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Report on Sampling of Commercial Landings of Haddock taken from Georges Bank, 1973

by

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Abstract

The U.S.A. procedure for estimating the numbers at age, N_a, of haddock landed monthly, is reviewed. Estimates of the variance and coefficient of variation of $\hat{N}_a$ are made from samples taken from the Georges Bank area in 1973 and landed at New Bedford or Boston, Massachusetts. Estimates of the number of samples (n) and the number of fish measured per sample (m) which yield given degrees of precision in estimates, are listed.

Introduction

The age composition of haddock caught by the U.S.A. fleet is presently estimated from samples of fish taken at ports where the bulk of the fish is landed. Port agents attempt to sample catches landed monthly from each sampling area 51, 52 and 53 (Figure 1) and market category: large haddock or scrod[1]. The fish are randomly selected from a bin, or a number of 125 lb. boxes are randomly selected, such that a minimum number of fish are sampled. Present instructions require that 100 large haddock or 60 scrod be taken per sample for length measurements, and 20 large haddock and 15 scrod be randomly selected from each of these samples, for aging (Schultz, Res. Doc. 74/). This amount of sampling results in less than 1% of the haddock landed monthly being measured for lengths, with a proportionate number being aged.

The present study examines the precision of the U.S.A. sampling scheme for estimating the age composition of the catch landed, by considering the 1973 sampling of haddock taken from the Georges Bank area (Figure 1, Areas 52 and 53).

Estimation of Na, the number at age.

Monthly estimates of the number landed at age a (large haddock plus scrod) are made using the following formula:

$$\hat{N}_a = \sum_{\ell=1}^{J} (\hat{p}_{\ell L} * \hat{N}_L + \hat{p}_{\ell S} * \hat{N}_S) * \hat{p}_{a/\ell}, \qquad (1)$$

where $\hat{N}_a$ = estimated number landed which are of age (group) a (large haddock plus scrod),

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[1] Large haddock is defined to be haddock weighing more than 2.5 lbs; scrod weighs between 1.5 and 2.5 lbs. (inclusive)

$\hat{p}_{\ell L}$ = estimated percent of large haddock landings which are of length (group) ℓ ,

$\hat{p}_{\ell S}$ = estimated percent of scrod landings which are of length (group) ℓ ,

$\hat{p}_{a/\ell}$ = estimated percent of landings (large haddock plus scrod) which are of age (group) a, out of those of length (group) ℓ,

$\hat{N}_L$ = estimated number of large haddock landed,

$\hat{N}_S$ = estimated number of scrod landed, and

J = number of length groups considered.

Estimation of p_ℓ, percent at length ℓ[1], and $p_{a/\ell}$, percent of age a at length ℓ, and estimated variances of each.

The estimators used in this study were: (2)

$\hat{p}_\ell$ = average percent at length ℓ over samples taken within the month for market category large haddock ($\hat{p}_{\ell L}$) or scrod ($\hat{p}_{\ell S}$); abbreviated as "$\bar{p}_\ell$".

$\hat{p}_{a/\ell}$ = average percent of age a which are at length ℓ over samples taken within a quarter; denoted by $\bar{p}_{a/\ell}$,

$\hat{N}_L$ = total weight of large haddock landed/average weight of large haddock sampled; denoted by $W_L/\bar{w}_L$, and

$\hat{N}_S$ = total weight of scrod landed/average weight of scrod sampled; denoted by $W_S/\bar{w}_S$.

The estimates of percent at age within a length group are made ignoring the market category stratification used to estimate percent at length.

Estimates of the variances of $\hat{p}_\ell$ and $\hat{p}_{a/\ell}$ were made as follows: For each market category (large haddock or scrod),

$$\widehat{\mathrm{Var}\,(\bar{p}_\ell)} = \frac{1}{n} * \frac{1}{(n-1)} * \sum_{i=1}^{n} (p_{\ell i} - \bar{p}_\ell)^2, \qquad (3)$$

where n = number of samples taken within the month,

$p_{\ell i}$ = percent of fish at length ℓ in sample i, where the number of fish in the sample which are of length ℓ is assumed to be a binomial variable, and

$\bar{p}_\ell$ = as explained earlier.

For each quarter. the variance of $\hat{p}_{a/\ell}$ is estimated as:

$$\widehat{\mathrm{Var}\,(\bar{p}_{a/\ell})} = \frac{1}{n} * \frac{1}{(n-1)} * \sum_{i=1}^{n} (p_{a/\ell i} - \bar{p}_{a/\ell})^2, \qquad (4)$$

where n = number of samples taken (large haddock and scrod) within the quarter,

$p_{a/\ell i}$ = percent of fish of age a out of those at length ℓ in sample i, where the number of fish in the sample which are at age a within the length group ℓ, is assumed to be a binomial variable, and

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[1] Henceforth the terms "length" and "age a" refer either to a single length or age, or to a respective group labelled "ℓ" or "a".

$\bar{p}_{a/\ell}$ = as explained earlier.

In both variance formulas (3) and (4), the ratio of the number of samples taken (n) to the number in the population (N) is assumed to be negligible; it is certainly true that $n/N < .01$. Calculated values of $\bar{p}_\ell$, $\bar{p}_{a/\ell}$, $\widehat{Var}(\bar{p}_\ell)$ and $\widehat{Var}(\bar{p}_{a/\ell})$, as well as values of the number of samples (n) and the number of fish per sample (m)[1] contributing to the estimates, are given in Tables 1-3.

In each market category, the differences in percent at length estimates within a month were as large as those of the samples between months. There was insufficient data to determine whether a source of these differences was due to a port difference, but in months where comparisons were possible, the within port differences in percent at length values were as substantial as those between ports. The percent of age at length estimates show a trend with time, i.e. the distribution within any length group has moved ahead to another length group with time.

Estimation of the variance of $\hat{N}_a$, the estimated number at age.

Estimates of the variance and coefficient of variation of the estimated number at age, $\hat{N}_a$, for each month were made using the following formulas:

$$\widehat{Var}(\hat{N}_a) = \widehat{Var}(\hat{N}_{aL}) + \widehat{Var}(\hat{N}_{aS})$$

$$= \sum_{\ell=1}^{J} \left[(\hat{p}_{\ell L}{}^2 * \widehat{Var}(\hat{p}_{a/\ell}) + \hat{p}_{a/\ell}^2 * \widehat{Var}(\hat{p}_{\ell L})) * N_L{}^2 \right]$$

$$+ \sum_{\ell=1}^{J} \left[(\hat{p}_{\ell S}{}^2 * \widehat{Var}(\hat{p}_{a/\ell}) + \hat{p}_{a/\ell}^2 * \widehat{Var}(\hat{p}_{\ell S})) * N_S{}^2 \right], \qquad (5)$$

where $\hat{N}_L$ and $\hat{N}_S$ are the estimated number of large haddock and scrod landed during the month, assumed to be known for this analysis (see (2)); $\hat{p}_{\ell L}$ and $\hat{p}_{\ell S}$ are the percent at length group ℓ for the market categories large haddock (L) and scrod (S); and other estimates are as outlined earlier. The coefficient of variation then is:

$$\text{c.v.}\ (\hat{N}_a) = \widehat{Var}(\hat{N}_a)^{\frac{1}{2}} / \hat{N}_a .$$

Estimates of these statistics from the samples taken each month of 1973 are given in Tables 4a-4d. Estimates of percent of age at length are given for age groups 2-3, 4-5, 10-11, and greater than 11 years old. Since in most cases these age groups accounted for as much as 80% of the estimated number landed, analyses of these subsets of data should be representative of the entire data base.

Change in precision of estimators as the number of samples (n) and the number of fish per sample (m) is varied.

The statistics in Tables 4a-4d overestimate or underestimate the true variance of the estimated number at age, depending in part on whether the variance of the estimated number landed within a month and market category is known and can consequently be treated as a constant. In any event it can easily be seen from (5) that improvement in the precision of $\hat{N}_a$ depends on improvement in the precision of $\hat{p}_{\ell L}$, $\hat{p}_{\ell S}$ and $\hat{p}_{a/\ell}$. The number of samples (n) and number of fish per sample (m) needed to improve the precision of these estimates were determined using the following form of the variance of a general p:

$$\widehat{Var}(\hat{p}) = \frac{\hat{S}_b{}^2}{n} + \frac{\hat{S}_w{}^2}{nm}, \qquad (6)$$

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[1] Where the number of fish per sample (m) varies from sample to sample, $m = (M - \sum_{i=1}^{n} m_i^2 / M) / (n-1)$, where n = number of samples, m_i = number of fish in i^{th} sample, and M = total number of fish sampled (Davies, p. 131).

where $\hat{p}$ = either $\hat{p}_{\ell L}$, $\hat{p}_{\ell S}$ or $\hat{p}_{a/\ell}$,

n = number of samples,

m = number of fish per sample,

$\widehat{Var(\hat{p})}$ = as outlined earlier for the respective estimates,

$\hat{S}_w^2$ = estimate of the within component of the variance of $\hat{p}$, namely

$$\hat{S}_w^2 = \frac{1}{n} * \sum_{i=1}^{n} \frac{m_i p_i q_i}{(m_i - 1)}$$

where m_i = number of fish in the i^{th} sample,

n = number of samples,

$q_i = 1.0 - p_i$, where, say, p_i = number of fish of length ℓ / m_i ,

$\hat{S}_b^2$ = estimate of the between component of the variance of $\hat{p}$, found by subtraction.

Formula (6) assumes the finite population correction to be 1, or that n/N is negligible. For the estimated variance of the percent at length estimates, it was further assumed that each sample contained the same number of elements (fish) from which a random number of fish were sampled. This assumption seems reasonable in view of the stratification by large haddock and scrod, and since the trips from which the samples are drawn are comparable. Since no one n-m combination will satisfy a given level of precision of percent at length for all length groups, n-m combinations required to satisfy levels of coefficient of variation were determined for the following categories of $\hat{p}_\ell$,

1) $\hat{p}_\ell < .10$

2) $.10 \leq \hat{p}_\ell < .20$

3) $.20 \leq \hat{p}_\ell < .30$

4) $\hat{p}_\ell \geq .30$

For each of these groups, the average estimate of S_b^2 and S_w^2 as calculated from Tables 1-3 was used to determine the coefficient of variation of $\hat{p}_\ell$ = .05, .15, .25, .35 using (6), for large haddock and scrod samples separately. The results are given in Table 5(a)-(h). The precision of the estimates of percent at length for the groups comprising the bulk of the distribution at length is the best, as expected. In all cases, the precision is affected more by an increase in the number of samples (n), than by an increase (or decrease) in the number of fish per sample (m).

A similar analysis was done on the estimates of the percent of age at length, for the age groups noted earlier. Since these age groups dominate the distribution, there are many estimates of percent at length accounting for $\geq 50\%$ of the age distribution within a length group (see Table 3). Accordingly, the n-m combinations (number of samples - number of fish per sample) satisfying the following groups were considered:

1.) $.70 < \hat{p}_{a/\ell} \leq 1.00$

2.) $.30 < \hat{p}_{a/\ell} \leq .70$

3.) $0 \leq \hat{p}_{a/\ell} \leq .30$

As before, the formula used here was (6), and the average values of the estimates of S_b^2 and S_w^2 were used (Tables 6a-6b), with the coefficients of variation being the calculated variable. Since the haddock samples are dominated by two strong age groups (3-5 year olds and 10-11 year olds), the most likely percent of age at length estimates are those in Table 6b and Table 6c for which the coefficients of variation are in an acceptable, if not desirable, range for the present sampling scheme. Of all length groups relevant to this study of age groups, only 2 out of the 35 length groups had an m value (number of fish per sample) greater than 5. Increasing the number of fish aged per sample (from 20 large haddock and 15 scrod) would improve the precision of the estimated percent of age at length values, without markedly affecting the present sampling scheme.

Prediction of the number of samples (n) and the number of fish per sample (m) needed to attain specified levels of precision.

Although Tables 5 and 6 adequately relate the change in precision (expressed in terms of the coefficient of variation) as the number of samples (n) and/or the number of fish per sample is modified, it is of interest to determine how good any of these n-m combinations are as a tool to achieve specified levels of precision on the estimates, when given levels of error are allowed. Snedecor (1967) suggests the following method to determine the sample size needed to achieve a desired level of precision, for preselected alpha(a) and beta(b) error rates:

$$(Z_a + Z_b)^2 * \frac{\widehat{\text{Var}}(p_\ell)}{n} = (d * \hat{p}_\ell)^2 \qquad (7)$$

where Z_a and Z_b are Student's-t variates corresponding to two-tailed significance levels for a and b,

$\widehat{\text{Var}}(p_\ell)/n$ = estimated variance of $\hat{p}_\ell$,

d = a preselected percentage difference between the estimated percent $\hat{p}_\ell$ and the true percent p_ℓ, which one would correctly like to detect at the alpha level a with power (1-b),

$\hat{p}_\ell$ = estimate of the population mean of length group ℓ.

The difference term $(\hat{p}_\ell - p_\ell) = d*p_\ell$ is assumed to be a normal variate. The estimated variance of $\hat{p}_\ell$ is estimated according to formula (6) depending on whether the exercise is being performed on the percent at length estimator ($\hat{p}_\ell$), or on the percent at age at length estimator ($\hat{p}_{a/\ell}$). As illustrative examples of the procedure, data of the large haddock samples of February for the 60-64 cm group and data of the second quarter, 4-5 year old haddock of the 55-59 cm group were analyzed, with d, a and b requirements varying for the estimated percent at length in the first case, and for the estimated percent of age at length in the second case. The estimated variance of each and associated statistics (as required in (7)) are given in Table 1 and Table 3, respectively. The results are given in Table 7. It is clear that at the present level of sampling of large haddock, the estimated percent at length for the group considered is only within ± 50% of the population mean percent. Further a halving of the number of fish measured per sampling would not markedly improve the precision. The results of the analysis for the percent of age (4-5 yrs) at length (50-54) is less ominous. The presence of two dominant age groups (3-5 year olds and 10-11 year olds) at the same length group, as is evident from Table 3, improves the precision of the estimator for each group. Further examination of the structure of the population, and perhaps post-stratification by dominant year classes, would enable better assessment of the precision of such estimates.

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Table 1. Statistics from 1973 commercial length samples of large haddock taken from Georges Bank. n=number of samples, m=number of fish/sample.

Month	length group (cm)	$\hat{p}_1$	$\mathrm{Var}(\hat{p}_1)$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
January	50-54	.058	.001207	.0761	.0041	4	105
	55-59	.257	.0102	.1708	.0392	4	105
	60-64	.170	.0022	.1452	.0074	4	105
	65-69	.204	.00365	.1484	.0132	4	105
	70-74	.186	.00591	.1264	.0225	4	105
	75-79	.111	.00169	.00169	.0058	4	105
	80-84	.014	.00014	.0138	.0004	4	105
February	40-44	.004	.000018	.00534	.000015	4	93
	45-49	.007	.000022	.00803	.000003	4	93
	50-54	.056	.000583	.05936	.00169	4	93
	55-59	.198	.005537	.14953	.02054	4	93
	60-64	.215	.001224	.16280	.00315	4	93
	65-69	.238	.00186	.18040	.00548	4	93
	70-74	.192	.00169	.15458	.00510	4	93
	75-79	.065	.00076	.05405	.00246	4	93
	80-84	.025	.000189	.02327	.00050	4	93
March	55-59	.052	.00052	.04981		1	96
	60-64	.156	.00139	.13305		1	96
	65-69	.208	.00173	.16647		1	96
	70-74	.344	.00238	.22804		1	96
	75-79	.208	.00173	.16647		1	96
	80-84	.031	.00032	.03036		1	96
April	50-54	.016	.00013	.01564	.00024	3	104
	55-59	.044	.00075	.04135	.00184	3	104
	60-64	.102	.00044	.09208	.00044	3	104
	65-69	.181	.00234	.14612	.00562	3	104
	70-74	.350	.00113	.22722	.00119	3	104
	75-79	.272	.00410	.19047	.01048	3	104
	80-84	.036	.00049	.03336	.00116	3	104
May	50-54	.009	.000017	.00924	.000032	7	107
	55-59	.096	.00153	.08311	.00993	7	107
	60-64	.145	.00148	.12019	.0092	7	107
	65-69	.210	.00076	.16376	.00379	7	107
	70-74	.271	.00107	.19186	.00571	7	107
	75-79	.202	.00208	.14719	.01321	7	107
	80-84	.054	.00037	.04757	.00218	7	107
	85-89	.014	.000033	.01310	.00011	7	107
June	50-54	.028	.00017	.02670	.00023	3	97
	55-59	.108	.00097	.07684	.02822	3	97
	60-64	.177	.01037	.12530	.02980	3	97
	65-69	.199	.00097	.15928	.00127	3	97

Table 1 (continued)

Month	length group (cm)	$\hat{p}_1$	$\hat{Var}(\hat{p}_1)$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
June (continued)	70-74	.270	.01028	.17931	.02899	3	97
	75-79	.164	.00419	.13076	.01124	3	97
	80-84	.051	.00072	.01042	.00205	3	97
	85-89	.003	.00001	.00344	(-)	3	97
July	45-49	.10	.01000	.00986	.01992	2	120
	50-54	.07	.00250	.06305	.00447	2	120
	55-59	.069	.00148	.06267	.00244	2	120
	60-64	.16	.01000	.12533	.018956	2	120
	65-69	.24	.00001	.18217	(-)	2	120
	70-74	.25	.00442	.18631	.00729	2	120
	75-79	.16	.01071	.12259	.02040	2	120
	80-84	.045	.00063	.04274	.00089	2	120
October	50-54	.057	.00325	.05109	.00601	2	105
	55-59	.086	.005776	.07325	.01085	2	105
	60-64	.110	.001120	.09733	.00132	2	105
	65-69	.205	.00384	.16053	.00616	2	105
	70-74	.315	.00226	.21530	.00246	2	105
	75-79	.162	.00081	.13620	.00033	2	105
	80-84	.058	.00081	.05361	.00111	2	105
	85-89	.010	.0				
November	50-54	.013	.000072	.01309	.00009	3	103
	55-59	.023	.00001	.02231	(-)	3	103
	60-64	.048	.00023	.04620	.00024	3	103
	65-69	.175	.00264	.14033	.00656	3	103
	70-74	.327	.00054	.22113	(-)	3	103
	75-79	.314	.00423	.20898	.01068	3	103
	80-84	.084	.00051	.07683	.00077	3	103
	85-89	.016	.00013	.01581	.00023	3	103
December	50-54	.090	.00731	.07167	.01382	2	90
	55-59	.060	.00303	.04945	.00550	2	90
	60-64	.22	.01199	.16126	.02219	2	90
	65-69	.18	.00016	.15073	(-)	2	90
	70-74	.26	.01254	.18095	.02308	2	90
	75-79	.15	.01638	.11387	.03150	2	90
	80-84	.05	.00053	.04493	.00056	2	90

Table 2. Statistics from 1973 commercial length samples of scrod taken from Georges Bank.

Month	length group (cm)	$\hat{p}_1$	$\hat{Var}(\hat{p}_1)$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
January	30-34	.010	.0001	.012	.000075	2	96
	35-39	.267	.00497	.1769	.00497	2	96
	40-44	.649	.00137	.2133	.00052	2	96
	45-49	.0675	.00093	.0620	.00121	2	96
	50-54	.006	.000036	.0052	.000018	2	96
February	35-39	.100	.01000	.1075	.01842	2	68
	40-44	.361	.0625	.1995	.1221	2	68
	45-49	.326	.01877	.2031	.0345	2	68
	50-54	.111	.01232	.0696	.02362	2	68
	55-59	.102	.01040	.0654	.01985	2	68
March	30-34	.005	.000025	.00611	(-)	2	81
	35-39	.213	.02387	.16384	.04572	2	81
	40-44	.503	.00462	.25280	.00613	2	81
	45-49	.250	.0396	.13465	.07754	2	81
	50-54	.0145	.000210	.01700	.00021	2	81
	55-59	.0145	.000210	.01700	.00021	2	81
April	35-39	.084	.00070	.07883	.00056	2	93
	40-44	.575	.00119	.24758	(-)	2	93
	45-49	.330	.00040	.22241	(-)	2	93
	50-54	.0115	.00013	.01064	.00015	2	93
May	35-39	.106	.00057	.09497	.00069	3	94
	40-44	.523	.00184	.25011	.00285	3	94
	45-49	.316	.00308	.21161	.00698	3	94
	50-54	.051	.00026	.05042	.00025	3	94
	55-59	.003	.00001	.00355	(-)	3	94
June	35-39	.082	.000316	.07509	.00021	3	102
	40-44	.413	.000677	.24269	(-)	3	102
	45-49	.416	.00657	.23129	.01744	3	102
	50-54	.062	.000685	.05739	.00149	3	102
	55-59	.023	.00020	.02217	.00039	3	102
	60-64	.003	.00001	.00326	(-)	3	102
July	30-34	.002	.000004	.00206	(-)	5	93
	35-39	.03	.000009	.02945	(-)	5	93
	40-44	.26	.00371	.18113	.01659	5	93
	45-49	.51	.00043	.25100	(-)	5	93
	50-54	.17	.00457	.12635	.02148	5	93
	55-59	.03	.00014	.02477	.00044	5	93
August	30-34	.006	.00004	.00647	.00057	3	102
	35-39	.048	.00038	.04588	.000697	3	102
	40-44	.335	.000478	.2249	(-)	3	102
	45-49	.457	.00284	.2457	.00612	3	102
	50-54	.150	.00279	.12339	.00718	3	102
	55-59	.003	.00001	.00108	.000023	3	102

Table 2. (continued)

Month	length group (cm)	$\hat{p}_1$	$\hat{\text{Var}}(\hat{p}_1)$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
September	30-34	.064	.00135	.05710	.00487	3	104
	35-39	.420	.04628	.10675	.18410	3	104
	40-44	.149	.00177	.12272	.00589	3	104
	45-49	.286	.02732	.12233	.10810	3	104
	50-54	.080	.00216	.06717	.00800	3	104
October	30-34	.026	.00066	.02367	.00175	3	105
	35-39	.186	.02636	.09883	.07814	3	105
	40-44	.083	.00029	.07599	.00015	3	105
	45-49	.319	.00571	.20822	.01514	3	105
	50-54	.365	.01150	.21102	.03250	3	105
	55-59	.023	.00007	.02178	.000013	3	105
December	30-34	.040	.00152	.03615	.00257	2	77
	35-39	.46	.07426	.17601	.14623	2	77
	40-44	.29	.01588	.19319	.02924	2	77
	45-49	.13	.01040	.10525	.01943	2	77
	50-54	.08	.00697	.07063	.01302	2	77

Table 3. Statistics from 1973 commercial age samples of large haddock and scrod (combined) taken from Georges Bank. n=number of samples. m=number of fish per sample.

January - March

Age group (yrs)	length group (cm)	$\hat{p}_{a/1}$	$\widehat{Var}(\hat{p}_{a/1})$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
2-3	35-39	1.0	.0			5	4
	40-44	1.0	.0			6	4
	45-49	1.0	.0			7	4
4-5	50-54	1.0	.0			5	2
	55-59	.72	.00400	.25487	(-)	11	4
	60-64	.36	.01948	.10370	.14940	9	4
	65-69	.09	.00157	.08797	(-)	9	5
10-11	55-59	.254	.00365	.24248	(-)	11	4
	60-64	.54	.01966	.12515	.14565	9	4
	65-69	.62	.00779	.22275	.02556	9	5
	70-74	.887	.00174	.11169	(-)	7	5
	75-79	.70	.02193	.14388	.10280	6	5
	80-84	.67	.11111	.0	.33333	3	2
≥12	65-69	.06	.00092	.05555	(-)	9	5
	70-74	.02	.00040	.02007	(-)	7	5
	75-79	.21	.01800	.10528	.08695	6	5
	80-84	.33	.11111	.0	.33333	3	2

April - June

Age group (yrs)	length group (cm)	$\hat{p}_{a/1}$	$\widehat{Var}(\hat{p}_{a/1})$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
2-3	35-39	1.0	.0			7	3
	40-44	1.0	.0			8	8
	50-54	.30	.02589	.03685	.21459	9	2
	45-49	.95	.001126	.04873	.000886	8	6
4-5	45-49	.048	.00113	.04873	.00089	8	6
	50-54	.37	.02607	.29304	.08811	9	2
	55-59	.81	.01166	.07667	.09104	10	3
	60-64	.43	.01244	.12348	.10813	12	3
	65-69	.31	.00980	.15306	.07933	12	4
10-11	50-54	.22	.02160	.0	.19440	9	2
	55-59	.15	.01139	.05000	.09722	10	3
	60-64	.46	.01328	.13576	.11407	12	3
	65-69	.54	.00943	.18976	.06575	12	4
	70-74	.86	.00298	.07150	.02444	13	5
	75-79	.82	.00324	.14382	.01334	13	5
	80-84	.74	.02096	.11881	.08734	7	2
	85-89	.625	.05729	.12500	.16667	4	2
≥12	70-74	.010	.00012	.01081	(-)	13	5
	75-79	.080	.00105	.07607	(-)	13	5
	80-84	.14	.02041	.0	.14287	7	2
	85-89	.375	.05729	.12500	.16667	4	2

Table 3. (continued)

July - September

Age group (yrs)	length group (cm)	$\hat{p}_{a/1}$	$\hat{Var}(\hat{p}_{a/1})$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
2-3	35-39	1.0	.0			1	2
	40-44	.86	.00548	.05372	.04685	11	4
	45-49	.90	.00620	.03032	.06516	11	10
	50-54	.76	.01582	.04667	.14656	10	4
	55-59	.08	.00694	.08313	(-)	3	3
4-5	40-44	.045	.00207	.04545	.01136	11	4
	45-49	.10	.00620	.03032	.06516	11	10
	50-54	.23	.01512	.04778	.13928	10	4
	55-59	.77	.02111	.17750	.00417	3	3
	60-64	.89	.01210	.08291	.01972	3	5
	65-69	.28	.02154	.25511	.00084	3	4
10-11	60-64	.04	.00134	.03671	(-)	3	5
	65-69	.47	.00111	.36667	(-)	3	4
	70-74	.28	.02154	.25511	.00086	3	4
	75-79	.79	.04623	.14298	.02096	2	2
≥ 12							

October - December

Age group (yrs)	length group	$\hat{p}_{a/1}$	$\hat{Var}(\hat{p}_{a/1})$	$\hat{s}_w^2$	$\hat{s}_b^2$	n	m
2-3	40-44	.73	.07111	.06667	.18000	3	2
	45-49	1.0	.0			5	5
	50-54	1.0	.0			7	4
	55-59	.50	.08333	.11083	.19458	3	2
4-5	55-59	.17	.02778	.11083	.02792	3	2
	60-64	.53	.05116	.06633	.22262	5	2
	65-69	.26	.02096	.10095	.11310	7	3
	70-74	.08	.00097	.07564	(-)	7	7
10-11	55-59	.33	.11111	.0	.33333	3	2
	60-64	.47	.05116	.06633	.22262	5	2
	65-69	.61	.01920	.16135	.08061	7	3
	70-74	.72	.00653	.18743	.01890	7	7
	75-79	.75	.00657	.19819	(-)	5	5
	80-84	.50	.08333	.0	.33332	4	2
≥ 12	70-74	.02	.00025	.01573	(-)	7	7
	75-79	.07	.00518	.05069	.01578	5	5
	80-84	.50	.08333	.0	.33332	4	2

Table 4a. Estimated number at age $(\hat{N}_a \times 10^{-2})$ and related statistics for large haddock samples and scrod samples, and total, derived from samples (commercial) of fish taken from the Georges Bank area for January - March, 1973.

	January			February			March		
Ages 2-3	Large		Scrod	Large		Scrod	Large		Scrod
$\hat{N}ax$			366	35		168			539
$\widehat{\mathrm{Var}}(\hat{N}ax)$			1,015	26		4,167			21,185
c.v. $(\hat{N}ax)$			.09	.15		.38			.27
$\hat{N}a$		336			203			539	
c.v. $(\hat{N}a)$		.09			.32			.27	
Ages 4-5									
$\hat{N}ax$	153		2	153		39	31		6
$\widehat{\mathrm{Var}}(\hat{N}ax)$	1,722		5	1,261		811	75		34
c.v. $(\hat{N}ax)$	.27		1.14	.23		.72	.28		1.00
$\hat{N}a$		155			192			37	
c.v. $(\hat{N}a)$		.27			.24			.28	
Ages 10-11									
$\hat{N}ax$	253			281		6	190		2
$\widehat{\mathrm{Var}}(\hat{N}ax)$	2,175			1,437		32	447		7
c.v. $(\hat{N}ax)$	.18			.13		.98	.11		1.08
$\hat{N}a$		253			287			192	
c.v. $(\hat{N}a)$		.18			.13			.11	
Ages $\geq$ 12									
$\hat{N}ax$	21			22			20		
$\widehat{\mathrm{Var}}(\hat{N}ax)$	90			72			80		
c.v. $(\hat{N}ax)$	.46			.39			.45		
$\hat{N}a$		21			22			20	
c.v. $(\hat{N}a)$		.46			.39			.45	
Total number		765			704			788	
Est. no. landed		847			728			830	
Percentage		90%			97%			95%	

Table 4b. Estimated number at age ($\hat{N}_a \times 10^{-2}$) and related statistics for large haddock samples and scrod samples, and total, derived from samples (commercial) of fish taken from the Georges Bank area, April - June, 1973.

	April			May			June		
Ages 2-3	Large		Scrod	Large		Scrod	Large		Scrod
$\hat{N}ax$	2		299	2		567	10		2,310
$\widehat{Var}(\hat{N}ax)$	3		393	1		1,867	42		47,024
c.v. $(\hat{N}ax)$	.85		.07	.47		.08	.71		.09
$\hat{N}a$		301			569			2,320	
c.v. $(\hat{N}a)$		.07			.08			.09	
Ages 4-5									
$\hat{N}ax$	61		9	178		32	257		160
$\widehat{Var}(\hat{N}ax)$	236		30	1,563		178	4,289		3,515
c.v. $(\hat{N}ax)$	.25		.58	.22		.42	.25		.37
$\hat{N}a$		70			210			317	
c.v. $(\hat{N}a)$		.23			.20			.28	
Ages 10-11									
$\hat{N}ax$	302		1	548		10	674		47
$\widehat{Var}(\hat{N}ax)$	1,057		2	3,045		53	17,401		833
c.v. $(\hat{N}ax)$	.11		1.34	.10		.71	.20		.62
$\hat{N}a$		303			558			721	
c.v. $(\hat{N}a)$		.11			.10			.19	
Ages ≥ 12									
$\hat{N}ax$	13			27			26		
$\widehat{Var}(\hat{N}ax)$	28			107			159		
c.v. $(\hat{N}ax)$	.41			.38			.48		
$\hat{N}a$		13			27			26	
c.v. $(\hat{N}a)$		.41			.38			.48	
Total number		687			1,364			3,384	
Est. no. landed		883			1,733			3,629	
Percentage		78%			79%			93%	

Table 4c. Estimated number at age ($\hat{N}_a$ x 10^{-2}) and related statistics for large haddock samples and scrod samples, and total, derived from samples (commercial) of fish taken from the Georges Bank area, July-September, 1973.

	July			August			September		
Ages 2-3	Large		Scrod	Large		Scrod	Large		Scrod
$\hat{N}_{ax}$	80		1,437			1,124			785
$\widehat{\mathrm{Var}}(\hat{N}_{ax})$	2,800		23,719			11,768			58,856
c.v. ($\hat{N}_{ax}$)	.66		.11			.10			.31
$\hat{N}_a$		1,517			1,124			785	
c.v. ($\hat{N}_a$)		.11			.10			.31	
Ages 4-5									
$\hat{N}_{ax}$	200		213			128			49
$\widehat{\mathrm{Var}}(\hat{N}_{ax})$	5,308		7,372			3,492			854
c.v. ($\hat{N}_{ax}$)	.36		.40			.46			.60
$\hat{N}_a$		413			128			49	
c.v. ($\hat{N}_a$)		.27			.46			.60	
Ages 10-11									
$\hat{N}_{ax}$	170								
$\widehat{\mathrm{Var}}(\hat{N}_{ax})$	2,816								
c.v. ($\hat{N}_{ax}$)	.31								
$\hat{N}_a$		170							
c.v. ($\hat{N}_a$)		.31							
Ages $\geq$ 12									
$\hat{N}_{ax}$									
$\widehat{\mathrm{Var}}(\hat{N}_{ax})$									
c.v. ($\hat{N}_{ax}$)									
$\hat{N}_a$									
c.v. ($\hat{N}_a$)									
Total number		2,100			1,252			834	
Est. no. landed		2,243			1,999			1,754	
Percentage		94%			63%			48%	

Table 4d. Estimated number at age ($\hat{N}_a \times 10^{-2}$) and related statistics for large haddock samples and scrod samples, and total, derived from samples (commercial) of fish taken from the Georges Bank area, October - December, 1973.

	October			November			December		
Ages 2-3	Large		Scrod	Large		Scrod	Large		Scrod
$\hat{N}ax$	30		1,011	3			30		45
Var $(\hat{N}ax)$	464		32,024	6			532		2,048
c.v. $(\hat{N}ax)$	.73		.18	.85			.77		1.00
$\hat{N}a$		1,041			3			75	
c.v. $(\hat{N}a)$		.17			.85			.68	
Ages 4-5									
$\hat{N}ax$	45		5	22			49		
Var $(\hat{N}ax)$	518		30	55			436		
c.v. $(\hat{N}ax)$	.51		1.05	.33			.43		
$\hat{N}a$		50			22			49	
c.v. $(\hat{N}a)$		.47			.33			.43	
Ages 10-11									
$\hat{N}ax$	172		10	144			141		
Var $(\hat{N}ax)$	655		119	320			1,473		
c.v. $(\hat{N}ax)$	.15		1.09	.12			.27		
$\hat{N}a$		182			144			141	
c.v. $(\hat{N}a)$		.15			.12			.27	
Ages $\geq$ 12									
$\hat{N}ax$	14			16			10		
Var $(\hat{N}ax)$	57			62			36		
c.v. $(\hat{N}ax)$	.55			.51			.60		
$\hat{N}a$		14			16			10	
c.v. $(\hat{N}a)$		.55			.51			.60	
Total number		1,287			185			275	
Est. no. landed		1,631			304			809	
Percentage		79%			61%			34%	

Table 5a. Coefficient of variation of percent at length of large haddock samples, for $\bar{p}_1 < .10$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2 = .00313$

$\hat{S}_w = .03\overset{8}{\cancel{3}}25$

$\bar{P}_1 = .05$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	1.42	1.00	.82	.71	.64	.50	.45	.41	.37	.32
50	1.25	.88	.72	.62	.56	.44	.40	.36	.32	.28
75	1.21	.85	.70	.60	.54	.43	.38	.35	.31	.27
100	1.18	.84	.68	.59	.53	.42	.37	.34	.31	.26
200	1.15	.82	.67	.58	.52	.41	.36	.33	.30	.26
500	1.13	.80	.65	.57	.51	.40	.36	.33	.29	.25

Table 5b. Coefficient of variation of percent at length of large haddock samples, for $.10 \leq \bar{p}_1 < .20$ n= number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2 = .01250$

$\hat{S}_w = .12524$

$\bar{p}_1 = .15$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	.91	.65	.53	.46	.41	.32	.29	.26	.24	.20
50	.82	.58	.47	.41	.37	.29	.26	.24	.23	.19
75	.79	.56	.46	.40	.35	.28	.25	.23	.22	.18
100	.78	.55	.45	.39	.35	.28	.25	.23	.20	.17
200	.76	.54	.44	.38	.34	.27	.24	.22	.20	.17
500	.75	.53	.44	.38	.34	.27	.24	.22	.19	.17

Table 5c. Coefficient of variation of percent at length of large haddock samples, for $.20 \leq \bar{p}_1 < .30$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2 = .01212$

$\hat{S}_w = .16933$

$\bar{p}_1 = .25$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	.57	.41	.33	.29	.26	.20	.18	.17	.15	.13
50	.50	.35	.29	.25	.22	.18	.16	.14	.13	.11
75	.48	.34	.28	.24	.21	.18	.16	.14	.12	.11
100	.47	.33	.27	.24	.21	.17	.15	.14	.12	.11
200	.46	.32	.27	.23	.21	.16	.15	.13	.12	.10
500	.45	.32	.26	.22	.20	.16	.14	.13	.12	.10

Table 5d. Coefficient of variation of percent at length of large haddock samples, for $\bar{p}_1 = \geq .30$. n = number of samples; m = number of fish per sample.

$\hat{S}_{2b}^2 = .00478$

$\hat{S}_w = .22013$

$\bar{p}_1 = .35$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	.36	.25	.21	.18	.16	.13	.11	.10	.09	.08
50	.27	.19	.16	.14	.12	.10	.09	.08	.07	.06
75	.25	.18	.15	.13	.11	.09	.08	.07	.06	.06
100	.24	.17	.14	.12	.11	.08	.07	.07	.06	.05
200	.22	.16	.13	.11	.10	.08	.07	.06	.05	.05
500	.21	.15	.12	.10	.09	.07	.07	.06	.05	.05

Table 5e. Coefficient of variation of percent at length of scrod samples for $\bar{p}_1 < .10$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2 = .00165$

$\hat{S}_w = .03269$

$\bar{p}_1 = .05$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	1.5	.81	.66	.57	.51	.41	.36	.33	.30	.26
50	.96	.68	.55	.48	.43	.34	.30	.28	.25	.21
75	.91	.65	.53	.46	.41	.32	.29	.27	.24	.20
100	.89	.63	.51	.44	.40	.31	.28	.26	.23	.20
200	.85	.60	.50	.43	.39	.30	.27	.25	.22	.19
500	.83	.59	.48	.41	.37	.29	.26	.24	.21	.19

Table 5f. Coefficient of variation of percent at length of scrod samples for $.10 < \bar{p}_1 < .20$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2 = .02163$

$\hat{S}_w = .10156$

$\bar{p}_1 = .15$

	n									
m	1	2	3	4	5	8	10	12	15	20
20	1.09	.77	.63	.54	.49	.39	.34	.31	.28	.24
50	1.03	.72	.59	.51	.46	.36	.32	.30	.27	.23
75	1.01	.71	.58	.50	.46	.35	.32	.30	.26	.23
100	1.00	.71	.58	.50	.45	.35	.32	.29	.26	.22
200	1.00	.70	.57	.50	.45	.35	.31	.28	.25	.22
500	.99	.70	.57	.49	.44	.35	.31	.28	.25	.22

Table 5g. Coefficient of variation of percent at length of scrod samples for $.20 \leq \bar{p}_1 < .30$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2$ = .04755

$\hat{S}_w$ = .21753

$\bar{p}_1$ = .25

	n									
m	1	2	3	4	5	8	10	12	15	20
20	.97	.68	.56	.48	.43	.34	.31	.28	.25	.22
50	.91	.64	.53	.46	.41	.32	.29	.26	.24	.20
75	.90	.63	.52	.45	.41	.32	.28	.26	.23	.20
100	.89	.63	.51	.45	.40	.32	.28	.26	.23	.20
200	.88	.62	.51	.44	.39	.31	.28	.25	.23	.20
500	.88	.62	.51	.44	.39	.31	.28	.25	.23	.20

Table 5h. Coefficient of variation of percent at length of scrod samples for $\bar{p}_2 \geq .30$. n = number of samples; m = number of fish per sample.

$\hat{S}_{b_2}^2$ = .05648

$\hat{S}_w$ = .24653

$\bar{p}_1$ = .35

	n									
m	1	2	3	4	5	8	10	12	15	20
20	.69	.49	.40	.35	.31	.24	.22	.20	.18	.15
50	.65	.46	.38	.33	.29	.23	.21	.19	.17	.14
75	.64	.45	.38	.32	.29	.23	.21	.19	.17	.14
100	.64	.45	.37	.32	.29	.23	.20	.18	.17	.14
200	.63	.45	.36	.31	.28	.22	.20	.18	.16	.14
500	.63	.44	.36	.31	.28	.22	.20	.18	.16	.14

Table 6a. Coefficient of variation of estimated percent of age at length for haddock samples, for $0 \leq \hat{p}_{a/1} < .30$.

$\hat{S}_b^2 = .05085$

$\hat{S}_w^2 = .08517$

$\hat{p}_{a/1} = .15$

	n									
m	1	2	3	4	5	8	10	12	15	20
2	2.03	1.44	1.20	1.01	.91	.72	.64	.59	.53	.46
5	1.73	1.22	1.00	.87	.78	.61	.55	.50	.45	.39
10	1.62	1.15	.94	.81	.73	.57	.51	.47	.42	.36
50	1.53	1.08	.88	.76	.68	.54	.48	.44	.39	.34

Table 6b. Coefficient of variation of estimated percent of age at length for haddock samples, for $.30 \leq \hat{p}_{a/1} < .70$.

$\hat{S}_b^2 = .10935$

$\hat{S}_w^2 = .09502$

$\hat{p}_{a/1} = .50$

	n									
m	1	2	3	4	5	8	10	12	15	20
2	.79	.56	.46	.40	.35	.28	.25	.23	.20	.18
5	.72	.51	.41	.36	.32	.25	.23	.21	.19	.16
10	.69	.49	.40	.34	.31	.24	.22	.20	.18	.16
50	.67	.47	.39	.33	.30	.24	.21	.19	.17	.15

Table 6c. Coefficient of variation of estimated percent of age at length for haddock samples, for $p_{a/1} \geq .70$.

$\hat{S}_b^2 = .03574$

$\hat{S}_w^2 = .07656$

$\hat{p}_{a/1} = .85$

	n									
m	1	2	3	4	5	8	10	12	15	20
2	.32	.23	.18	.16	.14	.11	.10	.09	.08	.07
5	.27	.19	.15	.13	.12	.09	.08	.08	.07	.06
10	.25	.17	.14	.12	.11	.09	.08	.07	.06	.05
50	.23	.16	.13	.11	.10	.08	.07	.06	.06	.05

Table 7. Examples showing number of samples (n) and number of fish per sample (m) required to satisfy given alpha (a) and beta (b) levels while detecting a difference of ± d * p (See text, p).

Example 1.	$\bar{p}_1$	$\mathrm{Var}(\hat{\bar{p}}_1)$	d(±)	a	(1-b)	n	m
February samples large haddock 60-64 cm.	.215	.001224	.10	.05	.95	>100	100
			.20			36	100
			.50			7	100
			.10	.10	.95	>100	100
			.20			30	100
			.50			6	100
			.10	.05	.90	>100	100
			.20			29	100
			.50			6	100
			.10	.10	.90	90	100
			.20			24	100
			.50			6	100
			.10	.05	.80	84	100
			.20			22	100
			.50			5	100
			.10	.10	.80	66	100
			.20			18	100
			.50			4	100
			.10	.05	.95	> 100	50
			.20			47	50
			.50			9	50
			.10	.10	.95	> 100	50
			.20			39	50
			.50			8	50
			.10	.05	.90	>100	50
			.20			38	50
			.50			8	50
			.10	.10	.90	>100	50
			.20			32	50
			.50			7	50
			.10	.05	.80	>100	50
			.20			29	50
			.50			6	50
			.10	.10	.80	88	50
			.20			23	50
			.50			5	50

Example 2.	$\bar{p}_{a/1}$	$\hat{Var}(\bar{p}_{a/1})$	d±	a	(1-b)	n	m
Ages 4-5 Quarter II 50-54 cm group	.37	.02607	.10	.05	.95	>100	5
			.20			>100	5
			.50			58	5
			.10	.10	.95	>100	5
			.20			>100	5
			.50			48	5
			.10	.05	.90	>100	5
			.20			>100	5
			.50			47	5
			.10	.10	.90	>100	5
			.20			>100	5
			.50			39	5
			.10	.05	.80	>100	5
			.20			>100	5
			.50			36	5
			.10	.10	.80	>100	5
			.20			>100	5
			.50			29	5
			.10	.05	.95	>100	10
			.20			>100	10
			.50			47	10
			.10	.10	.95	>100	10
			.20			>100	10
			.50			37	10
			.10	.05	.90	>100	10
			.20			>100	10
			.50			38	10
			.10	.10	.90	>100	10
			.20			>100	10
			.50			31	10
			.10	.05	.80	>100	10
			.20			>100	10
			.50			29	10
			.10	.10	.80	>100	10
			.20			>100	10
			.50			23	10

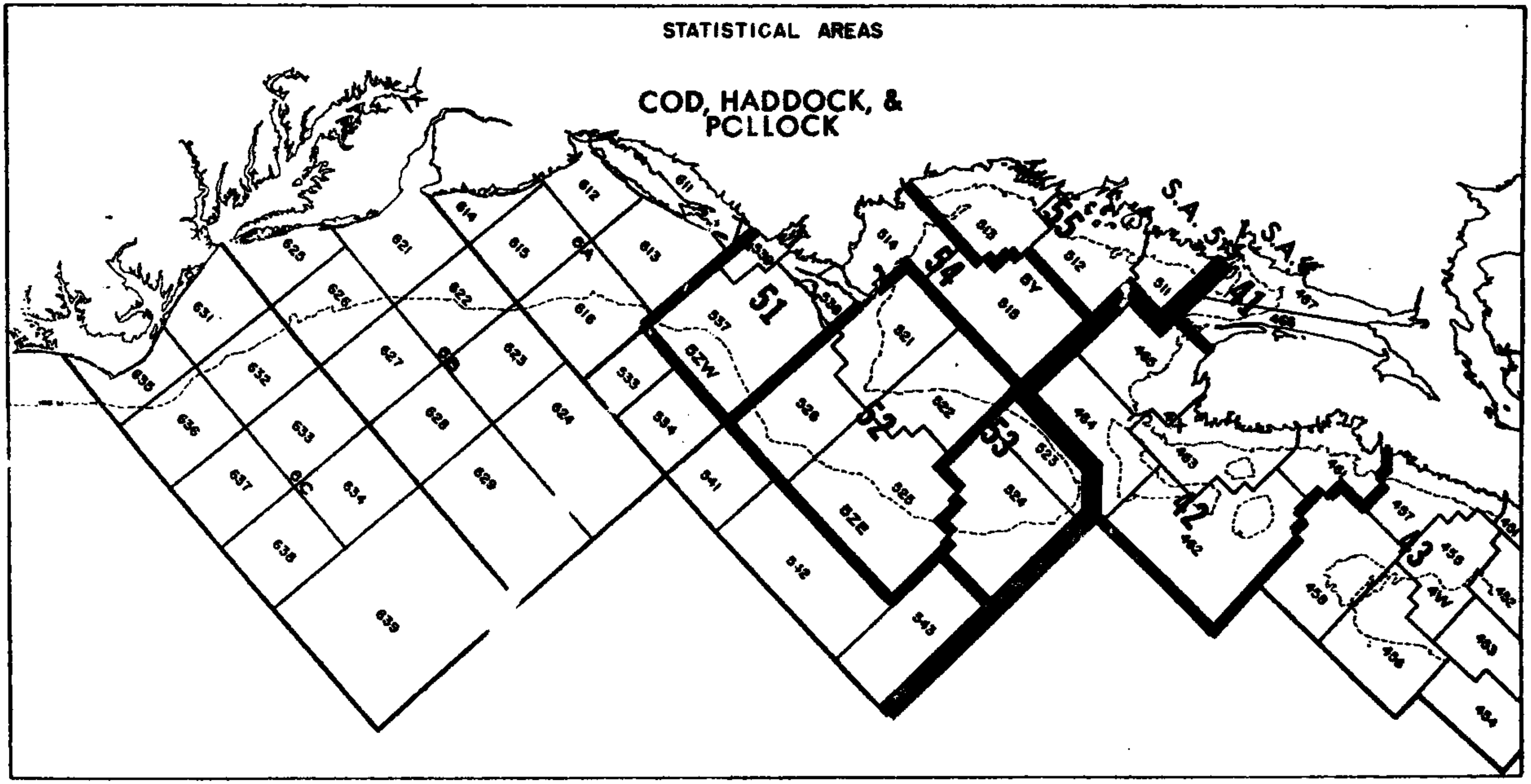

Figure 1. Northwest Atlantic Ocean partitioned into ICNAF areas 5 and 6, and also sampling areas for commercial samples.