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A note on yield allocation in multi-species fisheries[1]

by

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A system of national yield allocation, with bona fide reservations for possible new entries, has been established as one of the principal regulatory measures in the ICNAF area and it has been applied to more and more species stocks, where necessary, and rather independently. On the other hand, such difficulties have been rapidly prevailing that by-catches, essentially due to incomplete gear selectivity at the present stage of technology, coupled with a variety of fishing strategies, could be no more neglected for effective conservation of the stocks concerned.

This note, rather expository, is for better understanding of yield allocation in multi-species fisheries and especially at this moment for sound consideration of the United States proposal on total effort limitation in the ICNAF Subarea 5 and Statistical Area 6. It goes through simple, and numerical, cases to more general formulation of the model and its algorism for solution, indicating theoretically under what circumstances and how an additional measure such as reduction of overall quota is of prime necessity for conservation of the stocks.

1.1 For simplicity, let us start with the simplest case where two species stocks S_1 and S_2, are exploited by two groups of fishery G_1 and G_2, by-catching either species.

Assume the total allowable catches for both species estimated to be

TAC for S_1	5,000 tons,
TAC for S_2	7,000 tons,

and further that the by-catch rates (column vectors) of G_1 and G_2 are given as follows,

[1] Presented to the Special Commission Meeting, FAO, Rome, January 1974.

	G_1	G_2
S_1	0.90	0.15
S_2	0.10	0.85.

It may go without saying, but 1) in whatever way the total yield (12,000 tons) may be allocated between the two groups of fishery, their by-species catches shall not be summed up over the total allowable catch for the respective species, and 2) presumably it is intended to maximize the sum of their overall catches, as an agreed target for both groups of fishery, putting on the shelf bona-fide reservations for new entries.

Denote the overall catches for the two groups of fishery by C_1 and C_2, and then the conservation requirement in the above indicates that the following inequalities must be satisfied by C_1 and C_2

$$0.90\ C_1 + 0.15\ C_2 \leq 5{,}000$$

$$0.10\ C_1 + 0.85\ C_2 \leq 7{,}000.$$

Out of the infinitely many feasible sets (C_1, C_2)satisfying these inequalities and $C_1 \geq 0$ and $C_2 \geq 0$, the target in the above indicates to choose such a set as to maximize $C = C_1 + C_2$.

The solution, unique in this case, is summarized as follows,

	Overall Catch C_i	Breakdown by species S_1	S_2
G_1	4,267 tons	3,840	427
G_2	7,733 tons	1,160	6,573
Total	12,000 tons	5,000	7,000.

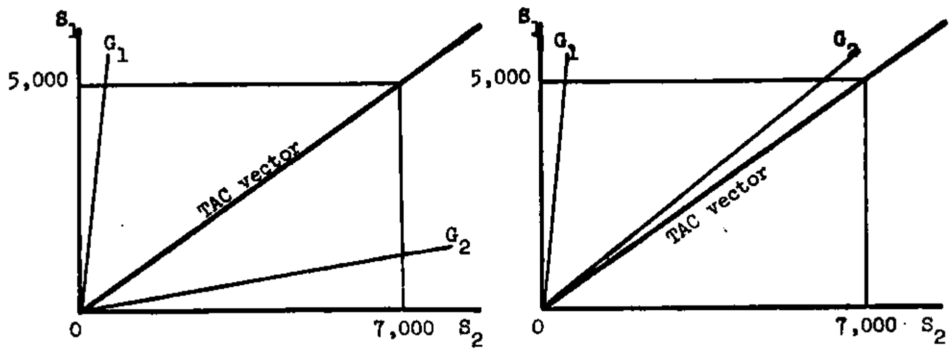
1.2 The sum of the total allowable catches for each species may not always be attainable, although it was in the numerical example in 1.1.

Let the by-catch matrix (a set of column vectors) be given as

	G_1	G_2
S_1	0.90	0.45
S_2	0.10	0.55,

and then all of the non-negative set (C_1,C_2) satisfying the conservation requirement for S_1, $0.90\ C_1 + 0.45\ C_2 \leq 5{,}000$, also meet the second inequality for S_2 and the feasible maximum overall catch amounts only to 11,111 tons. Further, it leaves nothing to the group G_1 under the framework of this model, that is, so far as the target of maximizing the combined catch of both species cannot be modified at all.

Why it happens, and its possible generalization to the case of m species stocks involved, may be more easily illustrated in the following figures.



The difference comes from where the two by-catch vectors are against the vector representing the proportion of the by-species total allowable catches, called TAC vector hereafter. Evidently a scalar product of the TAC vector can be found among weighted combinations of the two by-catch vectors in the left-hand figure, while it is not in the right-hand figure. It is also true in case of m species involved, and technological feasibility of the sum of the by-species TACs as a target depends upon where the by-catch vectors, inherent to the groups of fishery concerned, are against the TAC vector in the m-dimensional space.

To note, such a case as the above example should be regarded as one of the degenerated cases, because conservation of S_2 there is guaranteed by managing the fisheries so as to meet the conservation requirement for S_1, that is, it is essentially a single species case, although S_2 will be kept underexploited.

1.3 Now, go on to a case where two species stocks are exploited by more than two groups of fishery. Of course it happens that all of the by-catch vectors of n groups of fishery fall on the either side of the TAC vector as in 1.2, but if at least one of the by-catch vectors lies on the opposite side, then the sum of the by-species TACs can be attained as a target.

For simplicity here again, consider the simplest case of two species stocks by three groups of fishery. Let the by-catch matrix be given as follows,

	G_1	G_2	G_3
S_1	0.90	0.45	0.15
S_2	0.10	0.55	0.85.

The sum of the by-species TACs (12,000 tons) can be attained as a target, because the by-catch vector for G_3 lies on the other side of the TAC vector from the remaining two.

Denote the overall catches of G_1, G_2 and G_3 by C_1,C_2 and C_3 and then the problem is to choose a set of (C_1, C_2, C_3) which maximizes a linear functional $C = C_1 + C_2 + C_3$, out of the feasible sets which meet the following requirements

$$0.90\ C_1 + 0.45\ C_2 + 0.15\ C_3 \leq 5,000$$
$$0.10\ C_1 + 0.55\ C_2 + 0.85\ C_3 \leq 7,000$$
$$C_1 \geq 0,\ C_2 \geq 0, \text{ and } C_3 \geq 0.$$

Brief description of an algorism may serve for better understanding of the underlying principles. First, introduce two 'dummy' non-negative unknowns, λ_1 and λ_2, to delete the inequality signs from the conservation requirements, that is,

$$0.90\ C_1 + 0.45\ C_2 + 0.15\ C_3 = 5,000 - \lambda_1,$$
$$0.10\ C_1 + 0.55\ C_2 + 0.85\ C_3 = 7,000 - \lambda_2.$$

Then solve a set of these two linear equations for arbitrary two of the three variables, for example, for C_1 and C_2, and they are represented by linear functions of the remaining variable and unknowns as follows,

$$C_1 = -889 + 0.667\ C_3 - 1.222\ \lambda_1 + \lambda_2,$$
$$C_2 = 12,889 - 1.667\ C_3 + 0.222\ \lambda_1 - 2\lambda_2.$$

Summing them up,

$$C = C_1 + C_2 + C_3 = 12,000 - \lambda_1 - \lambda_2.$$

The target of maximizing a linear functional C suggests $\lambda_1 = \lambda_2 = 0$. Then

$$C_1 = -889 + 0.667\ C_3$$
$$C_2 = 12,889 - 1.667\ C_3.$$

That is, any set of non-negative C_1, C_2, and C_3 which meets these two linear relations is one of the solutions, that is, the yield allocation is indeterminate under these two linear constraints.

To obtain a unique solution, another linear relation can be introduced among the three variables. To note, however, it does not imply, any linear relation could be taken into account successfully without any modification of the target. Provided that any modification (practically a reduction) of the attainable target can be accepted, a solution satisfying an additional linear constraint may be obtained by going back to the original solution including 'dummy' parameters λ_1 and λ_2, and then any positive λ_1 and λ_2, if any, will represent the amounts of reduction, imposed by the additional constraint.

More generally, in case of n groups of fishery involved, a set of the feasible solutions satisfying the conservation requirements could be defined in the same way. What is essential there is that the 2 by n by-catch matrix determines two linear independent relations among the n variables, C_1, C_2------, C_n. In case of m species stocks involved, the m by n by-catch matrix determines m linear independent relations among them ($m \leq n$). Theoretically, then, (n - m) additional linear relationships can be introduced, so to speak on whatever outside basis, to obtain a unique solution. It is usually called 'degree of freedom', but practically, it seems, optional freedom,which has been apparently enjoyed in any mono-specific case, will not increase so much as expected, as theoretical 'degree of freedom' increases, although it depends on the intrinsic structure of the by-catch matrix.

1.4 Summing up the above, in case of m species stocks and n groups of fishery involved, the n by-catch vectors (assumed as fixed for each group) form a pyramid in the m-dimensional space, whose top is at the origin, although some vectors may be buried in it. If the TAC vector is in the interior of the pyramid, the sum of the TACs can be attained as a target. Otherwise, although there may happen a variety of cases generally, the sum of the TACs is evidently unattainable. In such cases, however, take some relevant species stocks out of consideration and the sum of the remaining TACs will turn to be attainable as a target. It does no harm to conservation of those species stocks, as it is evident in the case of 1.2. Practically speaking, therefore, the sum of the TACs, if the problem is reasonably framed, is always attainable as a target, although it may leave some other stocks underexploited.

If the number of groups of fishery, n, equals to the number of the species stocks, m, there exists a unique solution on yield allocation to attain the sum of the TACs, which is entirely determined, so to speak, technologically by the given by-catch matrix. If n increases over m, there exists, not a unique but, a set of solutions to attain the sum of the TACs. In other words, the yield allocation is indeterminate under the m linear technological constraints. Theoretically, (n - m) additional linear constraints can be introduced on whatever outside basis to arrive at a unique allocation, but practically, optional freedom would be very much limited unless appreciable reduction of the otherwise attainable target yields, of course different by species, can be accepted, because the given m by n by-catch matrix has imposed m linear independent constraints to be satisfied. When n is less than m, an appropriate reduction of the framework will lead to a satisfactory solution.

2. The by-catch rates have been rather unrealistically assumed in the above as fixed for each group of fishery, just for better understanding of basic framework and principles. It is never the case in practice. There are a great many factors involved in determining the by-catch rate; stock abundance, distribution and migration, coupled with gear selectivity, innovations, shift of the fishing ground and yearround fishing strategy. The underlying complexities will be considerably amplified as various species stocks are increasingly involved, so much that it is hardly possible to figure out in details. On the other hand, however, it appears to me, historical performances here and there indicate that the by-catch rates at the end of the season do vary from year to year, sometimes considerably, but they do not violently fluctuate from one of the extremes to another, excluding some exceptional cases. Therefore it can be reasonably assumed for the by-catch rates to vary, or to be controlled, within finite ranges. There are indeed many uncontrollable, or even difficult to predict, natural and some operational factors involved, but many others are undoubtedly controllable. And any satisfactory solution of the problem under consideration depends on how far and how much these factors can be controlled in practice, not only in quantity but in quality.

Theoretically, in the two species case, a by-catch rate of either species determines the other as a complement. In the three species case, a by-catch rate for a species will leave the rates for the other two species still indeterminate, because there remains one degree of freedom for determination. Thus, more generally, in the m species case, there are (m - 1) degrees of freedom, so to speak, to be expended to absorb the underlying complexities. Assuming they vary within finite ranges, their vectors make a pyramid* in the m dimensional space, as mentioned before. If n groups of fishery are concerned, there exist n such pyramids, one to each group, in the space. Too much complicated as it may appear, the key point is, it is not detailed configurations of these pyramids, nor all of their edge

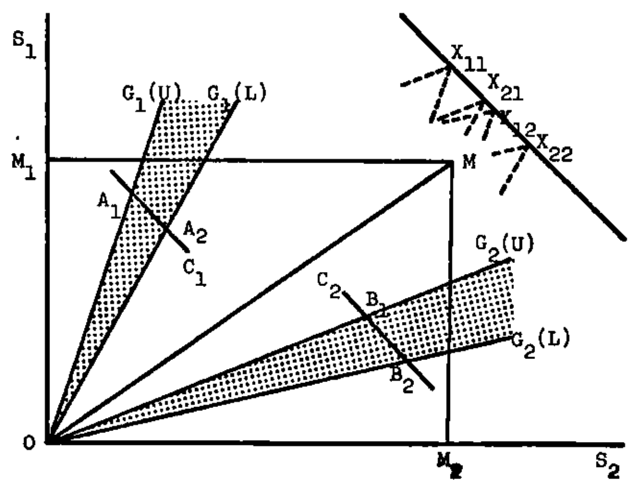
* convex cone, more generally if any continuity be assumed, with no change in the algorism.

vectors, but some components of the edge vectors and their configuration that play an essential role in solving the problem, as shown in what follows.

2.1 First, let us consider a case of two species stocks by two groups of fishery, whose by-catch rates are respectively indeterminate in finite ranges as follows,

	G_1		G_2	
	Upper	Lower	Upper	Lower
S_1	0.95	0.80	0.35	0.10
S_2	0.05	0.20	o.65	0.90.

Then, in the following figure, the actual catch of G_1 is represented by the point between two rays $OG_1(U)$ and $OG_1(L)$, and that of G_2 by the point between two rays $OG_2(U)$ and $OG_2(L)$. Denote the overall catches of G_1 and G_2 by C_1 and C_2, and then the actual catches of both groups will vary on the line segments A_1A_2 and B_1B_2 respectively.



Now, consider the four combinations of two vectors, one from each of the two groups, OA_1, OA_2 and OB_1, OB_2. The sums of two vectors in four combinations will be represented by the four vectors OX_{11}, OX_{21}, OX_{12}, and OX_{22}, whose endpoints are on the line segment $X_{11}X_{22}$, because they are all equal to $C_1 + C_2$. And it is evident, wherever the actual catches of G_1 and G_2 may fall on the line segments A_1A_2 and B_1B_2, that the point representing their sum is on the line segment $X_{11}X_{22}$.

To meet the conservation requirements, the line segment $X_{11}X_{22}$ must lie within the rectangle OM_1MM_2, which represents a set of the feasible by-species catches satisfying the conservation requirements, that is, not more than the TACs, 5,000 tons for S_1 and 7,000 tons for S_2. The figure indicates that it is necessary and sufficient for the S_1-coordinate of the end-point X_{11} not to exceed the TAC for S_1, 5,000 tons, and as well, for the S_2-coordinate of the other end-point X_{22} not to exceed the TAC for S_2, 7,000 tons, that is,

S_1-coordinate of X_{11} : $0.95\ C_1 + 0.35\ C_2 \leq 5{,}000$

S_2-coordinate of X_{22} : $0.20\ C_1 + 0.90\ C_2 \leq 7{,}000$.

Solving a set of these inequalities so as to maximize $C = C_1 + C_2$,

$C_1 = 2{,}611$ tons and $C_2 = 6{,}478$ tons.

And the yield allocation will be summarized as follows,

	G_1	G_2	Sub-total
S_1	≤2,480 tons	≤2,267 tons	≤ 4,747 tons
S_2	≤ 522 tons	≤5,830 tons	≤ 6,352 tons
Overall quota	2,611 tons	6,478 tons	Total quota 9,089 tons.

It is noted in this table that

1) The overall quotas for G_1 and G_2 are uniquely determined, while the by-species quotas are indeterminate with possible maximum allowable catches determined. That is, there is a room left for option of species, constrained by the fixed overall quotas.

2) The total quota allocated is appreciably reduced below the sum of the TACs, which has been shown in the above to be attainable if the by-catch rates can be fixed on the both sides of the TAC vector. Such a reduction of the total quota allocated depends on the ranges, within which the by-catch rates are indeterminate.

3) The possibly maximum by-species catches by both groups are not summed up to the by-species TACs respectively. This is because the target is assumed to be a maximum combined catch of both species. The catch of S_1, for instance, can be increased up to the TAC, but then decrease in the catch of S_2 will not only balance it out but further reduce the total quota below 9,089 tons.

2.2 Generally, in case of m species stocks by n groups of fishery, the by-catch vectors for each group, as metioned before, make a pyramid in the m dimensional space. The edge vectors can be arranged in a matrix form. Then, as in 2.1, the requirement inequality for S_i ($i = 1, 2, ---, m$) can be constructed by picking up the largest component on the i-th row in the by-catch matrix, one from each group, in addition to the TAC for S_i. Once a set of the requirement inequalities can be set up, it will be solved just as it is described in the above. Evidently any set of solution will retain all the features so far revealed.

Reviewing the algorism, it may be strange that use is made of only the largest component rate from each group's by-catch matrix in constructing the requirement inequalities. The ranges, within which the by-catch rates are indeterminate, however, have already been well represented by how much the sum of the largest components for each group exceeds unity, as it is strictly so in the two dimensional case in 2.1.

Summary Yield allocation problems in case, where m species stocks are exploited by n groups of fishery, have been rather theoretically considered in relation to conservation requirements, maximum utilization, by-catch structure and possible outside allocation formula.

1) Among the stocks concerned, there may be involved some stocks, technologically subordinate in the sense that their conservation is guaranteed by reasonably regulating the fisheries only on the other stocks although they may be left underexploited. Then they can be better considered separately.

2) Excluding such stocks, the sum of the biological TACs can be generally attainable as a target if the by-catch rates are assumed as fixed for each group of fishery. Otherwise, that is, if the by-catch rates are indeterminate within finite ranges on whatever reasons they may be, the sum of the biological TACs must not be taken as a reasonable target. In other words, not only overall but by-species reductions of the target below the biological TACs is then of prime necessity for effective conservation of the stocks concerned. The amounts of necessary reduction, different by stock, and a room for options on the fishery side as well, depend on the ranges within which the by-catch rates are indeterminate, in addition to the given by-catch structure and biological TACs.

3) In either case, the overall yield allocation among the n groups of fishery is uniquely determined (when n = m), or, more generally, indeterminate under the m linear constraints imposed (when n > m), technologically by the given by-catch structure. Theoretically, (n - m) additional linear constraints can be introduced on whatever outside considerations to arrive at a unique allocation, but optional freedom would be practically very much limited, in comparison with any single-species case, unless appreciable reduction of the otherwise attainable target yields can be agreed on.

4) Disregard of these structural features, especially of constraints imposed by the given by-catch structure, will probably result in false, or presently infeasible, allocation, increase of wasteful discard and other nuisances and finally disastrous failure of conservation. In this sense, the present system of allocation, applied to stock by stock rather independently, needs to be critically reviewed and reconsidered.