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ANNUAL MEETING - JUNE 1974

A possible alternative method to virtual population analysis for the calculation of fishing mortality from catch at age data[1]

by

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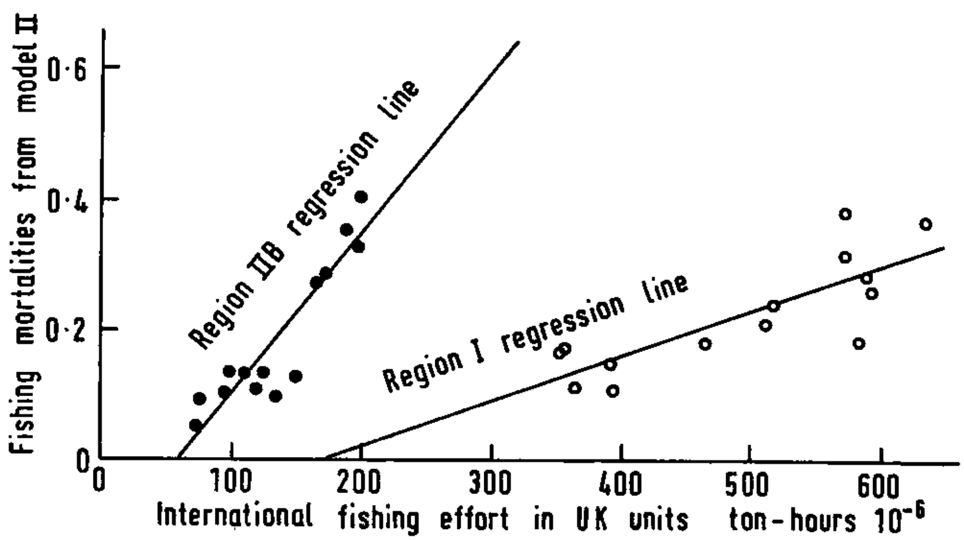

[1] Revision of Res.Doc. 74/20 presented to the Special Commission Meeting, FAO, Rome, January 1974.

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A possible alternative method to virtual population analysis for the calculation of fishing mortality from catch at age data[1]

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1 Introduction

Gulland's virtual population analysis is a method of estimating the fishing mortality experienced by year-classes of fish throughout their life. It has proved one of the most valuable ways of obtaining estimates of this parameter, but without accurate fishing effort data it cannot be used confidently for prediction. This paper suggests a new method developed from an idea of Agger et al. (1971) which may to some extent overcome this difficulty. However, this method is still in the process of development and is consequently described here in order that it may be further investigated by experts.

2 Gulland's virtual population analysis: advantages and disadvantages

Gulland's virtual population analysis (Gulland 1965) enables fisheries biologists to make estimates of population size and fishing mortality (F) at each age independently of measures of fishing effort. Since fishing effort data are often not proportional to F this is a very real advantage and is of great value in elucidating the structure of a fishery. A brief description of the method can be found in Pope (1972). Its chief disadvantage stems from the fact that n + 1 independent estimates (the n + 1 estimates are the population at the age of first capture and the fishing mortality at each of the n ages of the year-class) have to be obtained from n equations (the n equations are based on the catch at age data from the n ages of the year-class). Clearly it is not possible to solve these equations without making some assumptions about at least one of the parameters. Gulland's solution was to estimate or guess the fishing mortality in the most recent year and then to calculate the conse-

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quent populations and fishing mortalities in each of the preceding years. It has been shown by Pope (1972) and by Agger et al. (1971), that this assumption of the fishing mortality F_t (the fishing mortality of the oldest age) causes progressively smaller errors on the estimates of fishing mortality and population size calculated for the younger ages of the year-class. Thus Gulland's solution is extremely valuable in understanding a fishery in a historic sense and for elucidating its population dynamics. However, as the oldest age of each year-class of a fish stock is necessarily the most recent it follows that the estimate of the current population of fish is only as good as the current estimate or guess (usually guess) of the fishing mortality in the last year. While it is true that this assumption may be made more objective by making use of any fishing effort data available, the accuracy of this is rarely sufficient to give a very clear view of the current situation except in those fisheries that have a very constant exploitation rate. The relation of these problems to the setting of catch quotas has been discussed by Pope and Garrod (1973). It is clear that alternative methods of estimating fishing mortality and population size are urgently required.

3 A possible alternative to virtual population analysis

One attempt at developing an alternative to Gulland's method was made by Agger and his colleagues (Agger et al. 1971) who suggested a least squares solution which estimated the values of population at each age, the fishing mortality in each year and the natural mortality from the catch at age/year matrix. The author understands that this attempted to estimate these parameters for the fully-recruited ages only, and under the assumption that the error in catches was approximately normally distributed. Subsequent usage by the ICES North Sea Herring Assessments Working Group revealed problems and the method was rejected by this group. Nevertheless, despite the failure of this particular method the basic idea of using a non-linear least squares estimate of fishing mortality and population is one that is extremely attractive from a statistical standpoint and deserves close attention. It is therefore worth considering what problems were associated with the proposed method and seeing if they could be overcome. For example, consider the catch at age matrix given in Table 1A. If the natural mortality is assumed to be 0.2 the fishing mortalities at each year and age given in Table 1B will, apart from rounding errors, satisfy the catch data, but equally so will the different fishing mortalities given in Table 1C. In fact there are infinitely many possible solutions to this problem, because catch data from the fully-exploited ages of a year-class form a sequence which is identical except for random variations and a constant multiplier to that sequence formed by the adjacent year-classes but with successive terms displaced by one year. This suggests that a least squares solution would not be unique in this case and also that this problem might be overcome by considering ages of fish which are only partly recruited to the fishery.

Another part of the original model which is worth considering is the assumption that errors in catch samples are normal. Gulland (1955) suggests that the best form of age sampling would be one in which the numbers caught at each age had approximately the same coefficient of variation. While practical sampling schemes rarely achieve this ideal nevertheless as an assumption it is probably better than the assumption that each catch has an equivalent variance. This suggests that a method based on a least squares solution of a logarithmic transformation might be more appropriate.

Bearing in mind the points raised in the last two paragraphs a new least squares model can be developed. For example, consider the catch ${}_aC_n$ generated by a fishing fleet on a homogeneous fish stock. Assume that in year n the fleet generates a fully-recruited fishing mortality of F_n and that this is then modified on the various ages by selection (partial recruitment) factors S_a, then fishing mortality at age a in year n will be $F_n.S_a$. If the natural mortality is M the relation of ${}_aC_n$ to ${}_aP_n$, the population of age a, at the beginning of year n, will be:

$$ {}_aC_n.\epsilon = {}_aP_n.F_n.S_a\left(1 - \exp\left\{-(F_n.S_a + M)\right\}\right) / (F_n.S_a + M). \tag{1} $$

ϵ in this equation is the sampling error and is assumed to be log normal. Apart from ϵ, equation 1 is the usual model assumed for most fisheries based on a mixture of partially-and fully-recruited fish. In practice ${}_aC_n$ is the only variable which can be directly measured and the matrix of ${}_aC_n$ for all a and all n forms the input. To stabilise the sampling variance a $\log_e$ (ln) transformation is made which gives

$$ \ln({}_aC_n + \epsilon' = \ln({}_aP_n) + \ln(F_n.S_a) - \ln(F_n.S_a + M) $$
$$ + \ln\left(1 - \exp\left\{-(F_n.S_a + M)\right\}\right), \tag{2} $$

where ϵ' is the transformed error term and is normally distributed $n(0, \sigma^2)$ and M is the instantaneous coefficient of natural mortality. The problem is to find values of ${}_aP_n$, F_n and S_a which satisfy the catch matrix in some sense; it can be simplified by eliminating ${}_aP_n$ by subtracting the next diagonal term from each form of equation 2 (i.e. subtracting equation 2 with a = i + 1, n = j + 1 from equation 2 with a = i, n = j). This yields

$$\ln({}_aC_n/{}_{a+1}.C_{n+1}) = \ln({}_aP_n/{}_{a+1}.P_{n+1}) + \ln(F_n.S_a/F_{n+1}.S_{a+1})$$
$$- \ln\left\{F_n.S_a + M\right\} + \ln\left\{F_{n+1}.S_{a+1} + M\right\}$$
$$+ \ln\left\{1 - \exp\left(-(F_n.S_a + M)\right)\right\}$$
$$- \ln\left\{1 - \exp\left(-(F_{n+1}.S_{a+1} + M)\right)\right\} + \epsilon'' \qquad (3)$$

where ϵ'' is $n(0, 2\sigma^2)$.

Since $\ln({}_aP_n/{}_{a+1}.P_{n+1}) = F_n.S_a + M \qquad (4)$

it follows that equation 3 can be reduced to a form which contains only ${}_aC_n$, F_n, S_a, M and ϵ''.

For a least squares solution the problem then becomes one of finding the values of F_n and S_a which minimize $\left(\ln({}_aC'_n/{}_{a+1}.C'_{n+1}) - \ln({}_aC_n/{}_{a+1}.C_{n+1})\right)^2$, where ${}_aC_n$ refers to observed values of the catch at age data and ${}_aC'_n$ to the values of catch calculated from the values of F_n and S_a by the combination of equations 3 and 4. This is made under the assumption that M is known.

The minimization is achieved by a modified form of the steepest descent method and a computer program listing is given in Appendix A. This minimization works but it could almost certainly be greatly improved to make the iterations converge more rapidly.

4 Testing the model

It should be stressed that this method is only tentatively suggested, although the results obtained so far are encouraging. The most obvious test of this model is to solve F_n and S_a from catch data generated from an imaginary stock with known populations, F_n, S_a and M. If the catch is generated with no sampling error the proposed method, if it is satisfactory, should be expected to output the values of F_n and S_a which were used to generate the catch data. Table 2 shows the results of one such test. Slight differences occur due to rounding errors, but it is clear that the method has effectively reproduced the values of the F_n and S_a.

The next form of test is similar to the last, but with the generated catch data moderated by a series of log normal random numbers. The results from such catch data should not, of course, be expected to reproduce the true values of F_n and S_a, but a series of runs with the same basic data and with differing random numbers should be expected to generate average answers which do not differ systematically from the true ones, i.e. the method should produce unbiassed estimates of F_n and S_a. Tables 3, 4, 5 and 6 show the results of such comparisons made in this form, in which the random numbers were chosen to give the catch data coefficients of variation of 5, 10, 20 and 40%. It was only possible to make 10 runs at each level in the time available, but it can be seen from the tables that the average of those 10 runs ($\bar{F}$) was close to the true value (F_t) and this result is encouraging. The F in the last (10th) year and the F of the last age (8th) seem to deviate most from the true value. The coefficient of variation (Tables 7, 8, 9 and 10) of the fully-recruited fishing mortalities obtained suggests that the accuracy of a realization is related to the accuracy of the catch data, the precision of the final year's fishing mortalities being approximately half that of the catch data. This suggests that the method's use should be restricted to well-sampled catch data.

The value of the sum of squares is presumably an indication of the degree of variability of the catch data. For example, the average least sum of squares obtained from the 10 runs shown in Table 4 was 0.9482. For the array of 8 ages and 10 years there were $(a-1).(n-1) - n - a + 1$ degrees of freedom. That is, 46 degrees of freedom. Hence the means square of residuals was 0.02061, and this divided by 2 (see equation 3) gives an estimate of the variance of $\log_e$ transformed catch data as 0.01030. This would seem consistent with the 0.01 which would be expected from catch data with a 10% coefficient of variation. Thus the least squares function appears to be an indication of variance, but methods of using this in an analysis of variance have still to be developed.

5 Discussion

The method described in this paper was designed to circumvent some of the problems encountered by the pioneering work of Agger and his colleagues (Agger et al. 1971). However, the success of the method must still remain in question until it has been checked on a far greater variety of test data. The purpose of this preliminary report is purely to enable the method to be examined critically by international experts. Clearly there are many areas of the model to be further investigated, and in particular the standard errors of the estimates obtained will be of prime importance if the method is to be used in practice. It is also obvious that solutions to similar but slightly more complicated models will be of considerable interest; for example, solutions to the problems of selection changing with time or of the coefficient of variation changing with age would perhaps be more appropriate for some fisheries. Assuming that the method does in fact give unbiassed estimates it should prove to be very useful, since it will enable catch quotas to be set without reference to fishing effort data. While it is true that any estimates made by this method will have variances associated with them, that is equally true of any other method and the choice of method will depend on the relative sizes of these variances.

6 Summary

This paper describes a new method for calculating from catch at age data the fishing mortality and selectivity experienced by a fish stock, estimated by the method of least squares. Preliminary results using the method are encouraging but extensive testing is required before it is used as a standard technique. A computer program which makes estimates based on the method is listed in the appendix.

7 References

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Table 1 An example of a non-unique solution

Year	Age 1	2	3	4
A Catch data from a homogeneous fish stock with M = 0.2				
1960	154	77	22	52
1961	108	97	48	14
1962	76	63	56	28
1963	61	69	57	51
1964	169	52	58	48
B Possible values of coefficient of fishing mortality				
1960	0.23	0.23	0.23	0.23
1961	0.22	0.22	0.22	0.22
1962	0.20	0.20	0.20	0.20
1963	0.27	0.27	0.27	0.27
1964	0.39	0.39	0.39	0.39
C Other possible values of coefficient of fishing mortality				
1960	0.25	0.25	0.25	0.25
1961	0.25	0.25	0.25	0.25
1962	0.22	0.22	0.22	0.22
1963	0.32	0.32	0.32	0.32
1964	0.49	0.49	0.49	0.49

Table 2 Comparison of the values of fishing mortality and selection coefficients used to generate catches from a simulated population and the corresponding values estimated by the proposed method

FISHING MORTALITY

	Year 1	2	3	4	5	6	7	8	9	10
True value	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
Value estimated by least squares	0.23	0.22	0.19	0.27	0.39	0.43	0.44	0.46	0.48	0.40

SELECTION COEFFICIENTS

	Age 1	2	3	4	5	6	7	8
True value	0.01	0.08	0.41	0.67	1.00	1.00	1.00	1.00
Value estimated by least squares	0.01	0.08	0.41	0.67	1.00	1.00	1.00	1.00

Table 3 Comparison of the average fishing mortality ($\bar{F}$), from 10 runs of the model with catch data having a coefficient of variation of 5%, with the true values of fishing mortality (F_t) for each age/year

Age		Year 1	2	3	4	5	6	7	8	9	10
1	$\bar{F}$	0.002	0.002	0.002	0.003	0.004	0.004	0.004	0.004	0.004	0.003
	F_t	0.002	0.002	0.002	0.003	0.004	0.004	0.005	0.005	0.005	0.004
2	$\bar{F}$	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03
	F_t	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.03
3	$\bar{F}$	0.10	0.10	0.08	0.11	0.16	0.17	0.17	0.17	0.17	0.13
	F_t	0.09	0.09	0.08	0.11	0.16	0.18	0.18	0.19	0.20	0.17
4	$\bar{F}$	0.17	0.16	0.14	0.19	0.27	0.29	0.29	0.29	0.29	0.23
	F_t	0.15	0.15	0.13	0.18	0.26	0.29	0.30	0.31	0.32	0.27
5	$\bar{F}$	0.26	0.24	0.21	0.28	0.41	0.44	0.44	0.44	0.44	0.34
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
6	$\bar{F}$	0.27	0.26	0.22	0.30	0.43	0.47	0.47	0.47	0.47	0.36
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
7	$\bar{F}$	0.29	0.27	0.23	0.32	0.46	0.49	0.49	0.50	0.50	0.38
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
8	$\bar{F}$	0.31	0.30	0.26	0.35	0.50	0.54	0.54	0.54	0.54	0.42
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41

Table 4 Comparison of the average fishing mortality ($\bar{F}$), from 10 runs of the model with catch data having a coefficient of variation of 10%, with the true values of fishing mortality (F_t) for each age/year

Age		Year 1	2	3	4	5	6	7	8	9	10
1	$\bar{F}$	0.002	0.002	0.002	0.003	0.004	0.004	0.004	0.004	0.004	0.003
	F_t	0.002	0.002	0.002	0.003	0.004	0.004	0.005	0.005	0.005	0.004
2	$\bar{F}$	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02
	F_t	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.03
3	$\bar{F}$	0.10	0.09	0.08	0.11	0.16	0.17	0.17	0.17	0.17	0.13
	F_t	0.09	0.09	0.08	0.11	0.16	0.18	0.18	0.19	0.20	0.17
4	$\bar{F}$	0.16	0.16	0.13	0.18	0.26	0.28	0.28	0.28	0.29	0.22
	F_t	0.15	0.15	0.13	0.18	0.26	0.29	0.30	0.31	0.32	0.27
5	$\bar{F}$	0.25	0.24	0.20	0.28	0.40	0.42	0.42	0.43	0.43	0.32
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
6	$\bar{F}$	0.27	0.25	0.22	0.30	0.43	0.46	0.46	0.46	0.46	0.35
	F_t	0.23	.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
7	$\bar{F}$	0.28	0.26	0.23	0.31	0.44	0.47	0.48	0.48	0.48	0.36
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
8	$\bar{F}$	0.30	0.29	0.25	0.34	0.48	0.51	0.51	0.52	0.52	0.39
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41

Table 5 Comparison of the average fishing mortality ($\bar{F}$), from 10 runs of the model with catch data having a coefficient of variation of 20%, with the true values of fishing mortality (F_t) for each age/year

Age		Year									
		1	2	3	4	5	6	7	8	9	10
1	$\bar{F}$	0.002	0.002	0.002	0.002	0.004	0.004	0.004	0.004	0.004	0.003
	F_t	0.002	0.002	0.002	0.003	0.004	0.004	0.005	0.005	0.005	0.004
2	$\bar{F}$	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02
	F_t	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.03
3	$\bar{F}$	0.10	0.09	0.08	0.11	0.15	0.16	0.16	0.16	0.16	0.12
	F_t	0.09	0.09	0.08	0.11	0.16	0.18	0.18	0.19	0.20	0.17
4	$\bar{F}$	0.16	0.15	0.13	0.18	0.26	0.28	0.28	0.27	0.28	0.21
	F_t	0.15	0.15	0.13	0.18	0.26	0.29	0.30	0.31	0.32	0.27
5	$\bar{F}$	0.24	0.23	0.20	0.27	0.39	0.40	0.40	0.40	0.41	0.29
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
6	$\bar{F}$	0.27	0.26	0.22	0.30	0.44	0.46	0.46	0.45	0.45	0.33
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
7	$\bar{F}$	0.29	0.27	0.23	0.32	0.46	0.48	0.47	0.47	0.48	0.34
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
8	$\bar{F}$	0.31	0.30	0.25	0.34	0.50	0.52	0.51	0.50	0.51	0.36
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41

Table 6 Comparison of the average fishing mortality ($\bar{F}$), from 10 runs of the model with catch data having a coefficient of variation of 40%, with the true values of fishing mortality (F_t) for each age/year

Age		Year									
		1	2	3	4	5	6	7	8	9	10
1	$\bar{F}$	0.002	0.002	0.002	0.002	0.004	0.004	0.004	0.004	0.004	0.003
	F_t	0.002	0.002	0.002	0.003	0.004	0.004	0.005	0.005	0.005	0.004
2	$\bar{F}$	0.02	0.02	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.02
	F_t	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.03
3	$\bar{F}$	0.09	0.09	0.08	0.10	0.15	0.15	0.15	0.14	0.15	0.11
	F_t	0.09	0.09	0.08	0.11	0.16	0.18	0.18	0.19	0.20	0.17
4	$\bar{F}$	0.16	0.15	0.13	0.18	0.26	0.26	0.27	0.25	0.28	0.21
	F_t	0.15	0.15	0.13	0.18	0.26	0.29	0.30	0.31	0.32	0.27
5	$\bar{F}$	0.23	0.22	0.18	0.25	0.38	0.37	0.37	0.36	0.39	0.27
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
6	$\bar{F}$	0.29	0.27	0.23	0.31	0.47	0.46	0.46	0.45	0.47	0.32
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
7	$\bar{F}$	0.30	0.28	0.24	0.33	0.51	0.49	0.50	0.48	0.53	0.37
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41
8	$\bar{F}$	0.34	0.32	0.27	0.37	0.56	0.55	0.56	0.53	0.59	0.41
	F_t	0.23	0.22	0.20	0.27	0.39	0.43	0.45	0.46	0.48	0.41

Table 7 Coefficients of variation of fishing mortality obtained from 10 runs of the model with catch data having a coefficient of variation of 5%. Results are for the fully-recruited ages only for each year

Age	Year 1	2	3	4	5	6	7	8	9	10
5	6	4	5	3	4	4	5	6	10	12
6	6	5	6	5	5	5	5	7	9	12
7	8	7	6	5	7	6	6	8	10	12
8	11	10	10	8	9	9	8	8	11	12

Table 8 Coefficients of variation of fishing mortality obtained from 10 runs of the model with catch data having a coefficient of variation of 10%. Results are for the fully-recruited ages only for each year

Age	Year 1	2	3	4	5	6	7	8	9	10
5	7	7	5	6	8	6	7	10	15	18
6	11	11	10	9	10	8	7	12	14	17
7	12	12	10	11	13	10	10	14	17	20
8	17	18	16	16	16	15	13	17	17	18

Table 9 Coefficients of variation of fishing mortality obtained from 10 runs of the model with catch data having a coefficient of variation of 20%. Results are for the fully-recruited ages only for each year

Age	Year 1	2	3	4	5	6	7	8	9	10
5	13	11	5	9	13	11	15	21	32	44
6	18	15	11	11	14	12	11	20	27	37
7	24	21	14	19	21	18	18	23	34	45
8	28	26	19	22	22	22	16	19	28	39

Table 10 Coefficients of variation of fishing mortality obtained from 10 runs of the model with catch data having a coefficient of variation of 40%. Results are for the fully-recruited ages only for each year

Age	Year									
	1	2	3	4	5	6	7	8	9	10
3	11	20	19	14	23	16	27	37	52	74
4	17	19	25	25	31	31	45	48	70	97
5	19	15	12	17	28	20	32	43	60	87
6	24	20	19	18	29	22	25	43	50	70
7	30	26	22	31	44	30	45	55	73	101
8	26	24	22	31	42	29	47	56	76	107

APPENDIX A

Program HPMX

Program HPMX produces a least squares estimate of the fishing mortality and the selection factors from an array of catch at age data at $9 \leq 10$ ages and $n \leq 14$ years. The program iterates towards a solution of

$$\text{Min of } \left\{ \left(\ln \frac{\text{true catch } (i,\ j)}{\text{true catch } (i+1,\ j+1)} \right) - \ln \left(\frac{\text{catch } (i,\ j)}{\text{catch } (i+1,\ j+1)} \text{ predicted by model} \right) \right\}^2 .$$

Since a ln transformation is made of catch data zero catches are unacceptable and should be modified to a small but positive value. Operating experience tends to suggest that this method does not work very satisfactorily if $n \leq 10$ years and the method would probably not be very successful in a developing fishery. In these cases there seems to be a least squares solution which gives an extremely high or extremely low value of F in the final year. In general the method appears to work better the greater number of years that are available. Another prerequisite of successful runs would seem to be a marked change in selection between different ages.

The data inputs required are shown on the next page. The program is written in FORTRAN IV for a Hewlett Packard 2100A computer. The only complication in the program are the exec calls which transfer control from one segment to another. Thus CALL EXEC (8, IJA) transfers control to program HPMXA. Similarly, CALL EXEC (8, IJB) transfers control to program HPMXB and CALL EXEC (8, IJD) to program HPMXD. An annotated output of the program is shown following the program listing.

Data input

The listing given reads data in in free format. This means that numbers in any format can be read in providing they are separated by one or more blank columns. The data to be input are arranged in the following lns for a problem of a ages and n years.

Line no.			
1	a	n	
2	F_1	F_2 ...	F_n (initial estimates of F at age a)
3	S_1	S_2 .	S_a (initial estimates of selection coeff. at each age)
4	M		Natural mortality
5	C_{11}	C_{12}	C_{1a} Catches in 1st year in at each age
6	C_{21}	C_{22}	C_{2a} Catches in 2nd
4 + n	C_{n1}	C_{n2}	C_{na} .... nth ....

5 + n	D	Program step size a value of 0.01 is often satisfactory
6 + n	I_1	If this is set to zero F_1 will be modified; if set to 1 F_1 is held unchanged
	I_2	If this is set to zero F_2 will be modified; if set to 1 F_2 is held unchanged
4 + 2n	I_n	If this is set to zero F_n will be modified; if set to 1 F_n is held unchanged
	J_1	If this is set to zero S_1 will be modified; if set to 1 S_1 is held unchanged
	J_2	If this is set to zero S_2 will be modified; if set to 1 S_2 is held unchanged
3 + 2n + a	J_{a-1}	If this is set to zero S_{a-1} will be modified; if set to 1 S_{a-1} is held unchanged

N.B. S_a is always held constant

```
FTN4
      PROGRAM HPMX
      DIMENSION IJ(2)
      COMMON IJA(3),IJB(3),IJC(3),IJD(3),
     1B(24),F(14),S(10),C(14,10),
     2CL(13,9),ZSQ(10)
     3,AM,IK,INDEX
     4,IAA,IT,BZ(24),D,IND,E
      DATA IJ/2HHP,2HMX/,IA/1HA/,IB/1HB/,IC/1HC/,ID/1HD/
      DO 1 I=1,2
      IJA(I)=IJ(I)
      IJB(I)=IJ(I)
      IJC(I)=IJ(I)
      IJD(I)=IJ(I)
1     CONTINUE
      IJA(3)=IA
      IJB(3)=IB
      IJC(3)=IC
      IJD(3)=ID
      CALL EXEC(8,IJA)
      STOP
      END
      PROGRAM HPMXA,5
      COMMON IJA(3),IJB(3),IJC(3),IJD(3),
     1B(24),F(14),S(10),C(14,10),
     2CL(13,9),ZSQ(10)
     3,AM,IK,INDEX,IAA,IT
     4,BZ(24),D,IND,E
      READ(5,*) IAA,IT
      WRITE(6,7) IAA,IT
7     FORMAT(2I10)
      READ (5,*)(F(I),I=1,IT)
      WRITE(6,8)(F(I),I=1,IT)
8     FORMAT(5F10.4)
      READ(5,*)(S(I),I=1,IAA)
      WRITE(6,8)(S(I),I=1,IAA)
      READ(5,*)AM
      WRITE(6,8) AM
      DO 10 I=1,IT
10    READ(5,*)(C(I,J),J=1,IAA)
      DO 12 I=1,IT
      WRITE(6,14)(C(I,J),J=1,IAA)
12    CONTINUE
14    FORMAT(5F10.1)
      INDEX=1
      IND=100
      READ(5,*)D
      E=1.0
      DO 2001 I=1,IT-1
      DO 2001 J=1,IAA-1
      XX=C(I,J)/C(I+1,J+1)
2001  C(I,J) =ALOG(XX)
      DO 2002 I=1,IT
      READ(5,*)C(I,IAA)
2002  CONTINUE
      DO 2000 J=1,IAA-1
      READ(5,*)C(IT,J)
2000  CONTINUE
      IK=IAA+IT
      DO 15 I=1,IK
15    B(I)=0.0
      CALL EXEC(8,IJB)
      CALL HPMX
```

```
      STOP
      END
      PROGRAM HPMXB,5
      COMMON IJA(3),IJB(3),IJC(3),IJD(3),
     1B(24),F(14),S(10),C(14,10),
     2CL(13,9),ZSQ(10),AM,IK,INDEX
     3,IAA,IT,BZ(24),D,IND,E
      IK=IAA+IT
 98   CONTINUE
      Z=0
      DO 2003 I=1,IK
 2003 BZ(I)=0.0
      DO 2010 I=1,IT-1
      DO 2010 J=1,IAA-1
      FSA=S(J)*F(I)
      FSB=S(J+1)*F(I+1)
      ZA=FSA*1.0+AM
      ZB=FSB+AM
      XX=FSA/FSB
      CL(I,J)=ALOG(XX)+ALOG(ZB)-ALOG(ZA)+ALOG(1.0-EXP(-ZA)
     1)-ALOG(1.0-EXP(-ZB))
     2 +ZA
      Y=C(I,J)-CL(I,J)
      Z=Z+Y*Y
      L=IT+J
      X=S(J)*F(I)*(0.5+ZA/12.0) +1.0
      BZ(I)=BZ(I)+Y*X*F(I)
      BZ(L)=BZ(L)+Y*X*S(J)
      X=S(J+1)*F(I+1)*(0.5-ZB/12.0)-1.0
      BZ(I+1)=BZ(I+1)+Y*X*F(I+1)
      BZ(L+1)=BZ(L+1)+Y*X*S(J+1)
 2010 CONTINUE
      ZSQ(INDEX)=Z
      IF(INDEX.EQ.1) GO TO 53
      Z1=ZSQ(1)
      Z2=ZSQ(INDEX)
      IF(Z1.GE.Z2) GO TO 53
 521  CONTINUE
      DO 522 I=1,IT
      F(I)=F(I)-B(I)
  522 B(I)=B(I)*0.5
      DO 523 J=1,IAA-1
      K=IT+J
      S(J)=S(J)-B(K)
 523  B(K)=B(K)*0.5
      GO TO 54
 53   CONTINUE
      IND=IND+1
      IF(IND.LT.50)GO TO 2019
      CALL EXEC(8,IJD)
 2019 CONTINUE
      ZSQ(1)=ZSQ(INDEX)
      INDEX=1
      DO 2020 I=1,IT
      B(I)=B(I)*E+BZ(I)*D
      IF(C(I,IAA).GE.0.5) B(I)=0.0
 2020 CONTINUE
      DO 2021 J=1,IAA-1
      K=IT+J
      B(K)=B(K)*E+BZ(K)*D
      IF(C(IT,J).GE.0.5) B(K)=0.0
 2021 CONTINUE
      D=D*3.0
 54   INDEX=INDEX+1
      D=D*0.5
      IF(INDEX.GE.10) GO TO 99
```

```
55      PERCY=-0.99
        DØ 2040 I=1,IT
        ERC=B(I)/F(I)
        IF(ERC.LT.PERCY) PERCY=ERC
2040    F(I)=F(I)+B(I)
        DØ 2045 J=1,IAA-1
        K=IT+J
        ERC=B(K)/S(J)
        IF(ERC.LT.PERCY) PERCY=ERC
2045    S(J)=S(J)+B(K)
        IF(PERCY.GE.-1.0) GØ TØ 99
        IF(INDEX.LT.100) GØ TØ 521
        CALL HPMX
99      E=E*0.5
        CALL EXEC(8,IJD)
        STØP
        END
        PRØGRAM HPMXD,5
        CØMMØN IJA(3),IJB(3),IJC(3),IJD(3),
       1B(24),F(14),S(10),C(14,10),
       2CL(13,9),ZSQ(10)
       3,AM,IK,INDEX,IAA,IT,BZ(24),D,IND,E
        WRITE(6,1000) ZSQ(INDEX)
        WRITE(6,1000)(F(I),I=1,IT)
        WRITE(6,1000)(S(I),I=1,IAA)
        WRITE(6,1000)(BZ(I),I=1,IK)
1000    FØRMAT(5F10.4)
        ZSQ(1)=ZSQ(INDEX)
        INDEX=1
        IND=1
        CALL EXEC(8,IJB)
        CALL HPMX
        STØP
        END
        ENDF
```