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Estimation Using Research Survey Data and Commercial Catch Data

by

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SPECIAL SESSION ON BIOLOGICAL SURVEYS

The problem of combining estimates of abundance from different sources (commercial catch per unit effort, cohort analysis, research vessel surveys) is analysed using structural and functional maximum-likelihood models. The functional formulation requires more information than is generally available and cannot be widely applied to fisheries data. However, many sets of abundance data can be suitably transformed to meet the assumptions of the structural model. The structural model is a generalization of confirmatory factors analysis. These methods allow the history of a population to be reconstructed from a time-series of measurements using different estimators, as well as obtaining a better estimate of the abundance at any time. Estimates of the precision of the various abundance estimators can also be calculated. The method is very general and can be applied to a wide range of data. An example for a Newfoundland Atlantic salmon population is presented.

Introduction

Fisheries scientists continually encounter the problem of measurement error in the collection and analysis of assessment data. One of the goals of most assessments is to determine the abundance of a given stock or part of a stock such as the recruits. However, we cannot directly observe the true abundance of a stock. Rather, we must rely on indirect methods such as those described by Collie and Sissenwine (1983) for utilizing research vessel survey data, or cohort analysis using commercial catch data (Pope 1972). Usually, these methods yield estimates or indices which are related to the true abundance, but there is error in the relationships. As noted by Fournier and Archibald (1982) and Collie and Sissenwine (1983), early methods did not specify observational or measurement error in the relationship between the abundance index and the true abundance.

Because observational error is likely to occur with any particular method for describing stock abundance, we consider estimates from methods such as cohort analysis and research vessel surveys to be indicators of abundance. Often, several abundance indicators may be available for a given stock. This paper describes methods for combining the information on stock abundance contained in the available indicators. We obtain estimates of the measurement error variance associated with each indicator, the relationship of the indicator to the true abundance, and an overall estimate of abundance using several sources of information. This problem can be viewed as comparative calibration (Theobald and Mallinson 1978). That is, we wish to intercalibrate different methods of measuring abundance in cases where the true abundance is unknown. We describe two alternative maximum-likelihood formulations of the problem. An example of the use of the methods on a Newfoundland population of Atlantic salmon and recommendations for their application are discussed.

Model Formulation

We consider the case in which the true abundance in year i, x_i, is measured by p "indicators" that are linearly related to the true abundance. Possible indicators may be results from research vessel cruises, sequential population analysis (SPA), or commercial catch per unit effort. Note that estimates of relative abundance, such as catch per unit effort (CPUE), are considered indicators of abundance in this terminology. The model we consider is

$$y_{ij} = \lambda_j x_i + e_{ij} + \mu_j \qquad \text{where } i = 1, 2, \ldots, N \quad \text{(year)} \qquad j = 1, 2, \ldots, p \tag{1a}$$

or in vector notation

$$\underline{y}_i = \underline{\Lambda} x_i + \underline{e}_i + \underline{\mu}, \tag{1b}$$

where $\underline{y}_i$, $\underline{\Lambda}$, $\underline{e}_i$, and $\underline{\mu}$ are column vectors with p components, comprising elements y_{ij}, λ_j, e_{ij}, and μ_j ($j = 1 \ldots P$), respectively. Here the y_{ij} are the observed indicators of abundance, e_{ij} is interpreted as measurement error, μ_j and λ_j are intercept and slope parameters to be estimated. We assume that for each i ($i = 1, \ldots N$) $\underline{e}_i$ is distributed independently of x_i, with mean zero, and that the covariance matrix of the measurement errors, $\underline{\Psi}^2$, is diagonal (unless otherwise stated) with diagonal elements given by $\psi_{jj}{}^2$, which will be the same for each year. The measurement errors, e_{ij}, are assumed to be normally distributed. If the measurement errors are lognormally distributed (Peterman 1981), then the analysis would proceed using the natural logarithm of the indicators. However, in this case the model would be multiplicative.

It is necessary to specify the units of true abundance in model (1) by scaling x_i in terms of the indicators. This is accomplished by fixing one of the λ parameters equal to one. Preferably the units of x_i should be absolute (e.g. SPA) rather than relative (e.g. CPUE) abundance.

The Functional Case

Consider the case where the true abundances are fixed values to be estimated. Equation (1) is termed a functional relationship in this case (Kendall and Stuart 1973). The likelihood function is

$$L = \frac{1}{(2\pi)^{pN/2} \quad \psi^N} \quad \exp \quad -\tfrac{1}{2} \sum_i (\underline{y}_i - \underline{\Lambda}\, x_i - \underline{\mu})' \;\psi^{-2}(\underline{y}_i - \underline{\Lambda} x_i - \underline{\mu}), \qquad (2)$$

Note that all the x_i are parameters to be estimated. However, Anderson and Rubin (1956) have shown that this likelihood fluction does not yield valid estimates without additional information being incorporated into the model because it does not have a unique maximum. To see this, consider the simple case where for one particular j, $\psi_{jj} = 0$, $\mu_j = 0$, and $y_{ij} = \lambda_j x_i$ for all i. In other words, where all the information is contained in one of the indicators and so its measurement error approaches zero. The likelihood function (2) goes to infinity as the estimate of ψ_{jj} goes to zero. Thus, (2) has multiple unbounded "peaks". While this may seem a trivial case, it applies in situations where one indicator is very much better than the others. Ideally, this should be known in advance. Often the investigator does have a substantial amount of information on the values of the slopes λ_j and variances ψ_{jj}, e.g. from replicate observations. The model may still be useful for purposes of calibrating the poorer indicators to the best estimate of abundance. However, the measurement error of the best indicator may need to be fixed at some small value for the analyses to proceed, i.e. more information must be included in the model.

There is a second difficulty with the model as presently formulated. The property of consistency (that is as the sample size increases the estimator converges in probability to the true value) is no longer assured if the number of parameters to be estimated increases to infinity as the number of samples increases to infinity (Neyman and Scott 1948). In the case of (1), if $\lambda_j = 1$ for all j, and $\psi_{jj}{}^2 = \sigma^2$ for all j, the maximum likelihood estimator exists, but underestimates the true σ by $\frac{p}{p-1}$ for any sample size. Models such as that proposed by Ludwig and Walters (1981) must be treated carefully to avoid bias even for large samples. Methods with this difficulty (e.g. Ludwig and Walters 1981) can thus yield misleading results.

Well-behaved likelihood functions that yield consistent estimators are possible if there is more information available than was used in (2). There are two possible sources of extra information: (i) information may be available on the distribution of the x_i's, and (ii) information may be available on the parameters of $\underline{\Psi}$.

There are two approaches that incorporate additional information and lead to consistent estimators in the functional case (2). First, prior information on the measurement error variances of the indicators can be included, and the analysis proceeds using Bayesian methods (Lindley and El-Sayyad 1968). Second, we can use the observation of Anderson and Rubin (1956) that the sample means and dispersion matrix are sufficient statistics in the structural model, i.e. where the x_i are considered realizations of random variables (see below) and probability limits are of the same form in both models. That is, for more than two indicators, the parameters in the structural case can replace their corresponding parameters in the functional case, and the individual abundances can then be estimated. Although the functional model can be used if empirically derived prior distributions for the error variances can be obtained, this is rarely true for all indicators. Furthermore, the computational burden in the functional case can be overwhelming for even short time series of data. We shall present an alternative that sometimes overcomes the difficulties with the functional model.

Structural Case

An alternative formulation of the problem is to consider the true abundances as random variables such that (1) is a structural relationship (Kendall and Stuart 1973). In this case there is additional structure imposed on the model, i.e. the form of the probability density function of the true abundance. This adds an additional assumption to the model. It is an assumption that can, and should, be checked for agreement with the data collected.

In overview, if the estimation of abundance is treated as a structural problem, then the estimation procedure is a two-stage process. First, the model parameters, $\underline{\mu}$ and $\underline{\lambda}$, are estimated and then the individual abundances, the x_i's, are estimated utilizing these estimates of the structural parameters. Since the true abundance does not fully account for the total variance of the indicators of abundance, the true abundance cannot be estimated in the usual statistical sense. A minimum variance or least-squares principle must be invoked to obtain reasonable estimates (Anderson and Rubin 1956).

Specifically, we now assume that x_i is a realization of normal, independently distributed random variable, with mean 0. We now rewrite (1) as

$$\underline{y}_i = \underline{\Lambda} x_i + \underline{e}_i. \tag{3}$$

Model (3) corresponds to the model used in the well-known statistical method of factor analysis with a single unobserved factor (Lawley and Maxwell 1977). Note that although the population mean can be estimated using the sample mean, (3) is useful only in those cases where deviations from the population mean is of interest. Intuitively, factor analysis hypothesizes that the covariance between the observed variables results from some underlying common factor. Each observed variable is then an indicator of this common factor and each also contains a unique factor, e_{ij}. For the problem considered here, we assume that the underlying factor responsible for the covariance of the abundance indicators is the true abundance, and the e_{ij} can be interpreted as measurement or observational error.

If there is more than one age class that contributes significantly to the true abundance, then the abundance at time i will no longer be uncorrelated with the abundance at time i+1. An approximation for many populations is to replace the assumption that true abundances are independently distributed random variables with the assumption that

$$x_{i+1} = \gamma x_i + \zeta \tag{4}$$

where γ is a new structural parameter (interpretable as a constant survival probability) to be estimated and ζ are independent normal random variables (interpretable as a recruitment).

Other models may provide better representations for the stochastic variations in fish abundance. For example, two time lags may be preferable than the one used in (4), or if there is a linear trend to the data it can be removed by assuming the true abundance has the form

$$x_i = \alpha + \beta i + \zeta \tag{5}$$

where ζ is an independent normal random variable as before, and α and β are parameters to be estimated. This is preferable to detrending each of the abundance indicators because fewer parameters are estimated. Similarly, it may be necessary to assume that the technological efficiency of a fleet increases with time (Pope and Shepherd 1983).

The structural parameter estimates can be obtained from the likelihood function if the model is identified (see below). The actual computation may be more complex with certain models, eg. a model assuming (5) would use a covariance matrix $\underline{\Sigma}$ of dimensions 2p x 2p.

Estimation of Structural Parameters

For the parameters to be estimated there must be sufficient information so that the model is identified. That is, it must be possible to deduce uniquely the values of the structural parameters from the observed covariance between the indicators. The conditions for identification will not be given here in detail (see Judge et al. 1980 or Anderson and Rubin 1956). However, there are some simple rules that are useful. If only two indicators are available, the system is not identified unless subsidiary information is available such as an estimate of one of the measurement error variances. Such estimates may be obtained, for example, from replicate samples in research surveys. If three indicators are available, the system is identified unless there are correlated residuals. With four or more indicators, the system is overidentified and the hypothesis of correlated residuals can be tested. In general, it is not possible to estimate more parameters than there are unique elements in the covariance matrix of the indicators, $\underline{\Sigma}$. However, the model is not necessarily identified, even if the number of unique elements in $\underline{\Sigma}$, $\frac{p(p+1)}{2}$, is greater than or equal to the number of parameters to be estimated.

We proceed to estimate the parameters of $\underline{\Lambda}$ and $\underline{\Psi}$. The covariance matrix of the indicators is defined as the expectation $E(\underline{y}\underline{y}')$ and denoted by $\underline{\Sigma}$. It is written in terms of the model parameters $\underline{\Lambda}$ and $\underline{\Psi}$. (Since the sample mean is a sufficient statistic for the population mean we do not discuss its estimation.) Given the assumptions of the model we have

$$\underline{\Sigma} = \underline{\Lambda}\sigma_*^2\,\underline{\Lambda}' + \underline{\Psi}^2\ , \tag{6}$$

where σ_*^2 is the variance in the true abundance (Lawley and Maxwell 1971). The sample covariance matrix $\underline{S}$ based on N observations is defined by

$$\underline{S} = (1/n)\sum_i (\underline{y}_i-\bar{y})(\underline{y}_i-\bar{y})', \tag{7}$$

where $n = N-1$ and $\bar{y}$ is the sample mean of the vector of observed indicators. The $\log_e$ likelihood function is

$$\log_e L = -\tfrac{1}{2}\,[pN\,\log_e 2\pi - N\,\log_e|\underline{\Sigma}| - (y'\,\underline{\Sigma}^{-1}\,y)]. \tag{8}$$

Maximizing $\log_e L$ is equivalent to minimizing

$$F = \log_e|\underline{\Sigma}| + \mathrm{tr}(\underline{S}\underline{\Sigma}^{-1})$$

(Jöreskog 1973). The likelihood function is maximized over the elements of $\underline{\Sigma}$, i.e. the elements of $\underline{\Lambda}$ and $\underline{\Psi}$. The maximization of the likelihood function is described in Lawley and Maxwell (1971) and Jöreskog and Sörbom (1983).

Other formulations for the estimation of over-identified models are possible, such as an ordinary least squares or generalized least squares minimization criterion. An advantage of the maximum likelihood approach is that standard errors for the parameter estimates can be obtained and the assumptions of the model, e.g. independence of measurement errors, can be tested using likelihood ratio techniques or the χ^2 goodness of fit statistic (Jöreskog and Sörbom 1983).

Often one or more of the measurement error variances can be estimated independently of the above model. For example, the estimates of the measurement error variance for commercial catch-effort data can often be calculated if the different gear types and seasonal effects are combined via a multiplicative model with lognormal error (Gavaris 1980). Similarly, estimates of measurement error variances are often available for research vessel survey cruises. In either case the above likelihood function can be estimated with one or more of these model parameters held constant (Jöreskog 1973).

A useful special case occurs when the structural model is "just identified", i.e. contains just sufficient information to uniquely estimate each parameter. For example, consider (3) for a three-indicator model in which the true abundance is an independently distributed normal random variable with mean 0 and variance σ_*^2. In this case the maximum-likelihood estimate of $\underline{\Sigma}$ is simply $\underline{S}$, and is given by

$$\begin{matrix} \sigma_*^2 + \psi_{11}^2 & \lambda_2\sigma_*^2 & \lambda_3\sigma_*^2 \\ \cdot & \lambda_2^2\sigma_*^2 + \psi_{22}^2 & \lambda_2\lambda_3\sigma_*^2 \\ \cdot & \cdot & \lambda_3^2\sigma_*^2 + \psi_{33}^2. \end{matrix}$$

Since $\underline{\Sigma}$ is symmetric only the upper triangular elements are given.

If $\underline{S}$ is equated with the above $\underline{\Sigma}$ we have 6 equations with 6 unknowns. The estimates are

$$\begin{aligned} \hat{\lambda}_2 &= s_{12}/\sigma_*^2 \\ \hat{\lambda}_3 &= s_{13}/\sigma_*^2 \\ \hat{\psi}_{11}^2 &= s_{11} - \sigma_*^2 \\ \hat{\psi}_{22}^2 &= s_{22} - \lambda_2^2\sigma_*^2 \\ \hat{\psi}_{33}^2 &= s_{33} - \lambda_3^2\sigma_*^2 \\ \hat{\sigma}_*^2 &= s_{13}s_{12}/s_{23}. \end{aligned} \tag{9}$$

Barnett (1969) derives the asymptotic variances for the above estimators. Alternatively, bootstrap confidence intervals can be constructed (Effron 1979).

Estimating Individual Abundances

If abundance estimation is treated as a structural model then the individual true abundances are not parameters in the usual statistical sense but rather values ascribed to unobservable random variables. In factor analysis this problem is known as estimation of factor scores (Lawley and Maxwell 1971). The method of estimating individual abundances will depend upon the goals of the researcher. In some circumstances minimum variance may be the most appropriate criteria, while in other cases lack of bias may be viewed as a necessary prerequisite for the estimator.

If the error variance of abundance estimates, given by

$$E\,(\hat{x}_i - x_i)^2$$

where x_i is the estimator, is minimized we have

$$\hat{x}_i = \sigma_*^2\,\underline{\Lambda}'\underline{\Sigma}^{-1}\underline{y}_i \qquad (10)$$

(Lawley and Maxwell 1971). This is basically a regression estimator, and is biased.

An alternative approach that yields unbiased estimates was first suggested by Bartlett (see Lawley and Maxwell 1971). In this estimator the sums of squares of the standardized residuals,

$$(\underline{y}_i - \underline{\Lambda}\,\hat{x}_i)'\,\underline{\Psi}^{-1}\,(\underline{y}_i - \underline{\Lambda}\hat{x}_i)$$

is minimized. The resulting unbiased estimator for x_i is

$$x_i = (\underline{\Lambda}'\underline{\Psi}^{-1}\underline{\Lambda})^{-1}\sigma_*^2\underline{\Lambda}'\underline{\Psi}^{-1}\underline{y}_i. \qquad (11)$$

The choice of estimator depends on the particular application. If abundance is to be used in calculations of sustainable yield from a fishery, it seems most critical to have an unbiased estimator, sacrificing the minimum variance property. In other cases, such as studies relating abundance to other variables such as growth or recruitment, the achievement of minimum variance may be more desirable.

The most generally available computer program for analyzing structural models is LISREL VI (linear structural relationships) (Jöreskog and Sörbom 1983). This program is available as an extra cost option on SPSSX (Statistical Package for Social Sciences X).

Difficulties in the Analysis

The investigator must always determine what indicators are appropriate and useful for the analysis of abundance. Such judgments require both detailed knowledge of the data set and biology of the population and also consideration of the ability of the method to use a given indicator effectively.

In certain cases, the problem can be approached by looking for corroborating evidence that an indicator truly contains information on abundance. For example, if density dependent growth is hypothesized such that body size could be used as an indicator of abundance, the size of an individual may be compared to indicators of cohort and standing stock abundance. Plots of one indicator versus another are useful for assessing the quality of the indicators. Heteroscedasticity and the presence of outliers should be scrutinized in particular.

Other problems arise when one indicator is very much better than the others. The measurement error variance for this indicator may be close to zero and the minimization procedure may encounter difficulties. This is particularly true when the number of time periods in which abundance is measured is small (< 50). In this situation it may be necessary to fix the value of the measurement error variance for the dominant indicator. Note that the method is still very useful for calibrating the other, poorer indicators.

Sample size is an important problem, especially for fisheries where relatively few years of data are available. One reason small samples may cause difficulties is because the model assumes that the sample covariance matrix is estimated adequately, i.e. is a good estimate of the population covariance matrix for the indicators. With small samples this assumption may not be well met. The most common result in this case is the parameter estimates for the model are outside admissible parameter space, e.g. one of the variances is estimated to be negative.

Van Driel (1978) notes that improper solutions have three main causes:

1a) sampling fluctuations such as often occur with small sample sizes;

2a) the data come from a model which does not meet the assumptions of the method, such as linearity and normality;

3a) the model is not identified (see Anderson and Rubin 1956).

She recommends procedures for overcoming these difficulties as follows:

1b) If the improper solution is due to (1a), then fix the value of that parameter at its nearest interpretable value.

2b) Under (2a), omitting the appropriate value is often helpful.

3b) Under (3a), adding variables (indicators) may properly identify the model.

As a diagnostic aid, we have often found it useful to construct the model for the correlation matrix first, and when it is satisfactory, proceed to the model for the covariance matrix, which should have the same structure. When working with a correlation matrix, the units are standardized and it is often easier to detect a poorly estimated value or an inappropriate model. In addition, one knows that the variance of the unobserved abundance equals one in this case and the estimated variance value can be compared with this as a check on the model.

There are two types of correlations in the error structure: measurement error correlations between indicators at a given time and longitudinally (i.e. over time) correlated errors. There are situations in which it is possible that measurement error in one indicator of abundance may not be independent of the measurement error in other indicators. For example, CPUE calculated from a subset of the commercial vessels and the SPA from the total commercial catch are both indicators of abundance. The measurement errors may not be independent in these two cases unless the CPUE is from vessels that are responsible for only a small fraction of the total catch. The degree of correlation in the measurement error can be estimated if there are four indicators of abundance. The hypothesis that the correlation is nonzero can also be rigorously tested using likelihood ratio methods if the model is over-identified.

If the residuals in the analysis are not independent over time, then the standard errors of the estimates may be seriously underestimated (Judge et al. 1980). Such correlations may be induced by the age structured model used in SPA. That is, because an error in the estimation of catch in one year will affect estimates of abundance for more than one year, the residuals may not be longitudinally uncorrelated. There are formal statistical tests to detect such correlations (Sörbom 1975). An alternative means of deciding which errors are most highly correlated in SPA is to simulate the propagation of errors through the analysis. This can be done by fixing the catches in all years but one, which is varied in the simulation. Both the total catch and the age distribution of the catch in the 'test' year should be varied to determine how errors affect the SPA results. Relevant information on the accuracy of the data can then be appropriately included in the measurement error model. See Sörbom (1975) for examples of such models.

It is possible to include auto-correlation structure for the true abundance in the analyses as described above. However, this does not ensure the errors will be uncorrelated, but violation of this assumption does not necessarily invalidate the procedure (Jöreskog and Sörbom 1983).

An Example

We shall illustrate the preceding methods and indicate uses of the approach by an examination of empirical data on spawning escapement of Atlantic salmon in Newfoundland. The data sets consist of recreational catch and effort data for the adjacent Gambo and Terra Nova rivers which drain into the same inlet and counts from two fishways on Terra Nova River (Table 2). We seek an index of spawning escapement in the Terra Nova inlet. Note that an index is all that is possible here because we have no measure of absolute abundance, i.e. not all salmon pass through the fishways so the fishway counts will in general be an underestimate. Fishway counts before 1963 are available but are not comparable to those after 1963 because of changes in the construction of the fishways. Only data from years in which all four indicators were available were used for parameter estimation. We use these data because they allow us to illustrate several difficulties that can be encountered in using these techniques.

Initially we used catch per unit effort as an index of abundance. However, catch per unit effort was negatively correlated with the fishway counts. This was presumably due to two factors: (i) the increase of inexperienced fishermen in recent years during a time when abundance appeared to increase, and (ii) the dependence of fishing effort on abundance, i.e. in years of greater salmon abundance recreational fishermen tended to fish more. We thus used total catch as an index of abundance.

It is clear from the data that all indicators are generally increasing with time. This is probably due to the effect of the fishway and a decrease in commercial fishing in the inlet. Thus, the true abundance is probably not independent over time.

We begin by examining the correlation matrix of the four indicators of abundance

	y_1	y_2	y_3	y_4
y_1	1	0.82	0.51	0.37
y_2	.	1	0.61	0.39
y_3	.	.	1	0.78
y_4	.	.	.	1

where the subscript 1 refers to the Terra Nova lower fishway counts, 2 refers to the Terra Nova upper fishway counts, 3 refers to the Terra Nova recreational catch, and 4 will refer to the Gambo River recreational catch. We first examine the three indicators for Terra Nova River. The good correlation of the upper fishway with the other two indicators shows it perhaps has the least measurement error. This is also indicated when the parameters are estimated (Table 3; Model 1) using (9). Subsequent analyses utilized the LISREL VI program.

The Gambo River catches are next included as an indicator, with the assumption that the measurement errors are independent (Table 3; Model 2). The χ^2 goodness of fit is too large, indicating that at least one of the assumptions is wrong. An examination of the residuals of the fitted covariance matrix, $\underline{\Sigma}$, with the observed sample covariance matrix, $\underline{S}$, indicates that the measurement errors of the two catches may not be independent. The inclusion and estimation of such a measurement error covariance yields an excellent fit (Table 3; Model 3). One estimated measurement error variance is negative, but this could be due to sampling error. This parameter could be set to some small value and the model refit.

The model assumptions should now be checked. First, the abundance appears to increase over time (Table 2), invalidating our assumption that the stochastic process generating the true abundance is stationary. We thus assume the true abundance is given by (5) and assume the measurement errors are independent (Table 3; Model 4). The fit is not adequate, and an examination of the residuals indicates that the measurement errors of the fishway counts are not independent (Table 3; Model 5). The inclusion of a second error covariance term marginally increases the χ^2 goodness of fit and is of questionable significance.

A second assumption that is questionable is that the errors are normal. The model was fit using log abundances and the assumption is that log abundance increases linearly with time (Eq. 5). The resulting fit is a slight improvement over the assumption of normal errors, and there is no evidence that the two catch rate errors are correlated (Table 3; Models 7, 8, 9).

We have three models, 3, 6, and 9, that fit the data adequately. The inclusion of a linear trend leads to very different conclusions about the sizes of the measurement errors, i.e. σ_2 was lowest in Model 3, whereas σ_3 was lowest in models 6 and 9. The assumption of normal vs. log-normal measurement errors had only minor effects on the relative sizes of the measurement error variances by comparison.

Estimates of the true abundances showed increases over time for all models (Fig. 1). Note that we can estimate the true abundance in years in which not all the indicators are available, i.e. 1974-77. Given the adequate fit of Model 6 it appears that relatively little can be inferred from the data other than there has been an approximately linear increase in salmon abundance in the inlet. Note also that the estimates obtained from Model 1 are sometimes below those actually observed; however, this is because the estimates were scaled to the mean abundance of the lower fishway which in general underestimates the number of fish passing through the river. As new data become available, yearly abundance estimators can be made using the measurement error variances calculated in Model 6. Since we do not know if future abundance will continue to increase, we would recommend not including the linear trend in a new yearly abundance estimate.

Conclusions

The structural equation modelling method described above is a very general approach. It may be used to combine the information in a variety of indicators in a coherent and repeatable way. Obviously, the method is not restricted to abundance estimation, but can be used to estimate any quantity where more than one type of measurement is available.

Some of the assumptions of the method, particularly linearity and normality, must be carefully assessed for a given application. Similarly, sampling problems may often arise. Even in these cases, however, the method may serve as a guide to particular problems with the data or model (Van Driel 1978). An advantage of this method is that alternative hypotheses for measurement error structure and the distribution of the true abundances can be rigorously tested to determine if they are consistent with the available data. Often more than one set of hypotheses are consistent with the data, as in our example. It is necessary for rational management to be aware of the range of abundance estimates that are based on viable alternative hypotheses about the measurement error structure.

Information on the measurement errors inherent in a particular type of measurement are essential for further data analyses. For example, if abundance is related to other variables such as recruitment (which may have its own measurement model), then estimates of the measurement error variances are required to obtain unbiased parameter estimates for the stock recruitment relationship (Kendall and Stuart 1973; Ludwig and Walters 1981).

The method can be used to reconstruct the history of the population from information on the indicators for periods when less information was available. In other words the various indicators can be calibrated over a period when all were measured, and then, assuming the relationships are stationary in time, the true abundance in other periods can be inferred from the model. Such a method may be particularly useful for time series data containing gaps in the record.

Finally, these methods can extend to far more complicated problems, where the measurement error models described above are parts of a model relating several unobservable variables (Jöreskog 1973). For fisheries data, such models have been used to describe the interactions of catches in multispecies fisheries and to describe the mechanism underlying recruitment variability (Rosenberg 1984).

If the desired relationship is linear, then the complete data set on all variances and the dependent variable of interest can be analyzed using extensions of the methods described here, i.e. the analysis of covariance structures (Jöreskog and Sörbom 1983). If, however, the relationship is nonlinear, then the estimates of the true abundance and the associated measurement error variance estimate can be utilized in alternative procedures, e.g. Chandler (1972).

Although we have found factor analysis to be a useful adjunct to standard methods of population analysis; however, its usefulness should not be overestimated. Potential users should heed the warning of Lawley and Maxwell (1971, p. 38) that factor analysis "is useful only as an approximation to reality . . . [and] . . . should not be taken too seriously."

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Table 1.

Notation:

e_{ij}	error in indicator j at time i
N	number of observation
n	N-1
p	number of indicators
$\underline{S}$	sample covariance matrix
x_i	true abundance at time i
$\underline{y}_i$	vector of indicators at time i
β	slope of true abundance on year; see (5)
$\underline{\Lambda}$	vector of parameters
$\underline{\Psi}^2$	covariance matrix of measurement errors
$\underline{\Sigma}$	dispersion matrix of $\underline{y}$
σ_*^2	variance in the true abundance used in the structural model
χ^2	"chi square" goodness of fit

Table 2. Data used in the example.

Year	Terra Nova lower fishway	Terra Nova upper fishway	Terra Nova catch	Gambo catch
1963	871	407	303	170
1964	716	264	339	341
1965	728	385	337	169
1966	588	136	226	210
1967	972	415	339	259
1968	1089	437	331	245
1969	1051	599	523	239
1970	1224	733	461	314
1971	857	437	413	204
1972	957	532	478	149
1973	754	562	335	287
1974	-	283	248	257
1975	-	830	508	300
1976	-	383	431	359
1977	-	633	863	1059
1978	830	524	634	616
1979	739	485	552	245
1980	882	437	534	467
1981	1205	647	772	747
1982	983	633	489	379
1983	1267	892	529	447

Table 3. Maximum likelihood estimation of abundance model for data from the Terra Nova and Gambo Rivers in Newfoundland. The subscript 1 refers to Terra Nova lower fishway counts, 2 refers to Terra Nova upper fishway counts, 3 refers to Terra Nova catches, and 4 refers to Gambo River catches. χ^2 is the goodness of fit, d is the degrees of freedom, and p is the probability that the fit is acceptable. Other symbols are given in Table 1. Models 1, 2, and 3 assume that the true abundance is a i.i.d. normal random variable and that measurement errors are normal. Models 5 and 6 assume that the true abundance is given by Eq. 5, and measurement errors are normal. Models 7, 8, and 9 assume that the true log abundance is given by Eq. 5 and measurement errors are log normally distributed.

Model	1	2	3	4	5	6	7	8	9
λ_1	1	1	1	1	1	1	1	1	1
λ_2	1.08	0.97	1.13	1.09	1.13	1.46	2.46	2.6	2.57
λ_3	0.53	0.55	0.53	1.18	1.31	1.39	2.23	2.48	2.51
λ_4	-	0.46	0.39	1.18	1.31	1.41	2.38	2.67	2.74
σ_1	36	100	116	169	162	180	0.167	0.179	0.79
σ_2	16	66	(-)30	128	133	132	0.272	0.29	0.29
σ_3	34	103	110	47	44	85	0.10	0.1	0.09
σ_4	-	144	152	100	98	122	0.33	0.32	0.32
σ_{12}	-	-	-	-	127	138	-	0.19	0.19
σ_{34}	-	-	111	-	-	75	-	-	0.06
σ_*	35	167	158	60	55	10	0.71	0.063	0.066
β	-	-	-	12.38	13.6	11.4	0.016	0.015	0.015
χ^2		12.3	0.32	19.1	6.87	4.46	14.87	5.18	5.15
d	0	2	1	5	4	3	5	4	3
p		0.002	0.57	0.002	0.143	0.216	0.011	0.27	0.161

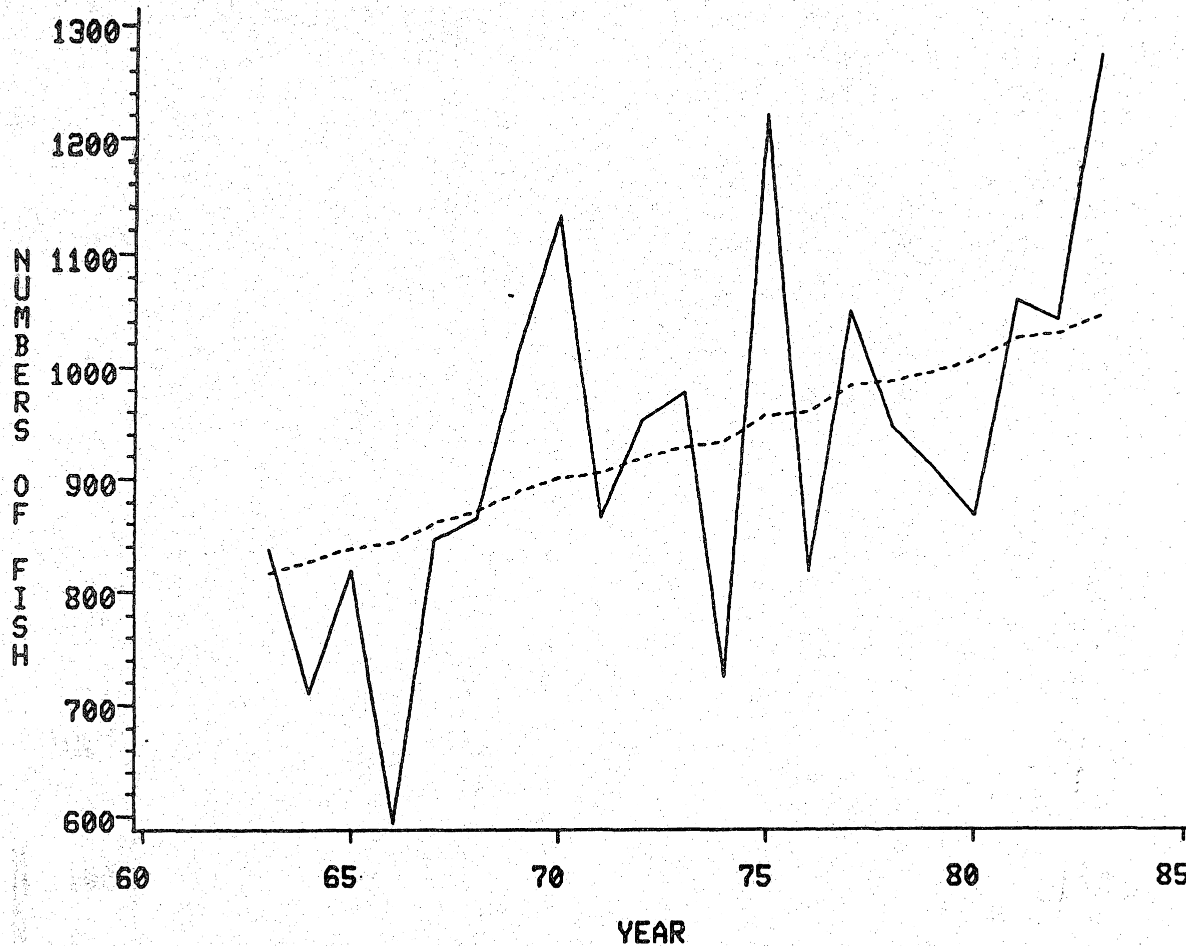

Figure 1. Estimated true abundance for the Terra Nova River salmon data using the minimum variance estimator (10) and the parameter estimates from model 1 (——) and 6 (----). Estimated abundance is scaled by the mean of the lower fishway.