



Estimation of Trawl Door Spread from Wing Spread

by

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Introduction

Very often trawl spread is used in the quantification of the fishing effort exerted by a trawl during a tow, expressed either as area of sea bed swept or as volume of water filtered. Usually the spread of the wing tips is used, even though some fish escape over the headline or under the footrope. However, there is definitely a herding action, driving fish into the path of the net from ahead of the ground warps (sweeps or cables) between the doors and the wing tips on either side of the net. The most convincing evidence of this is the fact that at one time commercial practice placed the trawl doors at the wing tips, but now, as a result of increased catches, virtually all trawls are fitted with ground warps. Treschev (1978) recognizes this action by defining the active region of a trawl to include both the fished region in the path of the footrope and the covered region in the path of the ground warps and wing bridles as shown in Fig. 1.

It is technically difficult but possible to measure the spread of the headline wing tips using hydroacoustic instruments. This is usually done during calibration tows because the vulnerable instruments otherwise interfere with shooting and hauling the gear during fishing tows. However, it is very impractical to measure the spread of the trawl doors, even during calibration tows. This was done by Crewe (1964) but the instruments were cumbersome. Any instruments on groundfish trawl doors are subject to very rough treatment, and any data link from the doors to the trawl or to the vessel is very exposed to damage. The alternative is to estimate door spread from measurements taken at the net or at the vessel.

Estimation of door spread from wing spread is basically an exercise in curve fitting. The trawl headline, footrope, wing bridles (legs) and ground warps (sweeps) are flexible members whose shape is governed by the equilibrium of forces on these lines and tensions in these lines. The procedure is to find, deductively, mathematical planforms and profiles for these lines which most closely satisfy these loading conditions. Crewe (1964) reports that a catenary fitted to the centre two-thirds of the headline, with tangential straight lines from this to the doors, fits his experimental measurements adequately. This is probably true for the relatively short wing bridles and ground warps in the UK fleet at that time, but hydrodynamic drag on the lines generates curvature so that Crewe's method results in an overestimate of door spread in Canadian trawls. J.J. Foster (1967) reports that, as a first approximation, the Marine Laboratory (Aberdeen) for simplicity sometimes fits the catenary to the full headline length and extrapolates the linear tangent at the wing tips to the doors. This results in a more realistic, narrower estimate of door spread for long ground warps than does the Crewe method, but it is relatively crude, not accounting specifically for various curvatures resulting from different ground warp lengths, diameters, and tensions.

The hydrodynamics of wire rope has been studied quite extensively, both by the U.S. Navy (Landweber and Protter 1944) and for the Canadian Navy (Eames 1967), particularly in relation to minesweeping gear. Analytical mathematical models of varying complexity have been developed. One of these describes the planform of a towed wire rope, secured at both ends, as a catenary whose parameter is a function of the tension in the line, its diameter and the hydrodynamic pressure. In the analysis of the data from our engineering study of groundfish trawls, Carrothers (unpublished) used this

fact, fitting one wire-rope catenary to the ground warps, other wire-rope catenaries to the upper and lower wing bridles and wing ends of the headline and footrope, and further catenaries to the bights of the headline and footrope. The starboard and port sides of the trawl were treated separately to account for asymmetry of the trawl. This method produced the door-spread estimates given in the seventh column of Table 1. It requires measurements of headline wing spread, wing bridle tensions, hydrodynamic pressure at the trawl and the diameters and lengths of all lines.

The method presented below for estimating door spread from headline wing spread as the only measured dimension is a simplified version of the above method. It has been applied to the trawls in our engineering study with the results given in the sixth column of Table 1 for comparison with the more rigorous method. Also for comparison, the door spreads calculated by means of a trawl warp analysis from measurements taken at the vessel during our engineering study are quoted in the eighth column of Table 1. This simplified method obviously can produce quite accurate results. For the averages quoted in Table 1, only data for hydrodynamic pressures between 25 and 70 pounds per square foot, corresponding to normal towing speeds between 3 and 5 knots, were used.

#### Description of the Method

This simplified method for estimating trawl door spread from headline wing spread first assumes that the trawl is symmetrical in planform so that only half the trawl need be treated. It then fits one wire-rope type catenary to the ground warp, upper wing leg and the forward one-eighth of the headline, and another catenary to the bight of the headline as shown in Fig. 2. The two catenaries are tangential where they touch one another.

Input data required are:

$$H_S = \text{headline wing spread} = 2 Y_W$$

$$H_L = \text{headline length} = 2(S_B + S_W)$$

$$S_L = \text{upper wing leg (bridle) length}$$

$$S_G = \text{ground warp (sweep) length}$$

$$A_W = \text{wire-rope catenary parameter}$$

The headline wing spread needs to be measured, for example by net sounder transducers mounted, facing inward, on the headline wing tips during a calibration tow, as described by Crewe (1964), Carrothers (1968), French (1968), and Acker and Brune (1974). The three line lengths can be taken from the trawl specification. The catenary parameter must be "guessed", but considerations for this are discussed in the next section. As shown in Fig. 3, the door spread estimate fortunately is relatively insensitive to bad "guesses" of the catenary parameter, but it is quite sensitive to errors in wing spread, 5% causing a 7% error in door spread estimate.

The procedure for estimating the spread of the trawl doors consists of the following 8 steps:

1. Calculate the length of the headline bight catenary as

$$S_B = 0.375 H_L$$

2. Calculate the length of the wing end of the headline as

$$S_W = 0.125 H_L$$

3. Calculate the offset of the headline wing tip from the trawl centre-line as

$$Y_W = H_S/2$$

4. Calculate the cotangent of the angle of incidence ( $\alpha$ ) of the headline at the point of contact of the two catenaries ( $C_A$ ) from

$$C_A = \sinh \left[ \left(1 - \frac{S_B}{A_W \cdot C_A}\right) \sinh^{-1} (C_A) + \frac{Y_W}{A_W} \right] - \frac{S_W}{A_W}$$

This can be done iteratively by the subroutine given in Appendix 1.

5. Calculate the distance of the centre-line of the wire-rope catenary from the trawl centre-line from

$$D_Y = \left(A_W - \frac{S_B}{C_A}\right) \cdot \ln(C_A + (C_A^2 + 1)^{\frac{1}{2}})$$

6. Calculate the door offset variable from

$$Z_D = C_A + (S_W + S_L + S_G)/A_W$$

7. Calculate the offset of the door from the wire-rope catenary centre-line from

$$Y_{DL} = A_W \cdot \ln(Z_D + (Z_D^2 + 1)^{\frac{1}{2}})$$

8. Calculate the door spread from

$$D_S = 2(Y_{DL} - D_Y)$$

For those who are interested, the rationale behind these equations is derived in Appendix 2.

#### Guesstimation of the Ground warp Catenary Parameter

This catenary parameter ( $A_W$ ) is a measure of the curvature of the ground warp, a high value representing little curvature (nearly straight) and a low value representing considerable curvature. Increased line tensions tend to straighten the ground warp and increase  $A_W$ , whereas increased drag acting across the line tends to bend the ground warp and decrease  $A_W$ .

Analytically, this parameter is given by:

$$A_W = \frac{T}{C_N \cdot \emptyset \cdot q}$$

where T = tension in the line

$C_N \approx 1.4$  = drag coefficient for wire rope when at right angles to the fluid flow

$\emptyset$  = diameter of the ground warp in the same length unit as for  $A_W$  and q

$q = \rho \frac{V^2}{2}$  = hydrodynamic pressure at the trawl

$\rho$  = mass density of sea water

V = trawl speed through the water

From this equation and as confirmed by experimental evidence in Fig. 4, anything added to the trawl which increases drag, such as a heavier footrope (Yankee trawls), a headline kite (Engel trawl), thicker netting twines or smaller meshes, thus increases the line tension (T) and consequently also increases the catenary parameter ( $A_W$ ). Also, a ground warp which is thinner, vis-à-vis the trawl drag, such as the Engel trawl compared to the Yankee trawls in our engineering study (Fig. 4; Table 1), thus increases the catenary parameter ( $A_W$ ).

The means and standard deviations for the ground-warp catenary parameters ( $A_W$ ) calculated from data measured during our trawl engineering study for hydrodynamic pressures between 25 and 70 lb/ft<sup>2</sup>, corresponding to normal towing speeds between 3 and 5 knots, are presented in the fifth column of Table 1 as a guide. If the specifications for a new trawl are compared with those for the trawls in our

engineering study, an educated guess can be made of the ground-warp catenary parameter. Diameters of ground warps for the trawls in our engineering study are given in the fourth column of Table 1 to facilitate appropriate adjustments to  $A_w$ .

The effect of increasing towing speed is to increase both the trawl drag (and hence also ground-warp tension) and the hydrodynamic pressure at about the same rate so that there is relatively little change in the ground-warp catenary parameter over the normal range of towing speeds. What happens is that, as towing speed increases, the increasing drag of the upper portion of the trawl forces the headline down and back so that the drag of the trawl as a whole increases somewhat more slowly with towing speed than does the hydrodynamic pressure and results in the slight negative regression displayed in Fig. 4.

It is significant that, as shown in Fig. 4, the difference between the ground-warp catenary parameters for the port and starboard sides of the trawl caused by cross-currents is of the same order of magnitude as differences caused by minor trawl appendages or changes in normal towing speed.

Considering that, as shown in Fig. 3, perturbations in  $A_w$  do not seriously affect the door-spread estimate, there should be no major difficulty in intuitively estimating adequately accurate values for the ground-warp catenary parameter ( $A_w$ ).

#### Acknowledgements

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#### Bibliography

- Acker, W.C., and F.A. Brune. 1974. Electroacoustic measurement of trawl parameters. *In Proc. "Ocean 74"*. IEEE publication 74C H0873-0 OCC, Vol. 2, pp. 75-77.
- Carrothers, P.J.G. 1968. Instrumentation for the engineering study of otter trawls. Fisheries Research Board of Canada, Bulletin 163.
- . 1979. The effect of load distribution on the mathematical planforms of trawl frame lines. ICES Document C.M. 1979/B:2.
- Crewe, P.R. 1964. Some of the general engineering principles of trawl gear design *in Modern Fishing Gear of the World 2*, p. 165.
- Eames, M.C. 1967. Steady-state theory of towing cables. Defence Research Establishment Atlantic, Report 67/5.
- Foster, J.J. 1967. Personal communication.
- French, Leon E. 1968. Sonic system for determining distances between selected points of an otter trawl. U.S. Fish and Wildlife Service, Fishery Industrial Research 4(3), p. 113.
- Landweber, L., and M.H. Protter. 1944. The shape and tension of a light, flexible cable in a uniform current. The David W. Taylor Model Basin, Report No. 533.
- Treschev, A.I. 1978. Application of the Fished Volume Method for Measuring Fishing Effort. ICES Cooperative Research Report No. 79.

Table 1. Measured dimensions of groundfish otter trawls and estimated trawl door spreads.

	HS WING SPREAD		HL HEADLINE LENGTH	SL WING BRIDLE LENGTH	SG GROUND WARP		AW CAT.PAR.		DS ESTIMATED DOOR SPREAD	DOOR SPREAD FROM VESSEL DATA		TOW NOS.	DATA POINTS FOR SPREAD		
	M	S			LENGTH	LENGTH	DIAM.	M		S	M			S	
ENGEL	145-F	3.3	96.	164.	120.	.880	1200.	90.	191.	198.	20.	149.	17.	1-2	9.
ENGEL	145-F	3.7	96.	164.	120.	.880	1432.	89.	194.	203.	22.	177.	19.	3	5.
ENGEL	145-F	.8	96.	164.	120.	.880	1209.	171.	205.	213.	5.	158.	8.	4	6.
ENGEL	145-F	1.3	96.	164.	120.	.880	1329.	255.	226.	233.	8.	166.	8.	5	7.
ENGEL	145-F	.8	96.	164.	120.	.880	979.	256.	217.	222.	5.	169.	31.	6-7	16.
WEST-COAST	POLYTHENE,	.4	77.	91.	120.	.750	1041.	168.	177.	183.	2.	209.	16.	8-9	13.
HIGH-LIFT	YANKEE 41,	.3	78.	90.	30.	.875	1021.	204.	127.	129.	1.	124.	11.	10-11	14.
YANKEE	35, POLYTHENE	1.4	52.	30.	90.	.500	619.	83.	100.	100.	7.	102.	6.	13-16	21.
YANKEE	36, POLYTHENE	1.7	60.	30.	120.	.625	647.	44.	131.	131.	9.	121.	10.	17-18	5.
YANKEE	41, POLYTHENE,	.8	79.	31.	180.	.875	779.	116.	171.	173.	4.	178.	10.	19-22	29.
YANKEE	41, POLYTHENE,	1.1	79.	31.	180.	.875	705.	81.	163.	163.	6.	172.	11.	23-26	31.
YANKEE	41, POLYTHENE,	.9	79.	31.	180.	.875	725.	66.	186.	187.	6.	182.	28.	27-28	18.
YANKEE	41, POLY BRAID,	1.1	79.	31.	180.	.875	707.	82.	173.	173.	6.	185.	11.	29-33	35.
YANKEE	41, POLY BRAID,	1.1	79.	31.	120.	.875	775.	157.	153.	153.	5.	160.	6.	35-36	14.
YANKEE	41, POLY BRAID,	.7	79.	7.	138.	.875	760.	213.	151.	147.	3.	173.	5.	37-39	16.
YANKEE	41, TREATED NYLON,	1.2	79.	31.	90.	.875	762.	96.	125.	127.	5.	144.	9.	40-43	21.
YANKEE	41, POLYTHENE,	1.7	79.	31.	180.	.875	731.	69.	200.	200.	11.	185.	22.	44-47	27.
YANKEE	41, POLYTHENE,	2.3	79.	31.	180.	.875	682.	71.	182.	184.	14.	194.	20.	48-51	34.
YANKEE	41, POLYPROPYLENE,	1.7	79.	31.	180.	.875	703.	91.	193.	195.	11.	198.	25.	52-55	28.
YANKEE	41, POLYPROPYLENE,	2.5	79.	31.	180.	.875	675.	91.	185.	187.	16.	195.	29.	57-59	25.
SKAGEN,	POLY BRAID,	1.8	82.	120.	180.	.875	642.	95.	199.	204.	12.	229.	14.	64-68	39.
GRANTON,	POLYTHENE,	1.2	79.	31.	120.	.875	792.	95.	140.	137.	5.	167.	9.	70-71	12.
ATLANTIC WESTERN III,	21-IN ROLLERS,	2.2	79.	91.	180.	.875	987.	247.	162.	165.	14.	141.	21.	72-75	30.

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10)

Note: All spreads and lengths are in feet. Warp diameter is in inches.

M = mean

S = standard deviation

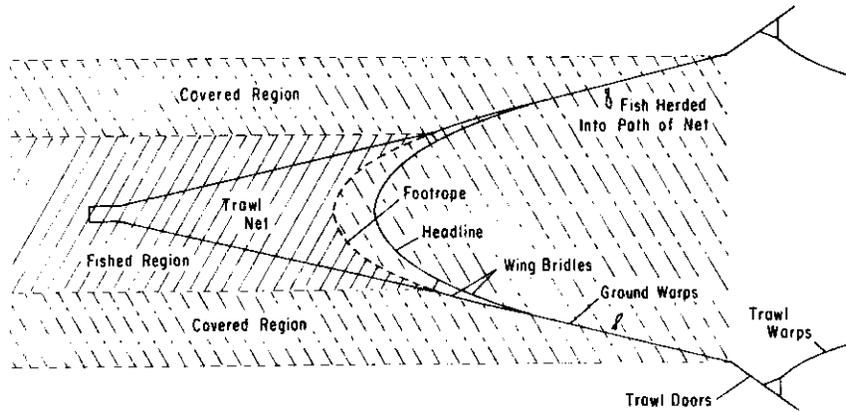


Fig. 1. The Active Region of a Groundfish Otter Trawl.

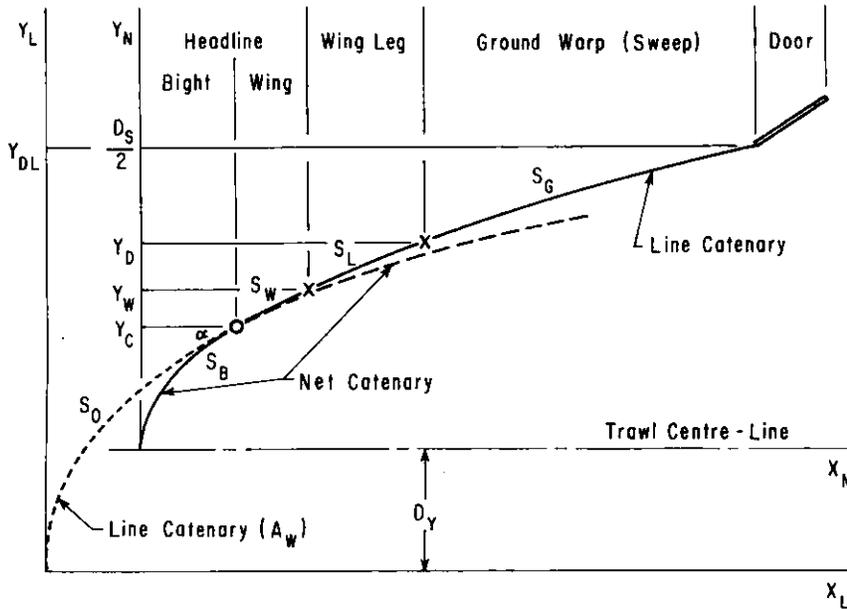
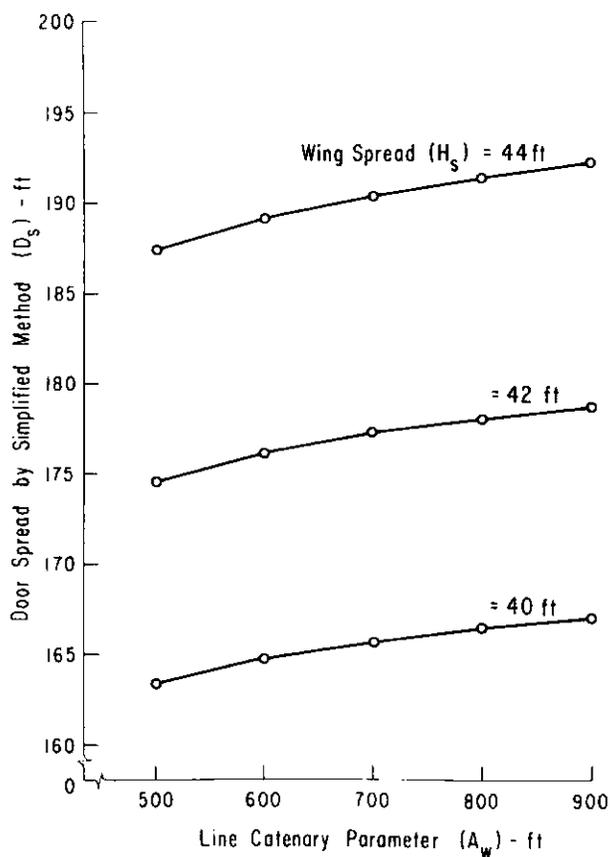


Fig. 2. Trawl Line Planform Geometry.



For Yankee 41 polythene trawl with  
7-in disc footrope  
79-ft headline ( $H_L$ )  
31-ft wing bridles ( $S_L$ )  
180-ft ground warps ( $S_G$ )  
42.5-ft measured wing spread ( $H_S$ )  
77.9-ft measured catenary parameter ( $A_W$ )

Fig. 3. Effect of Errors in Headline Wing Spread ( $H_S$ ) Measurement and Ground Warp Catenary Parameter ( $A_W$ ) Estimate on Door Spread ( $D_S$ ).

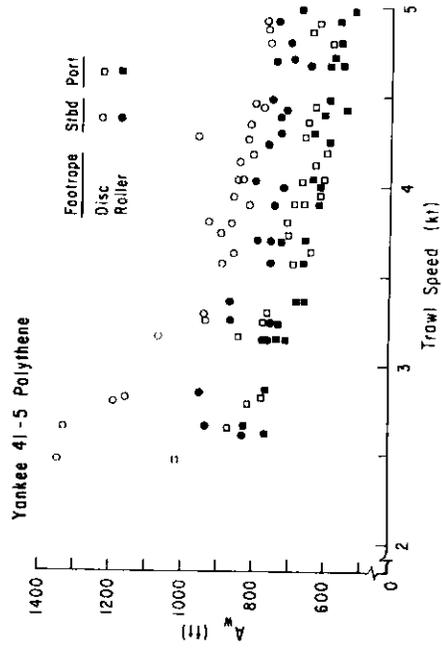
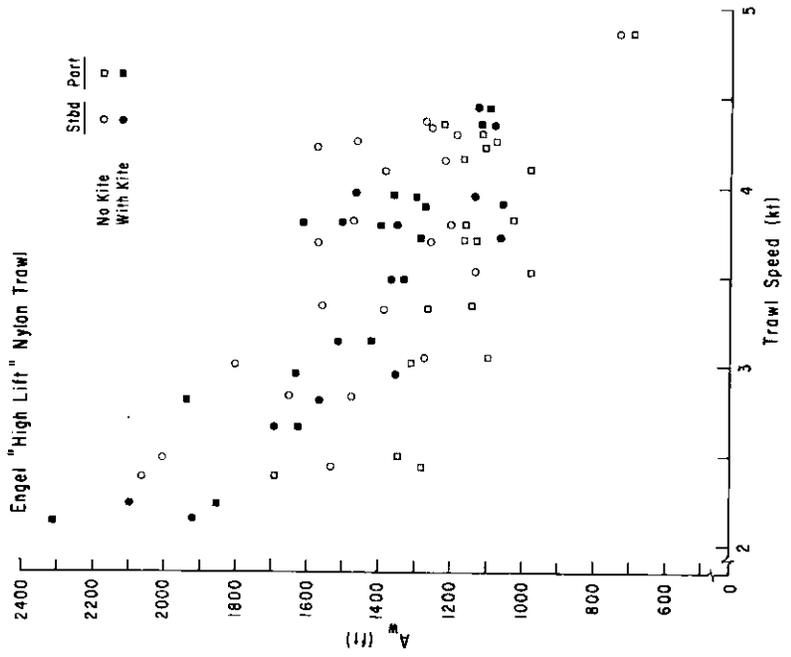


Fig. 4. Experimental Ground Warp Catenary Parameters ( $A_w$ ).

Computer Program for Estimating Door Spread by the Simplified Method

HP32102B.00.10 FORTRAN/3000 (C) HEWLETT-PACKARD CO. 1978 MON, APR 9, 1979

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$CONTROL INIT,LIST
C PROGRAM****TRAWL
C ESTIMATION OF TRAWL DOOR SPREAD FROM TRAWL DATA.
  CHARACTER TOWN*3,TYPE*60
100 FORMAT(6X,A60)
101 FORMAT(6X,2F5.1,3F5.0,F5.3,6F5.0,5X,A5,F4.0)
102 FORMAT(2X,A60)
103 FORMAT("      HS          HL          SL          SG          AW
1 DS          DOOR SPREAD FROM DATA")
104 FORMAT(" WING SPREAD HEADLINE WING GROUND WARP CAT.PAR.
1 ESTIMATED NET DATA VESSEL DATA TOW NOS. POINTS")
105 FORMAT(" M S LENGTH BRIDLE LENGTH DIAM. M S
* DOOR M S M S FOR")
106 FORMAT("          LENGTH          SPREAD /)
107 FORMAT(1X,F5.1,1X,F5.1,3X,2(F5.0,3X),3X,F5.0,1X,F5.3,4X,F5.0,F5
* .0,3X,F5.0,6X,F5.0,F3.0,3X,F5.0,F5.0,6X,A5,2X,F5.0/)
C INPUT TRAWL SPECIFICATION AND WRITE HEADINGS.
  WRITE(6,103)
  WRITE(6,104)
  WRITE(6,105)
  WRITE(6,106)
3 READ(5,100,END=99)TYPE
  WRITE(6,102)TYPE
C READ DATA
1 READ(5,101,END=99)HS,S,HL,SL,SG,DIAM,A2,SA2,DSN,SN,DSV,SV,TOWN,SPR
C CALCULATE LENGTHS OF BOSDM AND WING CATENARIES.
2 SB=0.375*HL
  SW=0.125*HL
C CALCULATE OFFSET OF WING TIP.
  YW=HS/2.
C CALCULATE THE COTANGENT OF ALPHA.
  COTA=CALCA(SB,SW,YW,A2)
C CALCULATE CATENARY CENTER-LINE SEPARATION.
  DELTAY=(A2-SB/COTA)*(ALOG(COTA+SQRT(COTA*COTA+1.0)))
C CALCULATE GROUND WARP FORWARD END OFFSET.
  XX=(COTA+(SW+SL+SG)/A2)
  Y2=A2*(ALOG(XX+SQRT(XX*XX+1.0)))
C CALCULATE DOOR SPREAD.
  SD=2.*(Y2-DELTAY)
C PRINT INPUT AND DOOR SPREAD.
  WRITE(6,107)HS,S,HL,SL,SG,DIAM,A2,SA2,SD,DSN,SN,DSV,SV,TOWN,SPR
C READ ANOTHER TRAWL SPECIFICATION AND DATA
  GO TO 3
99 CONTINUE
  STOP
  END

```

Iterative subroutine for step 4.

HEWLETT-PACKARD 32102B.00.10 FORTRAN/3000 MON, APR 9, 1979, 9:54 AM

```
      FUNCTION CALCA(SB, SW, YWB, AW)
      TCA=0.1
      DTCA=1.0
10  THX=TANH((1.-SB/(AW*TCA))*ALOG(TCA+SQRT(TCA*TCA+1.0)))+YWB/AW)
      FCA=THX/SQRT(1.0-THX*THX)-SW/AW
      IF(FCA-TCA)11,12,12
11  TCA1=TCA
      FCA1=FCA
      TCA=TCA+DTCA
      GO TO 10
12  IF(TCA1-FCA1-0.0005)13,14,14
13  CALCA=TCA1
      RETURN
14  IF(ABS(TCA-FCA)-0.0005)15,16,16
15  CALCA=TCA
      RETURN
16  IF(TCA-FCA)17,17,18
17  TCA2=TCA
      GO TO 19
18  TCA1=TCA
19  TCA=(TCA1+TCA2)/2.
      THX=TANH((1.-SB/(AW*TCA))*ALOG(TCA+SQRT(TCA*TCA+1.0)))+YWB/AW)
      FCA=THX/SQRT(1.-THX*THX)-SW/AW
      GO TO 14
      END
```

Derivation of Equations Used in the Simplified Method for Estimating Door Spread

It can be shown (Carrothers 1979) that the bight of the trawl headline can near enough be represented by a catenary of the form

$$X_N = A_N (\cosh \frac{Y_N}{A_N} - 1) \quad (1)$$

where  $X_N$  = distance ahead of the headline bosom

$Y_N$  = distance to port or starboard from the trawl centre-line

$$A_N = T_o / (C_N \cdot \phi_N \cdot q) \quad (2)$$

$T_o$  = headline tension at the trawl centre-line

$C_N \cdot \phi_N$  = effective hydrodynamic diameter of the loaded headline

$$q = \rho \frac{V^2}{2} = \text{hydrodynamic pressure at the trawl} \quad (3)$$

From the properties of the catenary, the angle of incidence ( $\alpha_N$ ) of the headline to the direction of tow at any point in this bight is given by

$$\cot \alpha_N = \frac{S_N}{A_N} = \sinh \frac{Y_N}{A_N} \quad (4)$$

where  $S_N$  = distance along the headline bight catenary from the trawl centre-line

It can be shown (Carrothers 1979) that the wing of the headline, the upper wing bridle and the ground warp (sweep line) can near enough be represented by a catenary of the form

$$X_L = A_W (\cosh \frac{Y_L}{A_W} - 1) \quad (5)$$

$$\text{where } A_W = T_L / (C_N \cdot \phi \cdot q) \quad (6)$$

$C_N \approx 1.4$  = hydrodynamic drag coefficient for wire rope with axis normal to the fluid flow

$\phi$  = diameter of the ground warp

As in (4), the angle of incidence ( $\alpha_L$ ) of these lines at any point is given by

$$\cot \alpha_L = \frac{S_C}{A_W} = \sinh \frac{Y_L}{A_W} \quad (7)$$

where  $S_C$  = distance along this line catenary from its origin at the intersection of the  $X_L$  and  $Y_L$  axes.

The axes for the line catenary are not coincident with the axes for the headline bight catenary.

As shown in Fig. 2, the principle of the method is to fit these two catenaries (1) and (5) to the known headline length and wing spread, making the two curves tangential at the point of contact, then extrapolating the line catenary (5) along the upper wing leg (bridle) and ground warp (sweep) to the door to get the door spread. For present purposes, the trawl is assumed to be near enough symmetrical about its centre-line, the  $X_N$ -axis of the headline bight catenary (1).

From these geometric constraints and the properties of the catenary, it can be shown (Carrothers 1979) that

$$\cot \alpha = \sinh \left[ \left( 1 - \frac{S_B}{A_W \cot \alpha} \right) \sinh^{-1} (\cot \alpha) + \frac{Y_W}{A_W} \right] - \frac{S_W}{A_W} \quad (8)$$

where  $\alpha$  = angle of incidence of the headline to the direction of tow at the point of contact of the two catenaries

$S_B$  = length of the headline bight catenary (1) from its origin to the point of contact

$S_W$  = length of the line catenary (5) from the point of contact to the wing tip

$Y_W$  = wing-tip offset from the trawl centre-line

This is the equation solved iteratively for  $C_A = \cot \alpha$  in step 4.

In the more rigorous method for estimating door spread from trawl-net data described in the Introduction and used to produce the data in the seventh column of Table 1, two-thirds of the headline and a similar length of the footrope were assigned to the net catenaries, resulting in the estimate of door spread from net data being only about 3% higher overall than that from vessel data. However, a similar proportion in the simplified method results in the door spread estimate being about 10% too high overall. This bias was corrected by assigning three-quarters of the headline to the net catenary. Then,

$$\begin{aligned} S_B &= 0.375 H_L \\ S_W &= 0.125 H_L \end{aligned} \quad (9)$$

where  $H_L$  = known headline length

Also, as the trawl is considered near enough symmetrical

$$Y_W = H_S/2 \quad (10)$$

where  $H_S$  = measured headline wing-tip spread

These are the equations used for steps 1, 2 and 3.

For the headline bight catenary at the point of contact between the two catenaries,  $\cot \alpha_N = \cot \alpha$  as found by (8),  $S_N = S_B$  and  $Y_N = Y_C$ .

Then equation (4) gives

$$\begin{aligned} A_N &= S_B / \cot \alpha \\ Y_C &= A_N \cdot \sinh^{-1} (\cot \alpha) \\ &= \frac{S_B \cdot \sinh^{-1} (\cot \alpha)}{\cot \alpha} \end{aligned} \quad (11)$$

And for the line catenary at the point of contact  $\cot \alpha_L = \cot \alpha$  as found by (8), whence (7) gives

$$Y_C + D_Y = A_W \cdot \sinh^{-1} (\cot \alpha) \quad (12)$$

From (11) and (12)

$$D_Y = (A_W - \frac{S_B}{\cot \alpha}) \sinh^{-1} (\cot \alpha) \quad (13)$$

which is the equation used for step 6, given the identity

$$\sinh^{-1}(C_A) = \ln(C_A + (C_A^2 + 1)^{\frac{1}{2}}) \quad (14)$$

Extrapolating the line catenary from the point of contact of the two catenaries to the forward end of the ground warp, at the doors  $Y_L = Y_{DL}$  and  $S_C = S_0 + S_W + S_L + S_G$  so that (7) gives

$$Y_{DL} = A_W \cdot \sinh^{-1} \left( \frac{S_0 + S_W + S_L + S_G}{A_W} \right) \quad (15)$$

But at the point of contact of the two catenaries,  $\alpha_L = \alpha$  and  $S_C = S_0$  so that (7) gives

$$S_0 = A_W \cdot \cot \alpha \quad (16)$$

Substituting (16) into (15)

$$Y_{DL} = A_W \cdot \sinh^{-1} \left( \cot \alpha + \frac{S_W + S_L + S_G}{A_W} \right) \quad (17)$$

Setting

$$Z_D = \cot \alpha + \frac{S_W + S_L + S_G}{A_W} \quad (18)$$

to simplify manipulation, (17) and (18) and the identity represented by (14) give the equations used in steps 6 and 7.

The geometry presented in Fig. 2 says that

$$\frac{D_S}{2} = Y_{DL} - D_Y \quad (19)$$

A simple transposition gives the equation used in step 8.

