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On estimating the relaibility of data on age composition of fish population obtained by means of the age key

by

E.I. Shemina
AtlantNIRO
Kaliningrad, USSR

In estimating the age composition of fish population be means of length-age key the initial data are as follows:

N - the number of fish with measured length,

N_l - the number of fish with length l,

P_l - the share of fish with length l,

$P_{t/l}$ - a table of length-age key, the share of age t individuals among the fish with length l.

In this case the number of fish in l size group and t age group is

$$N_{t/l} = N_l P_{t/l} = N P_l P_{t/l} \tag{1}$$

The ratio

$$\frac{\sum_l N_{t/l}}{N} = P_t \tag{2}$$

P_t - the share of t age group.

It can be suggested that

$$P_t = \sum_l P_l P_{t/l} \tag{3}$$

and approximately (1,2)

$$D(P_t) = \sum_l \left[P_l^2 D(P_{t/l}) + P^2_{t/l} D(P_l) \right] \tag{4}$$

Based on this formula, P_t, dispersion $D(P_t)$, mean-root-square error $S(P_t)$ and variation factor V_t at the value of P_t have been calculated by means of length-age key.

In tables 1 and 2 the examples of calculations for the most numerous silver hake age groups are given according to the data for 1972.

Table 1

The estimation of precision in age composition calculations

March 1972, 4W

Age t	P_t	$D(P_t)$	$S(P_t)$	$V_t = \frac{S(P_t)}{P_t}$	Confidence limits (0.95 probability)	
					lower limit	upper limit
2	0.346	0.00078	0.028	0.081	0.290	0.402
3	0.322	0.00075	0.027	0.085	0.268	0.376
4	0.103	0.00124	0.035	0.342	0.033	0.173
5	0.064	0.00015	0.012	0.192	0.192	0.088

May 1972, 5ZE

Table 2

4	0.560	0.03200	0.031	0.056	0.498	0.622
5	0.171	0.00097	0.034	0.199	0.103	0.239
6	0.068	0.00116	0.019	0.273	0.030	0.106

To minimize $D(P_t)$, the necessary measurement number and the number of age determinations for a table-key should be estimated.

Let us consider these 2 tasks separately.

1. Length composition estimation

The initial formula is given below:

$$S^2(P_l) = \frac{S_b^2(P_l)}{N} + \frac{S_w^2(P_l)}{n\,N} \quad (5)$$

where N is the number of samples,

n is the sample size,

$S_b^2(P_l)$ is an inter-sample dispersion

$S_w^2(P_l)$ is an intra-sample dispersion.

The analysis of material for 1971-72 has shown that in all cases $S_b^2(P_l) > S_w^2(P_l)$, therefore, for better precision the increase in sample number can be recommended without increase in sample volume which should remain as an existing one.

The minimum number of samples for estimating the length composition at confidence probability of 0.95 is

$$N_{min} \approx \frac{4S_b^2(P_l)}{E^2} \quad (6)$$

where E is an allowable difference between the true value of P_l and its estimate.

The size of one sample at that is

$$n_1 = \frac{4(1-P_l)}{\mathcal{L}^2 P_l N_{min}} \quad (7),$$

where $\mathcal{L}$ is a given relative error.

A similar error can be obtained at various combinations of N and n values.

If the number od samples is increased in V times as compared with N_{min}, the error being the same (4), we'll have:

$$n = \frac{P_l(1-P_l)}{(V-1)\,S_b^2} \quad (8)$$

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An example of similar calculations is given in table 3.

Table 3

The calculation of the number of observations for estimating the size-composition of silver hake stock

May, 5ZE $\alpha = 0.1$

l	P_l	$S_b^2(P_l)$	N_{min}	V=2		V=1.5	
				$N=VN_{min}$	n	$N=VN_{min}$	$n_k=1.5$
28-29	0.123	0.0034	91	182	32	137	64
30-31	0.296	0.0038	17	34	56	26	112
32-33	0.244	0.0035	24	48	53	36	106
34-35	0.153	0.0022	39	78	58	59	116
36-37	0.072	0.0019	144	288	37	216	74
38-39	0.032	0.0004	160	320	69	240	138

A large number of samples for size groups (36-37) and (38-39) is obtained, however, it is hardly necessary to assess these not numerous groups with a high precission. At $\alpha = 0.2$ the number of samples is decreased 4 times, at $\alpha = 0.5$ - 25 times etc.

2. The assessment of the number of age-determinations for length-age key

If a concentration is considered to be uniform and the error of table-key depends only on one sample size, the number of measurements can be determined by a formula:

$$n_{lt} = \frac{4K^2 P_l^2 P_{t/l}(1-P_{t/l})}{\alpha^2 P_t^2} \qquad (9),$$

which was drawn in (3).

For each l the number of age determinations is max. N_{tl} is the number of size groups in t age group.

The analysis of data shows that there exists a dispersion of $P_{t/l}$ values between different sample-keys. In this case the number of samples is estimated by the following formula:

$$N_{lt} = \frac{4K^2 P_l{}^2 S^2 (P_{t/l})_b}{\alpha^2 P_t{}^2} \quad (10)$$

The lesser α and P_t, the larger is the total number of age determinations and sample-keys. Moreover, the less significant the ratio of P_l to P_t, i.e. the greater the entropy of $P_{t/l}$, the larger is this value. Tho number of age determinations would have been zero, if only one value of l corresponded to each t.

Table 5

Length-age key for silver hake, 4W, March 1972

(the number of age determinations n = 260)

Age, t / Length, l	1	2	3	4	5	6
22 - 23		1				
24 - 25		0.833	0.167			
26 - 27		0.390	0.610			
28 -29		0.031	0.469	0.500		
30 - 31			0.161	0.821	0.018	
32 - 33			0.036	0.571	0.357	0.036
34 - 35				0.100	0.700	0.200
36 - 37					0.667	0.330

Table 6

Length-age key for silver hake, 5ZE, May 1972
(the number of age determinations n = 200)

Age / Length	2	3	4	5	6	7	8
28 -29		0.357	0.643				
30 -31			0.981	0.019			
32 -33			0.659	0.341			
34 - 35			0.404	0.532	0.064		
36 - 37				0.316	0.579	0.105	
38 - 39				0.250	0.417	0.333	
40 - 41					0.375	0.500	0.125
42 - 43						1.000	
44 - 45							

This can be exemplified by calculations made on the basis of tables 5 and 6.

Table 7

The example of calculation of age determinations number at $\alpha = 0.1$

May, 5ZE

Age / Length	3	4	5	6	Number of age determinations necessary for a size group
28 - 29	$\frac{43}{6}$	$\frac{73}{8}$			$\frac{73}{8} = 9$
30 - 31		$\frac{35}{1}$	$\frac{547}{3}$		$\frac{547}{3} = 182$
32 - 33		$\frac{288}{1}$	$\frac{4494}{3}$		$\frac{4494}{3} = 1498$
34 - 35		$\frac{118}{1}$	$\frac{1909}{300}$	$\frac{1760}{165}$	$\frac{1909}{300} = 6$
36 - 37			$\frac{360}{56}$	$\frac{1560}{138}$	$\frac{1560}{138} = 11$
38 - 39			$\frac{63}{7}$	$\frac{310}{115}$	$\frac{310}{115} = 3$
40 - 41				$\frac{90}{17}$	$\frac{90}{17} = 5$
42 - 43					

Table 8

March, 4W

Age / Length	2	3	4	5	Number of age determinations necessary for a size group
24 - 25	77/1	139/1			139/1 = 139
26 - 27	540/2	975/5			975/5 = 195
28 - 29	60/1	897/9	5634/244		5634/244 = 23
30 - 31		474/5	3224/69	1004/4	3224/69 = 47
32 - 33		46/1	2027/2	4920/7	4920/7 = 703
34 - 35			161/8	976/15	976/15 = 65
36 - 37				42/1	42/1 = 42

In numerator a total number of age determinations is given and in denominator - the number of sample-keys, while the fraction magnitude is equal to each sample size. All these indices differ in various size groups. It must be remembered that lowered values of α , say, 0.2 or 0.5 may reduce the number of observations 4 and 25 times, accordingly, compared with tables 7 and 8, which will suffice to make corresponding calculations.

In case of direct determination of age-composition without the length-age key, the calculation of the number of observations is made in the same way as in calculation of size-composition (formula (6),(7),(8)).

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