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A Simple Iterative Solution to the Catch Equation

by

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Abstract: A simple method of estimating stock size at age and fishing mortality rates from catch at age data based on the near linearity in fishing mortality of the square root of the ratio of catch at age n to the surviving number of fish at age n+1 is presented.

Definitions:

C_n — Catch of fish from a year-class at age n

F_n — Instantaneous rate of fishing mortality on the year-class in year n

M — Instantaneous rate of natural mortality

P_n — Stock size in numbers of the year-class at the beginning of year n

exp — Exponential function

Analysis:

The catch equations of Beverton and Holt (1957):

$$C_n = \frac{F_n}{F_n+M}(1- \exp(-F_n -M)P_{n+1}$$

and $P_{n+1} = P_n \exp(-F_n-M)$

allow C_n to be expressed as

$$C_n = \frac{F_n}{F_n+M} (\exp(+F_n +M)-1)P_{n+1} \qquad (1)$$

so that if P_n and C_n are known, F_n may be obtained by solving equation (1)

If a starting value F is assumed for the fishing mortality in the last year of fishing, a population estimate for the final year may be obtained by:

$$P = \frac{F+M}{F} C_{final} \qquad \text{if survival to the next year is negligible}$$

or $$P = \frac{F+M \times C_{final}}{F(1-\exp(-F-M))}$$ if some fish survive.

Figure 1 shows $\sqrt{C_n/P_{n+1}}$ as a function of F_n for differing values of M. Observe that for $F_n > 0.05$, the curves are nearly linear.

Because of the near linearity shown in Figure 1, it is possible to solve equation 1 for F_n if C_n, P_{n+1}, and M are known by taking the square root of both sides and applying successive linear interpolations or extrapolations (method of regula falsi) until F_n is obtained with sufficient accuracy.

When F_n is known, P_n may be obtained as

$$P_n = \frac{F_n+M}{F_n(1-\exp(-F_n-M))} C_n$$

so that it is possible to obtain successively estimates of P_n and F_n for all years beginning with the last.

A program has been written for the Hewlitt-Packard 9821A calculator employing this method. Initial estimates of $F_n = 0.3$ and $F_n = 1.3$ are used for the first interpolation. In successive iterations, the latest estimate of F_n replaces the previous estimate farthest from the new estimate. Iterations continue until $\sqrt{C_n/P_{n+1}}$ is estimated within 10^{-4}. Negative estimates of F_n are replaced by 0.001.

Tests of the program indicate very rapid convergence in estimates of F_n with only three to six iterations required in most cases.

<u>HP 65 Program:</u>

A program has been written for the HP65 programmable pocket calculator implementing the above method. The program is stored on two magnetic cards. The first contains the initialization for a year class. The estimated rates of natural and fishing mortalities and the catch of the oldest fish are input and the population size of oldest fish is output and stored for further analysis. The program on the second card calculates F_n and P_n the rate of fishing mortality in year n and population size of the year class in year n using the catch in year n and the population size in year n+1 for a given year class. The user inputs an underestimate and overestimate of F_n and the program improves these estimates by successive linear interpolations (method of regula falsi) until sufficient accuracy is obtained.

Tests of the program showed that in the range $0.01 \leq M \leq 1$ and $0.001 \leq F \leq 1$ convergence was achieved in about 30 seconds with a maximum error of one part in 10^4 for P_n and F_n if the initial estimates of F_n were within a factor of 10 of the true value.

The program listings follow.

<u>Conclusion:</u>

The new method enables precise solutions of the catch equation to be obtained quickly and easily.

<u>Reference:</u>

Beverton, R. J. H., and S. J. Holt (1957), On the dynamics of exploited fish populations. Fish Invest. Series II, Vo. XIX.

HP-65 Program Form

Title VP Initialize Doubleday's Method Page 1 of 2

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY.

KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	
A	11	
Disp	21	
.	83	
3	03	
STO 1	3301	M
R/S	84	
STO 2	3302	F
R/S	84	
STO 3	3303	C
1	01	
STO 5	3305	
RTN	24	
LBL	23	
B	12	
RCL 2	3402	
RCL 1	3401	
+	61	
CS	42	
f^{-1}	32	
ln	07	
-	51	
g	35	
1/X	04	
STO 5	3305	$(1-e^{-z})^{-1}$
LBL	23	
C	13	
RCL 5	3405	
RCL 3	3403	
X	71	
RCL 2	3402	
÷	81	
RCL 2	3402	
RCL 1	3401	
+	61	
X	71	
STO 3	3303	P
RTN		

REGISTERS

- R_1 M
- R_2 F
- R_3 C,P
- R_4
- R_5 1 $(1-e^{-z})^{-1}$
- R_6
- R_7
- R_8
- R_9

LABELS

- A Init
- B Incom
- C Compl.
- D
- E
- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9

FLAGS

- 1
- 2

9320-0616

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM.

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HP-65 User Instructions

Title V.P. Page 1 of 2

Programmer W. Doubleday Date

VP Set-up: INIT | INCOM | COMPL | |

VP: Run | | | |

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Initialize	M		A	M
		F		R/S	F
		C		R/S	1.000
	either				
2	fishing incomplete			B	P
	or				
2	fishing complete			C	P

HP-65 Program Form

Title Virtual Population: Doubleday's Method — Page 2 of 2

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY.

KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	
A	11	
RCL 6	3406	C_n
RCL 3	3403	P_{n+1}
÷	81	
f	31	
√	09	
STO 7	3307	
RCL 5	3405	
RCL 4	3404	
−	51	
X	71	
RCL 4	3404	
B	12	
RCL 5	3405	New F
X	71	
−	51	
RCL 5	3405	
B	12	
RCL 4	3404	
X	71	
+	61	
RCL 5	3405	
B	12	
RCL 4	3404	
B	12	
−	51	
÷	81	
STO 2	3302	
B	12	
RCL 7	3407	
−	51	
g	35	
ABS	06	
EEX	43	
CS	42	
5	05	
X>Y	3524	Convergence Test
GTO	22	
1	01	
RCL 2	3402	
RCL 4	3404	
−	51	
g	35	
ABS	06	
RCL 2	3402	
RCL 9	3405	
−	51	
g	35	
ABS	06	

KEY ENTRY	CODE SHOWN	COMMENTS
X≤Y	3522	update F
GTO	22	
2	02	
RCL 2	3402	
STO 5	3305	
GTO	22	
A	11	
LBL	23	
2	02	
RCL 2	3402	
STO 4	3304	
GTO	22	
A	11	
LBL	23	$\sqrt{C_n/P_{n+1}}$
B	12	
STO 2	3302	
RCL 1	3401	
+	61	
STO 8	3308	
f-1	32	
ln	07	
1	01	
−	51	
RCL 2	3402	
X	71	
RCL 8	3408	
÷	81	
f	31	
√	09	
RTN	24	
LBL	23	Output
1	01	
RCL 2	3402	
R/S	84	F
RCL 1	3401	
+	61	
ent	41	
CS	42	
f-1	32	
ln	07	
CS	42	
1	01	
+	61	
÷	81	
RCL 2	3402	
÷	81	
RCL 6	3406	
X	71	
STO 3	3303	P_n
RTN	24	

REGISTERS	
R_1	M
R_2	F
R_3	P_{n+1}
R_4	F_1
R_5	F_2
R_6	C_n
R_7	$\sqrt{C_n/P_{n+1}}$
R_8	used
R_9	

LABELS	
A	Run
B	Sub
C	
D	
E	
0	
1	update
2	update
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

HEWLETT PACKARD

9320-0616

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM.

HP-65 User Instructions

Title V.P. Analysis Page ___ of ___

Programmer W. Doubleday Date ___

V.P. Setup: Init | Incom | Comp | |

V.P.: Run | | | |

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Run V.P. Setup				
2.	ent starting values	F_1	STO	4	F_1
		F_2	STO	5	F_2
		C_n	STO	6	C_n
3.	Run			A	F
4.				R/S	P_n
	Go to 2 and repeat for earlier ages				

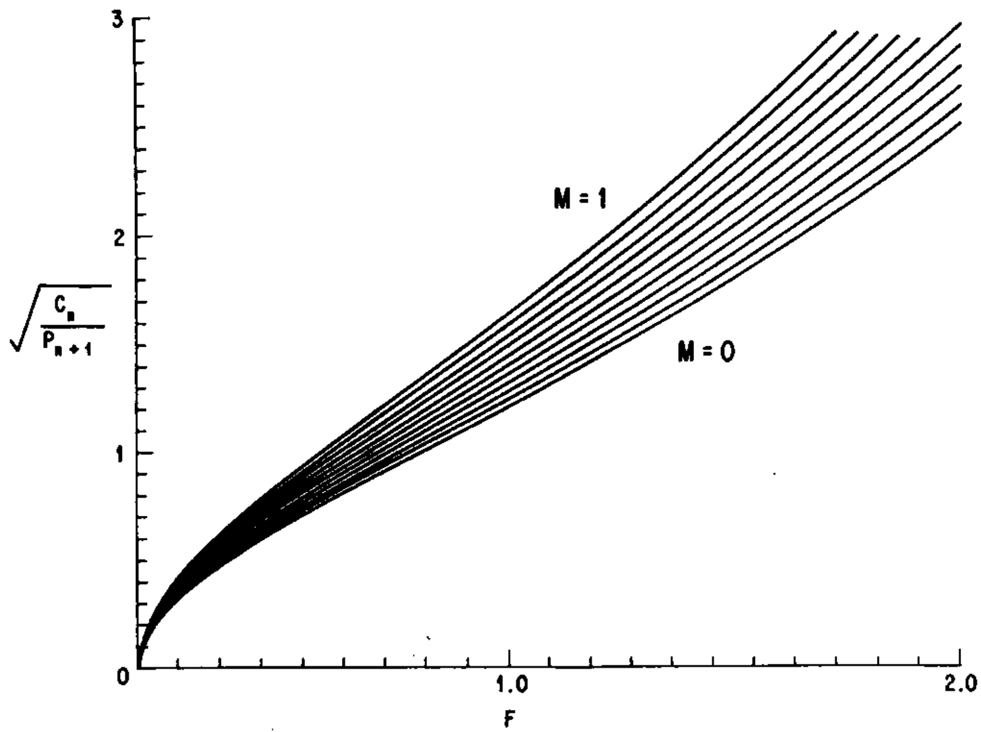

Fig. 1. $\sqrt{\frac{C_n}{P_{n+1}}}$ as a function of F for M between 0 and 1.