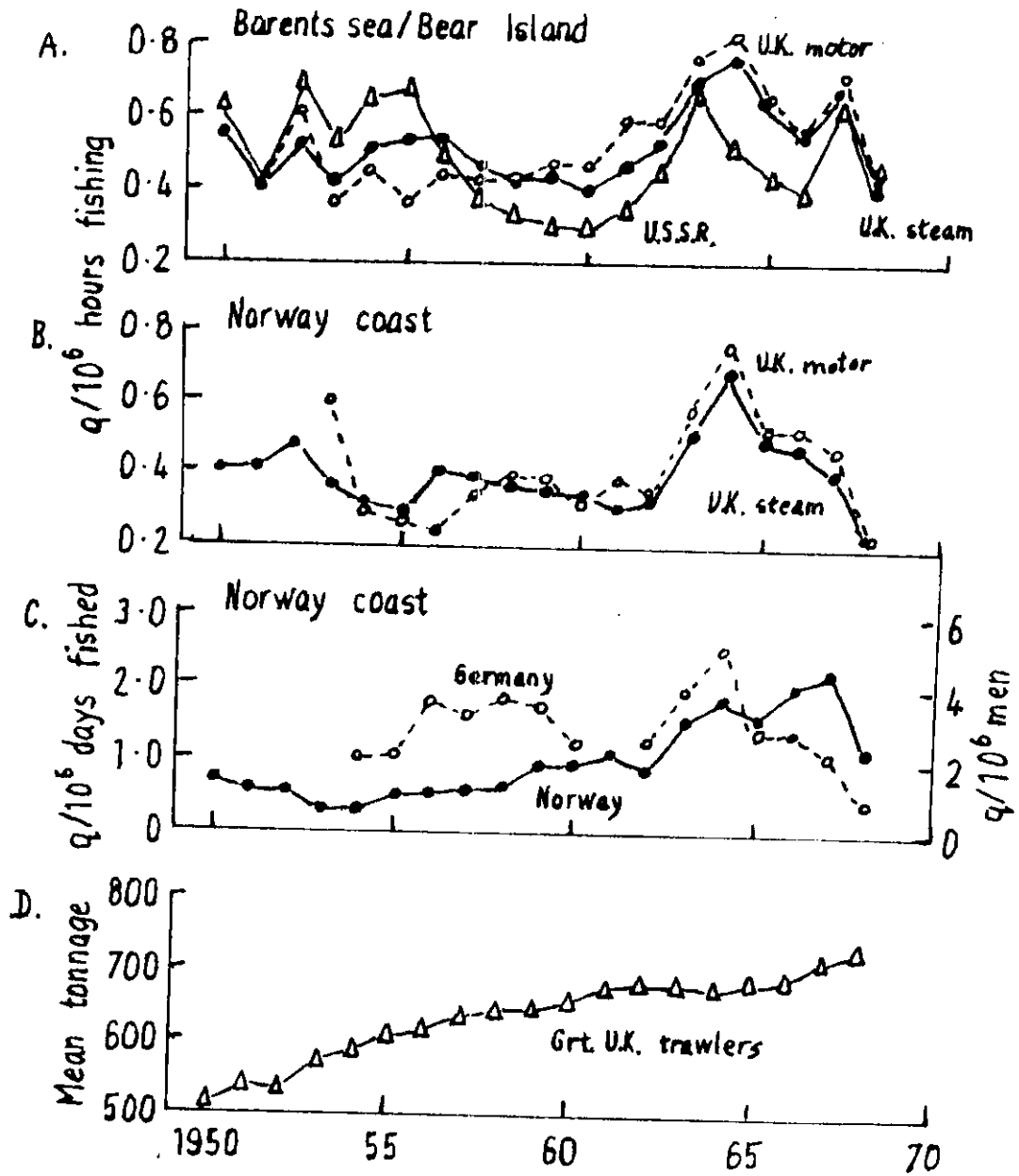


Figure 1 Trends in the catchability coefficient with time for various national fleets fishing at the north-east Arctic, together with the mean tonnage of UK trawlers.

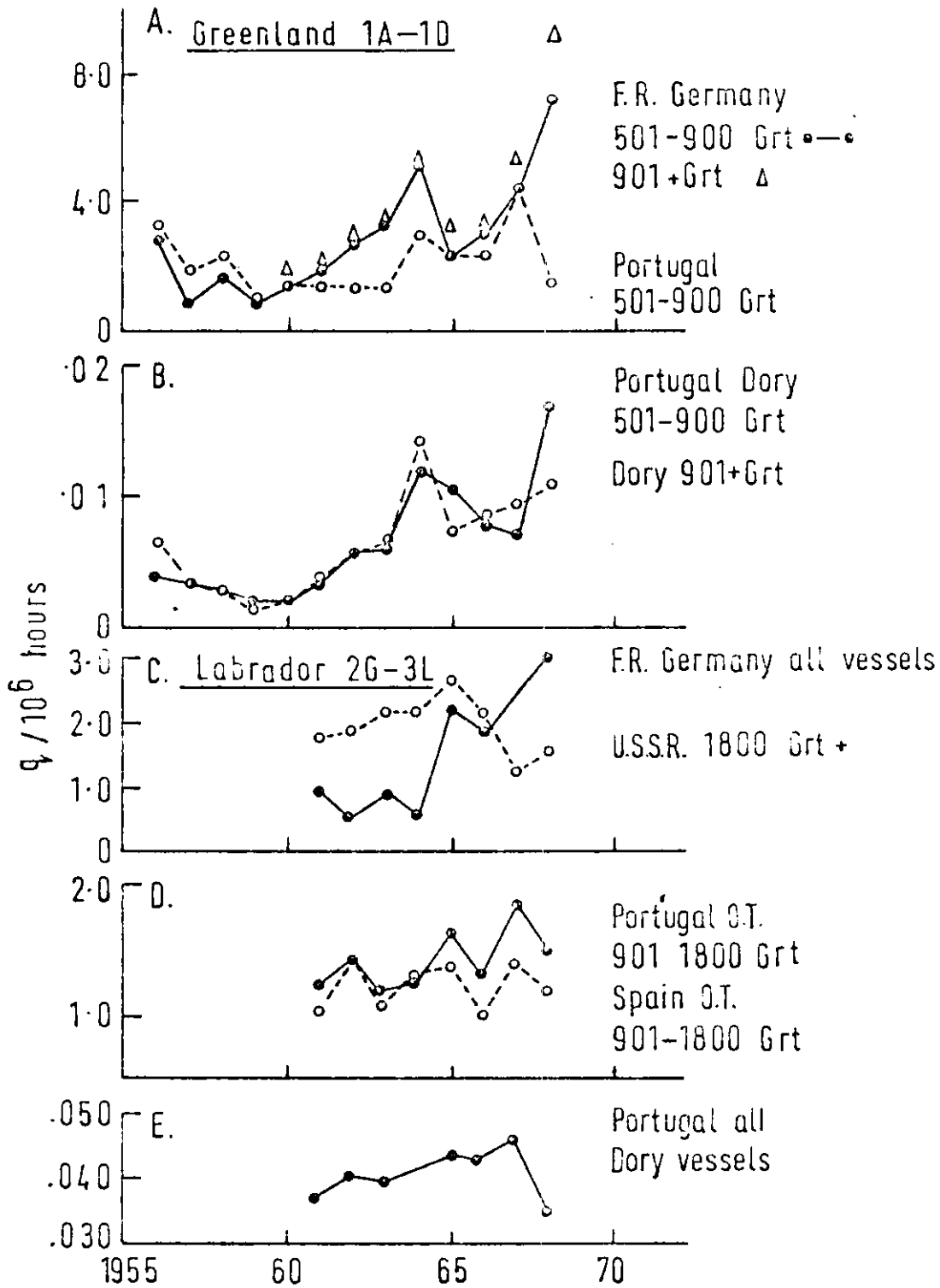


Figure 2 Trends in the catchability coefficient with time for various national fleets fishing at West Greenland and at Labrador.

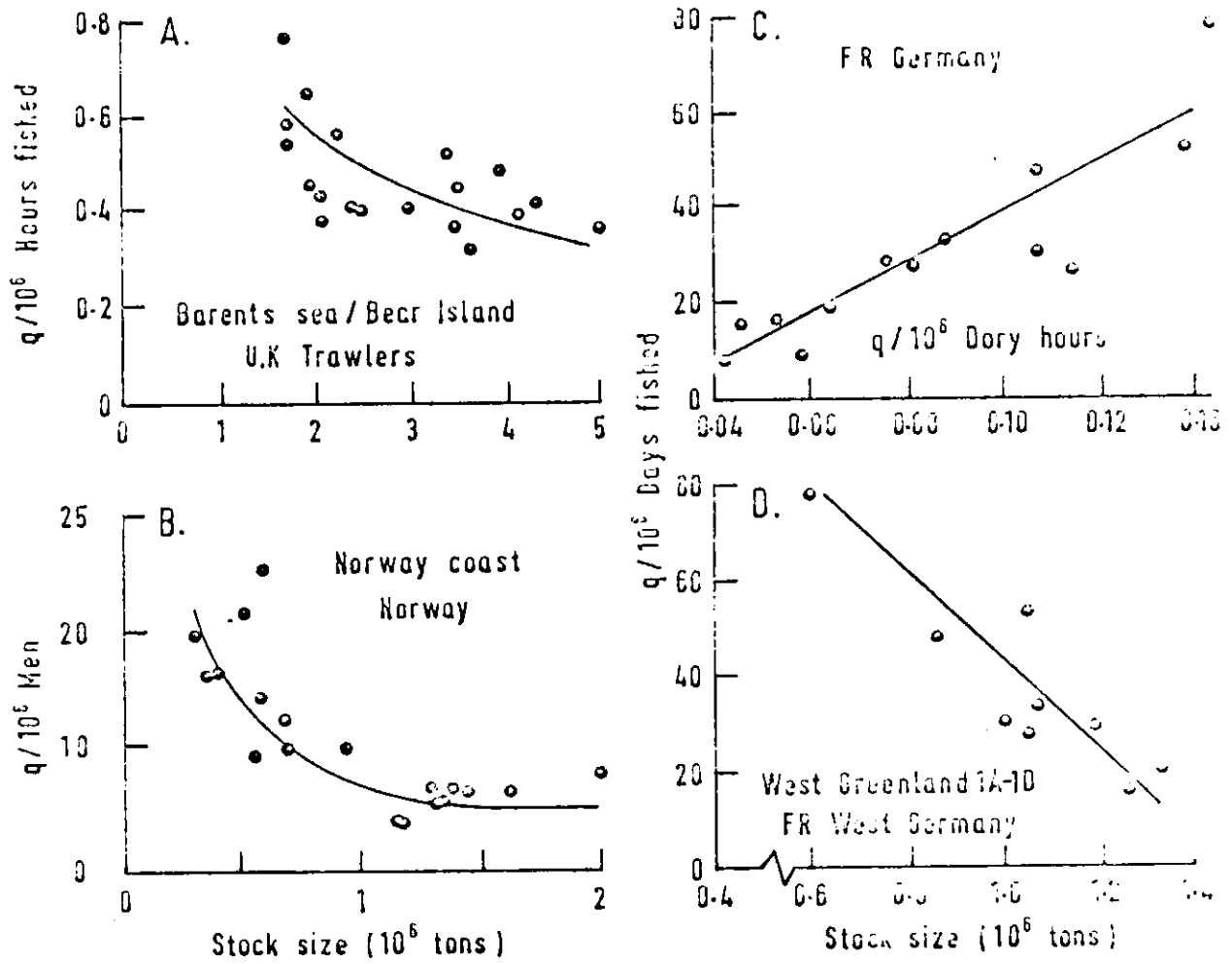


Figure 3 The relationship between the catchability coefficient and the biomass of the stock at the north-east Arctic and West Greenland.