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# Application of Multiplicative Model for Fishing Effort Standardization in a Special Case

by

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## INTRODUCTION

In 1989 the NAFO Sientific Council had no possibility to estimate silver hake stock in 4VWX subareas and postponed the solution of this problem for the meeting of Woking Group held in Copenhagen from 8 to 12 January 1990(Lassen H.1990).

However, during the meeting of Working Group the silver hake stock was not determined, as well. One of the reason was the complication in obtaining the standardizing values of fishing effort.

These complications were revealed in two aspects. The first one is non stable behaviour of standardized values of fishing effort obtained in the previous investigations. It turned out that in some cases the increments of the catch rates from year to year varied up to twenty per cent depending on the sample size, that is when information for one more year was added to the main sample used to determine the model parameters.

The second aspect of this problem was revealed in the fact that with the variation in sample size for the period from 1977 to 1989 from 154 units to 123 units (due to certain portion cutting) which had been proposed by the canadian scientists it was impossible to make calculations and obtain new estimates as the programm for the computer indicated the error and accidently stopped running.

This event was not a consequence of possible error in the program, but it may be considered as a special case of multiplicative model application. This requires an explication and ways of the problem solution when it occures.

## SHORT DESCRIPTION OF MULTIPLICATIVE MODEL

The multiplicative model for fishing effort standardization was suggested by Robson(1966) and Gavaris(1980).The model is based on the idea of standardized value of catch rate as the following equation

$$U = U_R \cdot P_{1J_1}^{X_{1J_1}} \cdot P_{2J_2}^{X_{2J_2}} \cdot P_{TJ_T}^{X_{TJ_T}} \qquad (1)$$

where $U$ - the standardized value of catch per fishing effort;

$U_R$ - catch rate for a particular combination of categories chosen as reference;

$P_{tj_t}$ - relative power of $j_t$ category in $t$ category type;

$X_{tj}$ = 1 if $u$ refers to $j$ category in $t$ category type;
= 0 in other cases;

$T_k$ - number of different category types.

Parameters of the multiplicative model are determined with the aid of the multiple linear regression after the logorithmic transformation

$$\text{LN } U = \text{LN } U_R + \sum_{t=1}^{T} \text{LN } P_{tj_t} \cdot X_{tj_t} \qquad (2)$$

where $\text{Ln}u_l$ - dependent variable;

$l$ - sample number, $l$=1,...,M;

$X_{tj_t}$ - independent variables.

With this the model parameters satisfy the following constraints

$$\sum_{l=1}^{K_l} \text{LN } P_{lj_l} = 0 \text{ , } t=1,2,\ldots,T \qquad (3)$$

where $j_t$ - number of categories in $t$ category type.

The sequence of calculations when determening model parameters is made according to the algoritm of multiplicative linear regression (Draper N.R.,Smith H.,1966). Quality of fitting is characterized by multiplicative correlation coefficient. Estimates of standard deviation values and confidence intervals have some characteristic features.These are described in detail by Gavaris(1980).

If **X** is the system matrix (2),unknown coefficients of model are determined by the following expression

$$\alpha = ( X^T \cdot X )^{-1} \cdot X^T Y \qquad (4)$$

where $\alpha = (U_R, \ldots, P_{ij}, \ldots)$ – vector of parameters;
$Y$ – column of dependent variable, $Y=(\ln U_1, \ln U_2, \ldots, \ln U_M)$;
$T$ – transpose sign.

The error induced by the computer with an accident completion of task when calculating in Copenhagen indicated that the difficulties with the calculations of inverse $(X^T \cdot X)^{-1}$ matrix had been arised. This fact corresponds to the step of model parameters determinations (4).

One may suppose that the system determinant is zero or close to zero. However, this hypothesis was not confirmed by the calculation: the determinant value of matrix corresponding to 123 sample size was $0.13 \cdot 10^{10}$.

More detailed investigation of this matrix indicated that it was ill-conditioned. This matrix property may be characterized by so called number of conditionality which is determined by the ratio of maximum eigen value to the the minimum eigen value (Maindonald J.H., 1984)

$$K = \frac{\lambda_1}{\lambda_n} \qquad (5)$$

where $K$ – number of conditionality;
$\lambda_i$ – eigen values of $X^T \cdot X$ matrix, $i=1,2, \ldots n$, sorted in a decreasing order.

If this number is rather great, the matrix is ill-conditioned what is revealed in a high sensitivity of system solution for variations in initial information. This is a typical property of incorrect problems (Tikhonov a.n., Arsenin V.I., 1974).

To have the most illustrative idea of importance of conditionality number of matrix let us cite J.H.Maindonald: "Number of conditionality, K, correlated the exactness of $\alpha$ elements calculated with a certain method using of which is justified to the precision of $X^T \cdot X$ elements. To say in a simple manner the number of conditionality is $10^h$, but $X^T \cdot X$ elements are determined with the precision up to $t$ figures, then $\alpha$ elements will be accurate of the order $t-h$ figures (Maindonald J.H., 1984).

When calculating the inverse matrix for system matrix constructed with the sample size of 123 units number of conditionality turned to be equal to $2.7 \cdot 10^{16}$.

It is evident that in similar special cases of determination of multiplicative model parameters it is nesessery to use some special methods.

# METHOD OF DETERMINATION OF MULTIPLICATIVE MODEL PARAMETERS WITH ILL - CONDITIONED $X^T \cdot X$ MATRIX

Method suggested is supposed for an incorrect problem solution(Tikhonov A.N.,Arsenin V.I.1974),(Morozov V.A.1987). The problem of ill- conditioned matrix inversion is the case.This method is based on search of solution $U$ system with **A** matrix

$$\mathbf{A} \cdot U = f \qquad (6)$$

when

$$||\mathbf{A} \cdot U - f|| = 0 \qquad (7)$$

where $|| \ ||$ - matrix norm;
$U$ - system solution;
$f$ - column of free members.

To have a single solution one may use the method of regularization.This method suggests the search of the following type solution

$$U_{\beta} = \arg \min_{u \in U} \{ ||\mathbf{A} \cdot U - f||^2 + \gamma \cdot |U||^{\ 2} \} \qquad (8)$$

where $\gamma$ - parameter of regularization.

A particular case when **A** matrix is non-negetively determined (8) will lead to the solution of equation system

$$(\gamma \cdot \mathbf{E} + \mathbf{A}) \cdot U_{\gamma}^{2} = \mathbf{A} \cdot f \qquad (9)$$

and,therefore is of

$$U_{\gamma} = (\gamma \cdot \mathbf{E} + \mathbf{A})^{-2} \cdot \mathbf{A}^{2} \cdot f \qquad (10)$$

where **E** - isolated matrix.

$U_{\gamma}$ depends on the parameter of regularization $\gamma$ . It may be selected using the algoritm of "moving check"(Vapnic V.N. *et al.*, 1984). The sense of this algoritm is in that from **X** matrix is removed one by one the $i$-th line and with a given coefficient the calculations of model coefficients are realized by (10).Then using the coefficients of model obtained and corresponding values of independent variables the value of dependent variables are calculated and the values of residuals are determined.

With calculations made for all $i$ one may define standard deviation of model using the given $\gamma$. Having carried out the calculations for a certain set of values $\gamma$ one may select final mean $\gamma_{min}$ which provides the minimum of standard deviation. The parameters of multiplicative model determined with $\gamma_{min}$ are to be selected as a final solution.

It may be demonstrated (Vapnic V.N. *et al.*,1984), that presented algoritm does not require multiple system solution with a given $\gamma$. For this purpose it is nesessery to use the following form of its writing

$$T_{\gamma} = \frac{1}{M} \sum_{l=1}^{M} \frac{(u_l - x_l^T \cdot B_{\gamma}^{-2} \cdot A_{\gamma}^{2} \cdot X^T \cdot Y)}{(1 - x_l^T \cdot B_{\gamma}^{-2} \cdot A_{\gamma}^{2} \cdot x_l)} \qquad (11)$$

where M - sample size;

$T_{\gamma}$ - value of standard deviation obtained with the procedure of "moving check';

$x_l$ $l$ line in **A** matrix;

$A_{\gamma}=X^T.X$;

$B_{\gamma}=(\gamma.E+X^T.X)$.

## THE DATA USED

To demonstrate the algoritm suggested we will use the same initial data which were used when standardizing a fishing effort for silver hake fisheries during the Working Group Meeting in January 1990 in Copenhagen (Lassen H.,1990). With a permission of D.E.Waldron this data are given in Table 1. Records which are to be removed to obtain the sample of 123 units are represented as marked block in the Table. These are the records with numbers of 68 through 98.

Let us denote the sample of 123 units by 1 and the sample of 154 units - by 2.

## CALCULATIONS RESULTS

The eigen values of system $X^t \cdot X$ matrix (2) for the samples 1 and 2 are given in Table 2. In the same Table values of determinant and value of numbers of conditionality are given.

It is evident that a system matrix for sample 1 is ill - conditioned that is why it was not possible to calculate its inverse by classic methods. This was the reason for a poor calculation in Copenhagen when standardizing fishing effort for silver hake fisheries.

Another is the case for the matrix of sample 2 (number of conditionality is equal to 544.2), that is why there are no difficulties in this case.

In Tables 3,4 and 5 the results of standardization of fishing effort with algoritm (10) for the sample 1 are represented. As it is obvious the algoritm suggested to determine the multiplicative model parameters allows for the solution in the case of ill - conditioned matrix of equation systems (2).

Results of standardization of fishing effort for silver hake by the algoritm (10) for sample 2 are given in Table 6,7 and 8. This case is not a special one, and therefore the calculation results are close to the results obtained with a common algoritm. The latter results are represented in Table 9.

The results of calculations to check up the stability of standardized value of estimates for catch per effort with somewhat changed sample size are given in Table 10. To obtain these figures all the data concerning 1989 were removed from the sample 1.Thus, the situation was imitated when to the main data file (1977 - 1988) a new portion of information for the next year (1989) was added. The calculation has been made by algoritm (10). The calculation results indicate that with the addition of new data previous estimates vary not more than 3.5 %. This allows for confirmation that the method of multiplicative model parameters determination (10) has stability.

## CONCLUSION

In practice, using of multiplicative model standardization for fishing effort indicated that the cases could be observed when the classic algoritm did not allow for this problem solution. Those special cases may arise when system matrix is ill - conditioned.

In such cases it is recommended to use a regularizing algoritm which allows in principle for the solution and in the case of non-singular matrix of system it will give the results which coincide with the results of a common algoritm.

Besides, algoritm suggested permits to obtain stable estimates.

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Table 1. Comertial silver hake catch and effort for multiplicative model

| nn | Catch | Effort | Source | Month | Year | Area | Reg. | Country |
|---|---|---|---|---|---|---|---|---|
| 1 | 14 | 10 | 1 | 8 | 77 | 450 | 2 | 1 |
| 2 | 3295 | 1841 | 1 | 4 | 77 | 460 | 2 | 1 |
| 3 | 3721 | 1933 | 1 | 5 | 77 | 460 | 2 | 1 |
| 4 | 1796 | 889 | 1 | 6 | 77 | 460 | 2 | 1 |
| 5 | 8261 | 4117 | 1 | 7 | 77 | 460 | 2 | 1 |
| 6 | 1704 | 435 | 1 | 8 | 77 | 460 | 2 | 1 |
| 7 | 470 | 332 | 1 | 9 | 77 | 460 | 2 | 1 |
| 8 | 2423 | 1083 | 1 | 4 | 77 | 470 | 2 | 1 |
| 9 | 1576 | 875 | 1 | 5 | 77 | 470 | 2 | 1 |
| 10 | 236 | 82 | 1 | 6 | 77 | 470 | 2 | 1 |
| 11 | 1051 | 524 | 1 | 7 | 77 | 470 | 2 | 1 |
| 12 | 232 | 186 | 1 | 4 | 78 | 450 | 2 | 1 |
| 13 | 2995 | 1406 | 1 | 5 | 78 | 450 | 2 | 1 |
| 14 | 326 | 203 | 1 | 6 | 78 | 450 | 2 | 1 |
| 15 | 1591 | 1087 | 1 | 4 | 78 | 460 | 2 | 1 |
| 16 | 5219 | 4812 | 1 | 5 | 78 | 460 | 2 | 1 |
| 17 | 6169 | 5196 | 1 | 6 | 78 | 460 | 2 | 1 |
| 18 | 7847 | 5626 | 1 | 7 | 78 | 460 | 2 | 1 |
| 19 | 4183 | 2031 | 1 | 8 | 78 | 460 | 2 | 1 |
| 20 | 330 | 247 | 1 | 4 | 78 | 470 | 2 | 1 |
| 21 | 133 | 88 | 1 | 5 | 78 | 470 | 2 | 1 |
| 22 | 1064 | 649 | 1 | 6 | 78 | 470 | 2 | 1 |
| 23 | 27 | 38 | 1 | 7 | 78 | 470 | 2 | 1 |
| 24 | 13 | 10 | 1 | 8 | 79 | 450 | 2 | 1 |
| 25 | 2103 | 1244 | 1 | 4 | 79 | 460 | 2 | 1 |
| 26 | 8847 | 4874 | 1 | 5 | 79 | 460 | 2 | 1 |
| 27 | 8390 | 4985 | 1 | 6 | 79 | 460 | 2 | 1 |
| 28 | 7470 | 3948 | 1 | 7 | 79 | 460 | 2 | 1 |
| 29 | 2014 | 1338 | 1 | 8 | 79 | 460 | 2 | 1 |
| 30 | 713 | 411 | 1 | 9 | 79 | 460 | 2 | 1 |
| 31 | 65 | 31 | 1 | 4 | 79 | 470 | 2 | 1 |
| 32 | 739 | 436 | 1 | 5 | 79 | 470 | 2 | 1 |
| 33 | 98 | 65 | 1 | 6 | 79 | 470 | 2 | 1 |
| 34 | 1531 | 1176 | 1 | 4 | 80 | 460 | 2 | 1 |
| 35 | 9033 | 7902 | 1 | 5 | 80 | 460 | 2 | 1 |
| 36 | 11333 | 9056 | 1 | 6 | 80 | 460 | 2 | 1 |
| 37 | 9018 | 7083 | 1 | 7 | 80 | 460 | 2 | 1 |
| 38 | 3665 | 5683 | 1 | 8 | 80 | 460 | 2 | 1 |
| 39 | 168 | 118 | 1 | 5 | 80 | 470 | 2 | 1 |
| 40 | 1639 | 906 | 1 | 6 | 80 | 470 | 2 | 1 |
| 41 | 4494 | 2725 | 1 | 7 | 80 | 470 | 2 | 1 |
| 42 | 66 | 113 | 1 | 8 | 80 | 470 | 2 | 1 |
| 43 | 117 | 80 | 1 | 6 | 81 | 450 | 2 | 1 |
| 44 | 363 | 275 | 1 | 7 | 81 | 450 | 2 | 1 |
| 45 | 601 | 490 | 1 | 4 | 81 | 460 | 2 | 1 |
| 46 | 13317 | 6091 | 1 | 5 | 81 | 460 | 2 | 1 |
| 47 | 11804 | 8717 | 1 | 6 | 81 | 460 | 2 | 1 |
| 48 | 9940 | 7100 | 1 | 7 | 81 | 460 | 2 | 1 |
| 49 | 763 | 543 | 1 | 8 | 81 | 460 | 2 | 1 |
| 50 | 220 | 192 | 1 | 6 | 81 | 470 | 2 | 1 |
| 51 | 17 | 14 | 1 | 8 | 81 | 470 | 2 | 1 |
| 52 | 2165 | 386 | 1 | 4 | 82 | 460 | 2 | 1 |
| 53 | 16644 | 3895 | 1 | 5 | 82 | 460 | 2 | 1 |
| 54 | 20985 | 5441 | 1 | 6 | 82 | 460 | 2 | 1 |
| 55 | 6653 | 2348 | 1 | 7 | 82 | 460 | 2 | 1 |
| 56 | 5134 | 1726 | 1 | 4 | 83 | 460 | 2 | 1 |
| 57 | 13127 | 6030 | 1 | 5 | 83 | 460 | 2 | 1 |
| 58 | 7110 | 4935 | 1 | 6 | 83 | 460 | 2 | 1 |
| 59 | 229 | 267 | 1 | 7 | 83 | 460 | 2 | 1 |
| 60 | 15732 | 3794 | 1 | 5 | 84 | 460 | 2 | 1 |
| 61 | 17276 | 7077 | 1 | 6 | 84 | 460 | 2 | 1 |
| 62 | 21453 | 8735 | 1 | 7 | 84 | 460 | 2 | 1 |
| 63 | 2838 | 904 | 1 | 8 | 84 | 460 | 2 | 1 |
| 64 | 8400 | 3702 | 1 | 5 | 85 | 460 | 2 | 1 |

Table 1.(Continued)

| nn | Catch | Effort | Source | Month | Year | Area | Reg. | Country |
|---|---|---|---|---|---|---|---|---|
| 65 | 26230 | 10541 | 1 | 6 | 85 | 460 | 2 | 1 |
| 66 | 15956 | 13623 | 1 | 7 | 85 | 460 | 2 | 1 |
| 67 | 5751 | 2069 | 1 | 8 | 85 | 460 | 2 | 1 |
| 68 | 435 | 184 | 2 | 7 | 77 | 460 | 1 | 1 |
| 69 | 50 | 41 | 2 | 7 | 77 | 460 | 2 | 1 |
| 70 | 771 | 712 | 2 | 8 | 77 | 460 | 1 | 1 |
| 71 | 16 | 72 | 2 | 8 | 77 | 460 | 2 | 1 |
| 72 | 18 | 16 | 2 | 8 | 77 | 470 | 1 | 1 |
| 73 | 225 | 128 | 2 | 9 | 77 | 460 | 1 | 1 |
| 74 | 527 | 408 | 2 | 4 | 78 | 460 | 1 | 1 |
| 75 | 1233 | 776 | 2 | 5 | 78 | 460 | 1 | 1 |
| 76 | 17 | 17 | 2 | 5 | 78 | 470 | 1 | 1 |
| 77 | 16 | 19 | 2 | 5 | 78 | 470 | 2 | 1 |
| 78 | 1375 | 493 | 2 | 6 | 78 | 450 | 1 | 1 |
| 79 | 2136 | 1557 | 2 | 6 | 78 | 460 | 1 | 1 |
| 80 | 116 | 63 | 2 | 6 | 78 | 460 | 2 | 1 |
| 81 | 72 | 40 | 2 | 6 | 78 | 470 | 1 | 1 |
| 82 | 774 | 310 | 2 | 7 | 78 | 450 | 1 | 1 |
| 83 | 2745 | 1645 | 2 | 7 | 78 | 460 | 1 | 1 |
| 84 | 47 | 32 | 2 | 7 | 78 | 470 | 1 | 1 |
| 85 | 74 | 29 | 2 | 8 | 78 | 450 | 1 | 1 |
| 86 | 3195 | 1316 | 2 | 8 | 78 | 460 | 1 | 1 |
| 87 | 110 | 102 | 2 | 9 | 78 | 460 | 1 | 1 |
| 88 | 1510 | 690 | 2 | 5 | 79 | 460 | 1 | 1 |
| 89 | 1684 | 847 | 2 | 5 | 79 | 460 | 2 | 1 |
| 90 | 105 | 81 | 2 | 5 | 79 | 470 | 1 | 1 |
| 91 | 11 | 15 | 2 | 5 | 79 | 470 | 2 | 1 |
| 92 | 3174 | 1604 | 2 | 6 | 79 | 460 | 1 | 1 |
| 93 | 2097 | 1188 | 2 | 6 | 79 | 460 | 2 | 1 |
| 94 | 41 | 46 | 2 | 6 | 79 | 470 | 1 | 1 |
| 95 | 83 | 87 | 2 | 6 | 79 | 470 | 2 | 1 |
| 96 | 2239 | 899 | 2 | 7 | 79 | 460 | 1 | 1 |
| 97 | 1017 | 455 | 2 | 7 | 79 | 460 | 2 | 1 |
| 98 | 392 | 259 | 2 | 8 | 79 | 460 | 2 | 1 |
| 99 | 9423 | 2240 | 2 | 5 | 86 | 460 | 2 | 1 |
| 100 | 11905 | 3292 | 2 | 6 | 86 | 460 | 2 | 1 |
| 101 | 5531 | 1803 | 2 | 7 | 86 | 460 | 2 | 1 |
| 102 | 63 | 16 | 2 | 6 | 86 | 470 | 2 | 1 |
| 103 | 1049 | 175 | 2 | 7 | 86 | 470 | 2 | 1 |
| 104 | 2300 | 638 | 1 | 5 | 82 | 460 | 2 | 2 |
| 105 | 3437 | 1491 | 1 | 6 | 82 | 460 | 2 | 2 |
| 106 | 5469 | 2542 | 1 | 7 | 82 | 460 | 2 | 2 |
| 107 | 1565 | 515 | 1 | 4 | 83 | 460 | 2 | 2 |
| 108 | 3003 | 2124 | 1 | 5 | 83 | 460 | 2 | 2 |
| 109 | 2564 | 2640 | 1 | 6 | 83 | 460 | 2 | 2 |
| 110 | 286 | 150 | 1 | 7 | 83 | 460 | 2 | 2 |
| 111 | 2614 | 724 | 1 | 4 | 84 | 460 | 2 | 2 |
| 112 | 6254 | 2364 | 1 | 5 | 84 | 460 | 2 | 2 |
| 113 | 5415 | 2351 | 1 | 6 | 84 | 460 | 2 | 2 |
| 114 | 213 | 192 | 1 | 7 | 84 | 460 | 2 | 2 |
| 115 | 2889 | 863 | 1 | 4 | 85 | 460 | 2 | 2 |
| 116 | 6098 | 3035 | 1 | 5 | 85 | 460 | 2 | 2 |
| 117 | 7014 | 2797 | 1 | 6 | 85 | 460 | 2 | 2 |
| 118 | 1682 | 831 | 1 | 7 | 85 | 460 | 2 | 2 |
| 119 | 869 | 744 | 2 | 8 | 86 | 460 | 2 | 2 |
| 120 | 2682 | 489 | 2 | 4 | 86 | 460 | 2 | 2 |
| 121 | 2482 | 850 | 2 | 5 | 86 | 460 | 2 | 2 |
| 122 | 950 | 503 | 2 | 6 | 86 | 460 | 2 | 2 |
| 123 | 342 | 353 | 2 | 9 | 86 | 460 | 2 | 2 |
| 124 | 5216 | 742 | 2 | 4 | 87 | 460 | 2 | 2 |
| 125 | 9411 | 1658 | 2 | 5 | 87 | 460 | 2 | 2 |
| 126 | 4641 | 2407 | 2 | 6 | 87 | 460 | 2 | 2 |
| 127 | 237 | 121 | 2 | 7 | 87 | 460 | 2 | 2 |
| 128 | 3173 | 597 | 2 | 5 | 87 | 460 | 2 | 1 |
| 129 | 15895 | 6758 | 2 | 6 | 87 | 460 | 2 | 1 |

Table 1.(Continued)

| nn | Catch | Effort | Source | Month | Year | Area | Reg. | Country |
|---|---|---|---|---|---|---|---|---|
| 130 | 18291 | 7438 | 2 | 7 | 87 | 460 | 2 | 1 |
| 131 | 24 | 11 | 2 | 7 | 87 | 470 | 2 | 1 |
| 132 | 247 | 139 | 2 | 8 | 87 | 470 | 2 | 1 |
| 133 | 20663 | 4789 | 2 | 4 | 88 | 460 | 2 | 1 |
| 134 | 782 | 274 | 2 | 4 | 88 | 460 | 2 | 2 |
| 135 | 18870 | 6050 | 2 | 5 | 88 | 460 | 2 | 1 |
| 136 | 2279 | 1079 | 2 | 5 | 88 | 460 | 2 | 2 |
| 137 | 12328 | 5864 | 2 | 6 | 88 | 460 | 2 | 1 |
| 138 | 4184 | 2384 | 2 | 6 | 88 | 460 | 2 | 2 |
| 139 | 3254 | 976 | 2 | 6 | 88 | 470 | 2 | 1 |
| 140 | 93 | 56 | 2 | 6 | 88 | 470 | 2 | 2 |
| 141 | 1233 | 1462 | 2 | 7 | 88 | 460 | 2 | 1 |
| 142 | 1796 | 973 | 2 | 7 | 88 | 460 | 2 | 2 |
| 143 | 650 | 60 | 2 | 3 | 89 | 460 | 2 | 1 |
| 144 | 30044 | 5110 | 2 | 4 | 89 | 460 | 2 | 1 |
| 145 | 19098 | 5255 | 2 | 5 | 89 | 460 | 2 | 1 |
| 146 | 9749 | 4245 | 2 | 6 | 89 | 460 | 2 | 1 |
| 147 | 4667 | 1552 | 2 | 7 | 89 | 460 | 2 | 1 |
| 148 | 4347 | 823 | 2 | 4 | 89 | 460 | 2 | 2 |
| 149 | 6844 | 2207 | 2 | 5 | 89 | 460 | 2 | 2 |
| 150 | 2999 | 1536 | 2 | 6 | 89 | 460 | 2 | 2 |
| 151 | 457 | 244 | 2 | 7 | 89 | 460 | 2 | 2 |
| 152 | 182 | 39 | 2 | 4 | 89 | 470 | 2 | 1 |
| 153 | 171 | 35 | 2 | 5 | 89 | 470 | 2 | 1 |
| 154 | 6128 | 1683 | 2 | 6 | 89 | 470 | 2 | 1 |

Table 2. $X^t \cdot X$ matrix eigen values for sample 1 and 2

| nn | Sample I | Sample 2 |
|---|---|---|
| I | 33I.2 | 462.6 |
| 2 | 58.8 | 83.4 |
| 3 | 32.6 | 41.5 |
| 4 | 27.2 | 37.7 |
| 5 | 23.7 | 28.7 |
| 6 | 21.7 | 26.6 |
| 7 | 15.0 | 25.1 |
| 8 | 13.4 | 19.4 |
| 9 | 11.8 | 18.7 |
| 10 | 11.0 | 13.9 |
| 11 | 10.4 | 11.3 |
| 12 | 9.6 | 10.3 |
| 13 | 8.6 | 9.7 |
| 14 | 8.3 | 8.9 |
| 15 | 8.0 | 8.4 |
| 16 | 7.8 | 8.0 |
| 17 | 7.2 | 7.9 |
| 18 | 6.3 | 7.3 |
| 19 | 4.3 | 7.0 |
| 20 | 2.8 | 5.7 |
| 21 | 2.1 | 4.4 |
| 22 | 0.9 | 2.9 |
| 23 | $0.12 \cdot 10^{-10}$ | 0.1 |
| 24 | | 0.9 |
| $\lvert X^t \cdot X \rvert$ | $0.13 \cdot 10^{10}$ | $0.11 \cdot 10^{27}$ |
| K | $2.7 \cdot 10^{16}$ | 544.2 |

Table 3. Statistical characteristics of catch rate standardization for silver hake fishing in the NAFO 4VWX subareas (sample 1)

REGRESSION OF MULTIPLICATIVE MODEL

MULTIPLE R............ .811
MULTIPLE R SQUARED..... .658

ANALYSIS OF VARIANCE

| SOURCE OF VARIATION | DF | SUMS OF SQUARES | MEAN SQUARES | F-VALUE |
|---|---|---|---|---|
| INTERCEPT | 1 | 6.877E0001 | 6.877E0001 | |
| REGRESSION | 22 | 2.210E0001 | 1.004E0000 | 8.733 |
| TYPE 1 | 1 | 1.264E-003 | 1.264E-003 | 0.011 |
| TYPE 2 | 6 | 5.886E0000 | 9.810E-001 | 8.531 |
| TYPE 3 | 12 | 8.896E0000 | 7.413E-001 | 6.446 |
| TYPE 4 | 2 | 1.287E-002 | 6.435E-003 | 0.056 |
| TYPE 5 | 0 | 0.000E0000 | 1.000E0000 | 8.696 |
| TYPE 6 | 1 | 8.554E-001 | 8.554E-001 | 7.438 |
| RESIDUALS | 100 | 1.150E0001 | 1.150E-001 | |
| TOTAL | 123 | 1.024E0002 | | |

Table 4. Coefficients of multiplicative model (sample 1)

REGRESSION COEFFICIENTS

| CATEGORY | CODE | VARIABLE | COEFFICIENT | STD. ERROR | NO. OBS. |
|---|---|---|---|---|---|
| 1 | 1 | INTERCEPT | 0.843 | 0.121 | 123 |
| 2 | 5 | | | | |
| 3 | 77 | | | | |
| 4 | 460 | | | | |
| 5 | 2 | | | | |
| 6 | 1 | | | | |
| 1 | 2 | 1 | 0.106 | 0.030 | 41 |
| 2 | 3 | 2 | 1.019 | 0.360 | 1 |
| | 4 | 3 | 0.192 | 0.099 | 21 |
| | 6 | 4 | -0.193 | 0.089 | 32 |
| | 7 | 5 | -0.336 | 0.093 | 26 |
| | 8 | 6 | -0.339 | 0.118 | 13 |
| | 9 | 7 | -0.593 | 0.216 | 3 |
| 3 | 78 | 8 | -0.441 | 0.144 | 12 |
| | 79 | 9 | -0.175 | 0.147 | 10 |
| | 80 | 10 | -0.535 | 0.153 | 9 |
| | 81 | 11 | -0.333 | 0.154 | 9 |
| | 82 | 12 | 0.585 | 0.172 | 7 |
| | 83 | 13 | -0.126 | 0.166 | 8 |
| | 84 | 14 | 0.365 | 0.166 | 8 |
| | 85 | 15 | 0.226 | 0.166 | 8 |
| | 86 | 16 | 0.412 | 0.131 | 10 |
| | 87 | 17 | 0.405 | 0.137 | 9 |
| | 88 | 18 | 0.051 | 0.133 | 10 |
| | 89 | 19 | 0.415 | 0.125 | 12 |
| 4 | 450 | 20 | 0.025 | 0.148 | 7 |
| | 470 | 21 | 0.027 | 0.084 | 26 |
| 6 | 2 | 22 | -0.226 | 0.083 | 33 |

Table 5.Standardized catch rate values for silver hake (sample 1)

PREDICTED CATCH RATE

STANDARDS USED VARIABLE NUMBERS: 1 5 450 2 1

| YEAR | TOTAL CATCH | PROP. | CATCH RATE MEAN | CATCH RATE S.E. | EFFORT |
|---|---|---|---|---|---|
| 77 | 37095 | 0.662 | 2.486 | 0.440 | 14924 |
| 78 | 48404 | 0.622 | 1.605 | 0.253 | 30149 |
| 79 | 51751 | 0.588 | 2.086 | 0.377 | 24808 |
| 80 | 44525 | 0.920 | 1.451 | 0.282 | 30688 |
| 81 | 44599 | 0.833 | 1.782 | 0.312 | 25027 |
| 82 | 60207 | 0.958 | 4.436 | 0.912 | 13573 |
| 83 | 35837 | 0.921 | 2.180 | 0.440 | 16441 |
| 84 | 74266 | 0.967 | 3.560 | 0.724 | 20858 |
| 85 | 75480 | 0.981 | 3.099 | 0.630 | 24354 |
| 86 | 82689 | 0.427 | 3.742 | 0.712 | 22096 |
| 87 | 61704 | 0.926 | 3.717 | 0.716 | 16601 |
| 88 | 74482 | 0.879 | 2.609 | 0.494 | 28549 |
| 89 | 86729 | 0.984 | 3.762 | 0.684 | 23055 |

AVERAGE C.V. FOR THE MEAN: .189

Table 6.Statistical characteristics of catch rate standardization for silver hake fishing in the NAFO 4VWX subareas(sample 2)

REGRESSION OF MULTIPLICATIVE MODEL

MULTIPLE R............. .758
MULTIPLE R SQUARED..... .574

ANALYSIS OF VARIANCE

| SOURCE OF VARIATION | DF | SUMS OF SQUARES | MEAN SQUARES | F-VALUE |
|---|---|---|---|---|
| INTERCEPT | 1 | 6.986E0001 | 6.986E0001 | |
| REGRESSION | 23 | 2.565E0001 | 1.115E0000 | 7.614 |
| TYPE 1 | 1 | 8.178E-001 | 8.178E-001 | 5.585 |
| TYPE 2 | 6 | 5.186E0000 | 8.643E-001 | 5.902 |
| TYPE 3 | 12 | 1.366E0001 | 1.138E0000 | 7.771 |
| TYPE 4 | 2 | 5.638E-001 | 2.819E-001 | 1.925 |
| TYPE 5 | 1 | 1.269E0000 | 1.269E0000 | 8.668 |
| TYPE 6 | 1 | 1.023E0000 | 1.023E0000 | 6.986 |
| RESIDUALS | 130 | 1.904E0001 | 1.464E-001 | |
| TOTAL | 154 | 1.145E0002 | | |

Table 7. Coefficients of multiplicative model

(sample 2)

REGRESSION COEFFICIENTS

| CATEGORY | CODE | VARIABLE | COEFFICIENT | STD. ERROR | NO. OBS. |
|---|---|---|---|---|---|
| 1 | 1 | INTERCEPT | 0.710 | 0.126 | 154 |
| 2 | 5 | | | | |
| 3 | 77 | | | | |
| 4 | 460 | | | | |
| 5 | 2 | | | | |
| 6 | 1 | | | | |
| 1 | 2 | 1 | -0.342 | 0.145 | 72 |
| 2 | 3 | 2 | 1.021 | 0.406 | 1 |
| | 4 | 3 | 0.206 | 0.107 | 22 |
| | 6 | 4 | -0.123 | 0.090 | 40 |
| | 7 | 5 | -0.214 | 0.095 | 33 |
| | 8 | 6 | -0.348 | 0.115 | 19 |
| | 9 | 7 | -0.480 | 0.193 | 5 |
| 3 | 78 | 8 | -0.227 | 0.128 | 26 |
| | 79 | 9 | -0.026 | 0.129 | 21 |
| | 80 | 10 | -0.386 | 0.163 | 9 |
| | 81 | 11 | -0.253 | 0.164 | 9 |
| | 82 | 12 | 0.670 | 0.186 | 7 |
| | 83 | 13 | -0.035 | 0.179 | 8 |
| | 84 | 14 | 0.459 | 0.178 | 8 |
| | 85 | 15 | 0.321 | 0.178 | 8 |
| | 86 | 16 | 0.972 | 0.201 | 10 |
| | 87 | 17 | 0.967 | 0.204 | 9 |
| | 88 | 18 | 0.612 | 0.203 | 10 |
| | 89 | 19 | 0.994 | 0.198 | 12 |
| 4 | 450 | 20 | 0.167 | 0.141 | 10 |
| | 470 | 21 | -0.103 | 0.082 | 35 |
| 5 | 1 | 22 | 0.456 | 0.155 | 21 |
| 6 | 2 | 23 | -0.246 | 0.093 | 33 |

Table 8. Standardized catch rate values for silver hake (sample 2)

PREDICTED CATCH RATE

STANDARDS USED VARIABLE NUMBERS: 1 5 450 2 1

| YEAR | TOTAL CATCH | PROP. | CATCH RATE MEAN | S.E. | EFFORT |
|---|---|---|---|---|---|
| 77 | 37095 | 0.703 | 2.546 | 0.452 | 14569 |
| 78 | 48404 | 0.879 | 2.039 | 0.308 | 23739 |
| 79 | 51751 | 0.827 | 2.483 | 0.423 | 20843 |
| 80 | 44525 | 0.920 | 1.723 | 0.344 | 25845 |
| 81 | 44599 | 0.833 | 1.976 | 0.361 | 22572 |
| 82 | 60207 | 0.958 | 4.940 | 1.052 | 12187 |
| 83 | 35837 | 0.921 | 2.444 | 0.510 | 14662 |
| 84 | 74266 | 0.967 | 4.005 | 0.840 | 18544 |
| 85 | 75480 | 0.981 | 3.486 | 0.731 | 21653 |
| 86 | 82689 | 0.427 | 6.644 | 1.562 | 12446 |
| 87 | 61704 | 0.926 | 6.613 | 1.567 | 9331 |
| 88 | 74482 | 0.879 | 4.639 | 1.090 | 16057 |
| 89 | 86729 | 0.984 | 6.804 | 1.559 | 12747 |

AVERAGE C.V. FOR THE MEAN: .205

Table 9.Standardized catch rate values for silver

hake (sample 2, common algoritm)

PREDICTED CATCH RATE

STANDARDS USED VARIABLE NUMBERS: 1 5 450 2 1

| YEAR | TOTAL CATCH | PROP. | CATCH RATE MEAN | S.E. | EFFORT |
|---|---|---|---|---|---|
| 77 | 37095 | 0.703 | 2.559 | 0.455 | 14497 |
| 78 | 48404 | 0.879 | 2.043 | 0.309 | 23695 |
| 79 | 51751 | 0.827 | 2.488 | 0.424 | 20800 |
| 80 | 44525 | 0.920 | 1.724 | 0.344 | 25820 |
| 81 | 44599 | 0.833 | 1.978 | 0.361 | 22547 |
| 82 | 60207 | 0.958 | 4.942 | 1.051 | 12182 |
| 83 | 35837 | 0.921 | 2.445 | 0.509 | 14655 |
| 84 | 74266 | 0.967 | 4.007 | 0.840 | 18533 |
| 85 | 75480 | 0.981 | 3.488 | 0.731 | 21641 |
| 86 | 82689 | 0.427 | 6.673 | 1.569 | 12392 |
| 87 | 61704 | 0.926 | 6.641 | 1.573 | 9292 |
| 88 | 74482 | 0.879 | 4.658 | 1.094 | 15991 |
| 89 | 86729 | 0.984 | 6.832 | 1.565 | 12695 |

AVERAGE C.V. FOR THE MEAN: .204

Table 10.Standardized catch rate values for silver

hake (sample 2, 1977-1988)

PREDICTED CATCH RATE

STANDARDS USED VARIABLE NUMBERS: 1 5 450 2 1

| YEAR | TOTAL CATCH | PROP. | CATCH RATE MEAN | S.E. | EFFORT |
|---|---|---|---|---|---|
| 77 | 37095 | 0.703 | 2.529 | 0.462 | 14665 |
| 78 | 48404 | 0.879 | 2.034 | 0.317 | 23799 |
| 79 | 51751 | 0.827 | 2.469 | 0.432 | 20964 |
| 80 | 44525 | 0.920 | 1.705 | 0.349 | 26111 |
| 81 | 44599 | 0.833 | 1.944 | 0.365 | 22937 |
| 82 | 60207 | 0.958 | 4.785 | 1.050 | 12582 |
| 83 | 35837 | 0.921 | 2.376 | 0.511 | 15084 |
| 84 | 74266 | 0.967 | 3.874 | 0.839 | 19171 |
| 85 | 75480 | 0.981 | 3.372 | 0.730 | 22385 |
| 86 | 82689 | 0.427 | 6.555 | 1.585 | 12615 |
| 87 | 61704 | 0.926 | 6.531 | 1.590 | 9448 |
| 88 | 74482 | 0.879 | 4.580 | 1.107 | 16263 |

AVERAGE C.V. FOR THE MEAN: .208