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The calculation of numbers caught at age from commercial catch landings.

by

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Introduction:

This submission proposes formulae for estimating numbers of fish caught at age on the basis of length sampling of commercial catches and an age-length key. Observations of length frequencies and age determination are pooled by means of weighted averages over a stratum of catches. The strata are chosen to represent all major sources of variation in catch composition. Sample variances are given as a rough guide to the precision of estimation.

Defining the Population:

Ordinarily, it is the numbers of fish at age caught by a national fleet from a single species stock during a three-month period that is to be estimated. In any case, it is necessary to define precisely the objective of the sampling and then to decide which catches or landings are to be considered. This should be done before sampling begins, so that sampling can be spread over the relevant catches.

Stratification:

Since practical difficulties usually prohibit a rigorous randomization of the choice of catches to be sampled, it is essential to divide the landings into non-overlapping strata representing the major sources of variation in the length composition of catches (such as different types of gear) and to ensure that samples are taken from each stratum. Thus, a stratum will consist of all catches by a particular gear (or perhaps by a particular gear and size of boat). The strata are treated separately and the resulting within stratum estimates of numbers caught are summed.

Notation:

The population is divided into I strata number 1, 2, . . . I and indicated by the subscript $_i$. Stratum $_i$ consists of L_i catches numbered 1, 2, . . . L_i with the subscript $_j$ (of which ℓ_i are included in the sample). The number of fish observed in length class k, for k = 1, 2, . . . K in landing $_{ij}$ is n_{ijk}. The number of fish (of those aged from landing $_{ij}$) that have age a and length class k is m_{ijak}. The total number of ages is A.

The Samples:

From a landing $_{ij}$ a volume v_{ij} of fish taken as a sample, all of which are measured. The total volume of the landing is V_{ij}. (One can use a weight w_{ij} and W_{ij}). If possible, the choice of a volume should be randomized. A stratified sub-sample of one or more fish per length group is aged.

Weighting Factors:

The weighting factor for landing $_{ij}$ is

$$f_{ij} = \frac{V_{ij}}{v_{ij}} \qquad \left(\text{or } \frac{W_{ij}}{w_{ij}} \right)$$

and the weighting factor for stratum $_i$ is

f_i = (total weight of catches on stratum $_i$)/weight of catches sampled in stratum $_i$.

Volumes may be used in place of weights in the calculation of f_i, however weights are more readily available.

The Estimates:

$$n_{ik} = f_i \sum_{j=1}^{\ell_i} f_{ij}\, n_{ijk}$$

n_{ik} estimates the number caught in length class k in stratum i.

$$p_{ika} = \sum_{j=1}^{\ell_i} m_{ijka} \Big/ \sum_{j=1}^{\ell_i} \sum_{a=1}^{A} m_{ijka}$$

p_{ika} estimates the proportion of fish in stratum i and length class k that have age a.

$$n'_{ia} = \sum_{k=1}^{K} n_{ik}\, p_{ika}$$

n'_{ia} estimates the number caught at age a in stratum i.

$$n_a = \sum_{i=1}^{I} n'_{ia}$$

n_a is the estimate of the number of fish with age a in the population.

Note: p_{ika} is the age-length key for stratum $_i$. It is assumed that the true age length keys for all catches within a stratum are identical.

Variance Estimates

The following are sample variances for the estimated quantities -

$$\text{Var}\,(n_{ik}) = f_i^{\,2}\,\ell_i \sum_{j=1}^{\ell_i} \frac{\left(f_{ij}\, n_{ijk} - \left(\sum_{j=1}^{\ell_i} f_{ij} n_{ijk} / \ell_i\right)\right)^2}{\ell_i - 1}$$

$$\text{Var}\,(p_{ika}) = \frac{\ell_i}{\left(\sum_{a=1}^{A} \sum_{j=1}^{\ell_i} m_{ijka}\right)^2} \sum_{j=1}^{\ell_i} \frac{\left(m_{ijka} - \left(\sum_{j=1}^{\ell_i} m_{ijka}/\ell_i\right)\right)^2}{\ell_i - 1}$$

$$\text{Var}\,(n_a) = \sum_{i=1}^{I} \sum_{k=1}^{K} \left(p^2_{ika} \;\; \text{Var}\,(n_{ik}) + n_{ik}^{\,2} \;\; \text{Var}\,(p_{ika})\right)$$

The sample variances are only a rough guide to the true precision of the estimates.

Examples:

The following is an idealized example to illustrate the calculations. There are two gear strata (I=2) and two ages (A = 2).

Catches							
	Gear 1			Gear 2			
Catch number	1	2	3	1	2	3	4
Volume	10^5	10^5	2×10^5	10^5	2×10^5	3×10^5	4×10^5
Weight	10^4	1.2×10^4	1.8×10^4	10^4	1.7×10^4	3.3×10^4	4×10^4
Sampled	Yes	No	Yes	No	Yes	Yes	Yes

One unit of volume was taken from each sampled landing and all fish were measured. One fish in each length group from each sampled catch was aged.

	Catch	Number of fish 10-15 cm, k=1	Age	Number of fish 15-20 cm, k=2	Age
Gear 1	1	10	1	15	2
	3	7	2	9	2
Gear 2	2	8	1	7	1
	3	9	1	5	2
	4	7	1	10	1

$f_{11} = 10^5$

$f_{13} = 2 \times 10^5$

$f_{22} = 2 \times 10^5$

$f_{23} = 3 \times 10^5$

$f_{24} = 4 \times 10^5$

$f_1 = 4 \times 10^4/3 \times 10^4 = 4/3$

$f_2 = 10 \times 10^4/9 \times 10^4 = 10/9$

$$n_{11} = \frac{4}{3}(10^5 \times 10 + 2 \times 10^5 \times 7) = 32 \times 10^5$$

$$n_{12} = \frac{4}{3}(10^5 \times 15 + 2 \times 10^5 \times 9) = 44 \times 10^5$$

$$n_{21} = \frac{10}{9}(2 \times 10^5 \times 8 + 3 \times 10^5 \times 9 + 4 \times 10^5 \times 7) = 79 \times 10^5$$

$$n_{12} = \frac{10}{9}(2 \times 10^5 \times 7 + 3 \times 10^5 \times 5 + 4 \times 10^5 \times 10) = 77 \times 10^5$$

$p_{111} = 1/2$ $\quad p_{112} = 1/2$

$p_{121} = 0$ $\quad p_{122} = 1$

$p_{211} = 1$ $\quad p_{212} = 0$

$p_{221} = 2/3$ $\quad p_{222} = 1/3$

$$n'_{11} = 32 \times 10^5 \times 1/2 + 44 \times 10^5 \times 0 = 16 \times 10^5$$

$$n'_{12} = 32 \times 10^5 \times 1/2 + 44 \times 10^5 \times 1 = 60 \times 10^5$$

$$n'_{21} = 79 \times 10^5 \times 1 + 77 \times 10^5 \times 2/3 = 130 \times 10^5$$

$$n'_{22} = 79 \times 10^5 \times 0 + 77 \times 10^5 \times 1/3 = 26 \times 10^5$$

$n_1 = 146 \times 10^5$

$n_2 = 86 \times 10^5$

$$Var\ (n_{11}) = \left(\frac{4}{3}\right)^2 \ 2\ (\ 8\ ) \times 10^{10}$$
$$= 28 \times 10^{10}$$

$$Var\ (n_{12}) = \left(\frac{4}{3}\right)^2 \ 2\ (4.5) \times 10^{10}$$
$$= 16 \times 10^{10}$$

$$Var\ (n_{21}) = \left(\frac{10}{9}\right)^2 \ 3\ (\ 44\ ) \times 10^{10}$$
$$= 163 \times 10^{10}$$

$$Var\ (n_{22}) = \left(\frac{10}{9}\right)^2 \ 3\ (\ 217\ ) \times 10^{10}$$
$$= 804 \times 10^{10}$$

$$Var\ (p_{111}) = 2\ \left(\frac{0.5}{4}\right)$$
$$= 0.25$$

$Var\ (p_{121}) = 0$

$Var\ (p_{112}) = 0.25$

$Var\ (p_{122}) = 0$

$$Var\ (p_{211}) = \frac{3\ (\ 0\ )}{2 \times 9}$$
$$= 0$$

$Var\ (p_{212}) = 0$

$Var\ (p_{221}) = 0.11$

$Var\ (p_{222}) = 0.11$

$$Var\ (n_1) = 10^{10}\ (28 \times 0.25 + 16 \times 0 + 164 \times 1 + 804 \times 0.11 + 256 \times 0.25 + 360 \times 0 + 16900 \times 0 + 676 \times 0.11)$$
$$= 397 \times 10^{10}$$

$Var\ (n_2) = 518 \times 10^{10}$

Summary:

The estimation formulae given above are valid whenever the age-length relationship within strata (e.g. within gears) is stable. The sample variances are crude approximations; if the choice of sampling were rigorously randomized, the use of more precise estimates of variance would be indicated. No attempt has been made to explain the mathematical derivation of the formulae. The author's intention is to provide a simple and sound guide to the estimation of numbers caught at age.