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Correlated Error Model Results for 2J3KL Cod in 1993

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Summary

The model that includes correlated errors developed by Myers and Cadigan was presented for 2J3KL cod. The correlated error model produces a much improved fit to the research vessel surveys than the model that assumed independent errors. The correlated error model produced lower estimates of abundance, i.e. the numbers went down from a total 82,145,000 fish of ages 3-12 to only 27,265,000. There are very large standard errors associated with both estimates. The fishing mortality estimated in the last two years is generally higher in the correlated error model. The fishing mortality in the in 1993 appears to be high both the independent and correlated error models because the Canadian commercial fishery was closed during 1993. In all models, with or without correlated errors, there is extraordinary patterns in the unstandarized residuals. There is generally a change in the sign of the residuals after 1988. This pattern implies that one of the assumptions of the model is badly wrong.

Introduction

We apply a statistical catch-at-age model with correlated to 2J3KL cod. We analyze the catch and fall research surveys for years up to 1993. Details of the methods are described in another document: "The Statistical Analysis of Catch-at-Age Data with Correlated Errors" (Myers and Cadigan 1994). Except for the assumption of the error distribution being correlated among ages within a year, we have followed all other assumptions traditionally used in the analysis as closely as possible. One minor change that we have made is the assumption used to estimate the numbers in the oldest age, A. We use the assumption that fishing mortality in the oldest age, A, is equal to that at age $A-1$, i.e. $F_{A,y} = F_{A-1,y}$.

The first difficulty that we had in the analysis was that there were no cod caught in 1993 over 8 years of age. We used two alternative strategies for dealing with this problem.

- We replaced the zero estimates of abundance for ages 9-12 in 1993 with half the minimum of the previously observed estimates. In our case we will replace the zero's with 0.005 (mean fish per tow).
- We gave the research surveys estimates zero weight for ages 9-12 in 1993. That is, we treated them as if this portion of the population was simply not surveyed.

Both strategies have obvious problems. In the first we constrain the estimates to be numbers that are without basis. Although they are probably within an order of magnitude of the correct numbers, that is all we can say. The second strategy is saying we know nothing about the abundance of these age groups. In fact we know quite a bit: they are so rare that none were caught in the surveys.

Methods

Population model

Some notation is first developed. $N_{a,y}$ is the number of age a fish in the population at the beginning of year y. $C_{a,y}$ is the number of age a fish in year y that are caught by the fishery. Lowercase letters are used to denote log transformations, e.g. $n_{a,y} = \log(N_{a,y})$. The number of ages and years modeled are A and Y, respectively.

The model for numbers at age is

$$N_{a,y} = C_{a,y}e^{m/2} + N_{a+1,y+1}e^{m}, \tag{1}$$

where m is the natural mortality rate which is usually assumed to be constant for all ages and years (Pope 1972). The catches are assumed to occur halfway through the year in (1).

Let $F_{a,y}$ denote the fishing mortality rate and $Z_{a,y} = m + F_{a,y}$ denote the total mortality. $F_{a,y}$ is defined as:

$$F_{a,y} = \log(\frac{N_{a,y}}{N_{a+1,y+1}}) - m.$$

Numbers at age A for all other years than Y are computed using $F_{A,y}$ and the solution for $N_{A+1,y+1}$ in (1):

$$\begin{aligned} N_{A,y} &= \frac{C_{A,y}\exp(M_{A,y}/2)}{1-\exp(-F_{A,y})}, \\ F_{A,y} &= F_{A-1,y}. \end{aligned} \tag{2}$$

We will call the numbers at age in the last year "survivors" and they will be denoted by $S_a \equiv N_{a,Y}$ and $s_a \equiv n_{a,Y}$. Given estimates of the S_a's, the numbers at age for all the cohorts represented in the last year can be reconstructed using Eq. (1). In Myers and Cadigan (1994) we considered several alternative models for $F_{A,y}$, and concluded that the above formulation gave the best fit.

Statistical models

Let $R_{a,y}$ be a random index of abundance from a research survey and let $r_{a,y} = \log(R_{a,y})$. We will not distinguish between a random variable and its observation by using the usual notation (normally upper and lower case is used for this purpose but the convention is in use here for logarithms); however, the distinction will be made when necessary.

Independent errors

The usual statistical model used in the analysis of commercial catch-at-age data assumes that the deviations in the log survey estimates from the population model are distributed as uncorrelated, normal random deviates with constant variance, i.e.

$$\begin{aligned} r_{a,y} &= q_a + n_{a,y} - tZ_{a,y} + \epsilon_{a,y}, \\ \epsilon_{a,y} &\overset{\text{ind}}{\sim} \mathrm{N}(0,\sigma^2), \end{aligned} \tag{3}$$

where q_a is the log catchability of the research surveys at age a and $\overset{\text{ind}}{\sim}$ denotes independently distributed over years and ages. The parameter t is the proportion of the year that has been completed when the survey takes place; for 2J3KL cod the surveys take place in November so we set $t = \frac{11}{12}$.

The parameters we estimate are the A survivors in year Y and the A survey catchability coefficients (by age). Note that the α_y's are constrained to be equal to one. The variance of the $\epsilon_{a,y}$ (σ^2) is also estimated. Let $\mathbf{r}$ be the random $A \times Y$ matrix of log survey numbers,

$$\mathbf{r} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdot & \cdot & \cdot & r_{1,Y} \\ r_{2,1} & r_{2,2} & \cdot & \cdot & \cdot & r_{2,Y} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ r_{A,1} & r_{A,2} & \cdot & \cdot & \cdot & r_{A,Y} \end{bmatrix} = [\mathbf{r}_{,1}, ..., \mathbf{r}_{,\mathrm{Y}}],$$

and let $\underline{\mathbf{r}}$ be the $AY \times 1$ vector of log survey numbers formed by stacking the columns of $\mathbf{r}$:

$$\underline{\mathbf{r}}_{\mathrm{AY}\times 1} = \begin{bmatrix} \mathbf{r}_{,1} \\ \mathbf{r}_{,2} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{r}_{,\mathrm{Y}} \end{bmatrix}.$$

The variance-covariance matrix is $\boldsymbol{\Sigma} = \mathrm{cov}(\underline{\mathbf{r}}) = \sigma^2\mathbf{I}$, where $\mathbf{I}$ is a $AY \times AY$ identity matrix.

Correlated errors

We consider an alternative statistical model which allows for correlated errors among ages in each year. We add a random effect (ξ) for years:

$$r_{a,y} = q_a + n_{a,y} - tZ_{a,y} + \epsilon_{a,y} + \xi_y, \quad (4)$$

$$\epsilon_{a,y} \overset{\text{ind}}{\sim} \mathrm{N}(0, \sigma^2), \quad (5)$$

$$\xi_y \overset{ind}{\sim} \mathrm{N}(0, \phi).$$

This is a mixed effects model in which all errors are equally correlated within a year but are independent between years. The correlation is $\rho = \phi/(\sigma^2 + \phi)$. The variance-covariance matrix is $\boldsymbol{\Sigma} = \mathrm{cov}(\underline{\mathbf{r}}) = \sigma^2\mathbf{I} + \phi(\mathbf{I}_\mathrm{Y} \otimes \mathbf{U}_\mathrm{A})$, where $\otimes$ denotes the Kronecker product, $\mathbf{I}_\mathrm{Y}$ is a $Y \times Y$ identity matrix and $\mathbf{U}_\mathrm{A}$ is an $A \times A$ matrix of 1's.

Maximum likelihood estimates

Maximum likelihood methods are used to estimate the model parameters under the two alternative error structures described above. The loglikelihood (l) is given by

$$l = k - \left[\log(\mid \boldsymbol{\Sigma} \mid) + (\underline{\mathbf{r}} - \mathrm{E}(\underline{\mathbf{r}}))'\boldsymbol{\Sigma}^{-1}(\underline{\mathbf{r}} - \mathrm{E}(\underline{\mathbf{r}}))\right]/2,$$

where k is a constant. The covariance matrix is combined with a weighting matrix to accomodate missing surveys and to extend the population beyond the oldest age that survey data is used for estimation. The covariance matrix actually modelled is:

$$\mathbf{W}^{-1}\boldsymbol{\Sigma}\mathbf{W}^{-1}$$

where $\mathbf{W}$ is a diagonal matrix of full rank. In this analysis the elements in $\mathbf{W}$ are either 1's or ∞'s; the ∞ elements correspond to missing surveys and *gives these surveys zero weight* in parameter estimation. Inferences are based on the marginal likelihoods of surveys with nonzero weights. Maximum likelihood parameter estimates are obtained using the iterative algorithms in Gumpertz and Pantula (1992). The derivatives of (3) with respect to the survivors, the q_a's, and the α_y's required in this algorithm are developed in (Myers and Cadigan 1994). The likelihood ratio test is used to test statistical hypotheses. To test that a subset of p parameters is equal to some specified value, models with and without the parameter constraints are fit yielding reduced and full log-likelihoods. Two times the difference between the reduced and full log-likelihoods is asymptotically a chi square random variable with p degrees of freedom (Cox and Hinkley 1974).

Results

We estimated parameters for four models.

For each of the above assumptions for the numbers of the older ages in 1993, we estimated models under the assumption that the errors are independent and are correlated.

The results for the case for which we replaced the zero in the RV surveys by 0.005 are shown in Tables 1-2. The results for the case where they were treated as missing was similar. For each run of the model we present:

- A summary table which includes the loglikelihood and the parameter estimates with standard standard errors. The estimated parameters are:
  1. The survivors are estimated from ages 3-12.
  2. The log catchabilities are estimated from ages 3-12 (the q_a's) .
  3. The independent estimation error variance, σ^2.
  4. The variance for the correlated errors, ϕ (for the correlated error model only).
- The numbers at age matrix (in thousands).
- The fishing mortality matrix.
- The residual matrix.
- The standarized residual matrix. This is done by multiplying the vector of differences between observed and predicted log survey numbers by the square-root inverse of the estimated covariance matrix.

Results

The following patterns can be seen from the data:

1. The correlated error model produces a much improved fit in both cases.
2. The correlated error model produces lower estimates of abundance under both assumptions about the older ages in 1992. In the model where we replaced the zero's by 0.005, the difference was very large, i.e. survivors were reduced from a total 82,145,000 fish of ages 3-12 to only 27,265,000.
3. The fishing mortality estimated in the last two years is generally higher in the correlated error model.
4. The fishing mortality in the in 1993 appears to be high in all cases, considering that the fishery was shut down.
5. In all cases there is extraordinary patterns in the unstandarized residuals. There is generally a change in the sign of the residuals after 1988.

Conclusions

We conclude that there is something fundamentally wrong with the assumptions used in the model. The use of correlated errors does not overcome this problem. It is unlikely that the problem is caused by an increase in natural mortality in 1991, the pattern in the residuals started in 1989.

It is possible that there has been large trends in misreporting that may be responsible for the pattern. This seems unlikely, but cannot be discounted at this time.

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Table 1. 2J3KL cod, Independent lognormal errors. Model bounds: ages 3 to 12, years 1978 to 1993.
Survey bounds: ages 3 to 12, years 1978 to 1993. Term F = alpha_y* average F for ages 11 to 11.

Age	Survivors	SE's	Catchabilities	SE's
3	21899.13	15480.51	-10.28	0.19
4	29510.23	13908.19	-9.66	0.18
5	18731.81	7779.66	-9.18	0.18
6	10232.21	4622.20	-8.77	0.18
7	1291.10	374.35	-8.60	0.18
8	269.25	77.49	-8.48	0.19
9	62.88	25.94	-8.36	0.19
10	40.86	22.11	-8.34	0.19
11	40.90	21.96	-8.47	0.19
12	66.67	28.63	-8.14	0.19

	Var est.	Loglik
sigma	0.5107	-163.26784

Numbers at age matrix

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	161277	359988	320678	350052	432743	338681	157784	129548	158288	172793	93839	36593	36519	21899
4	124128	129731	292756	261009	284260	353592	276700	128431	103957	127081	139937	69868	27145	29510
5	192330	90747	99726	211380	201375	219276	276083	212772	96811	71856	88084	77873	28561	18732
6	137966	131394	62361	64454	134515	136135	146398	186073	144941	60999	39694	39172	15072	10232
7	85457	85798	85136	38030	35553	75183	80764	78357	107952	75310	27129	11963	5331	1291
8	17886	53663	50326	46689	20239	17795	36214	42575	38393	47010	26634	7381	1593	269
9	4194	10273	33215	25885	24924	10014	8190	16328	17209	14597	12701	5574	465	63
10	1821	2333	5538	16394	13109	12344	4727	3897	8104	5922	4082	2114	270	41
11	1043	1021	1205	2794	7685	6908	5436	2061	1973	2711	1555	750	196	41
12	1030	644	556	681	1367	3997	3027	2661	1076	915	686	292	34	67
+	727132	865592	951498	1017368	1155769	1173925	995325	802704	678703	579194	434341	251581	115137	82145

Fishing mortality matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	0.00	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.01	0.02	0.02	0.01	0.09	0.10	0.01	0.05
4	0.07	0.06	0.11	0.06	0.13	0.06	0.06	0.05	0.06	0.08	0.17	0.17	0.39	0.69	0.17	0.21
5	0.23	0.22	0.18	0.18	0.24	0.25	0.19	0.20	0.19	0.18	0.26	0.39	0.61	1.44	0.83	0.22
6	0.43	0.27	0.28	0.23	0.29	0.39	0.38	0.32	0.43	0.34	0.45	0.61	1.00	1.79	2.26	0.24
7	0.58	0.44	0.27	0.33	0.40	0.43	0.49	0.53	0.44	0.51	0.63	0.84	1.10	1.82	2.79	0.91
8	0.57	0.56	0.35	0.28	0.46	0.43	0.50	0.58	0.60	0.71	0.77	1.11	1.36	2.56	3.03	0.92
9	0.50	0.50	0.39	0.42	0.51	0.48	0.50	0.55	0.54	0.50	0.87	1.07	1.59	2.83	2.23	0.46
10	0.42	0.50	0.38	0.46	0.48	0.56	0.44	0.62	0.63	0.48	0.90	1.14	1.49	2.18	1.69	0.00
11	0.50	0.40	0.28	0.41	0.37	0.52	0.45	0.63	0.51	0.45	0.57	1.17	1.47	2.89	0.88	0.00
12	0.50	0.40	0.28	0.41	0.37	0.52	0.45	0.63	0.51	0.45	0.57	1.17	1.47	2.89	0.88	0.00

Residual matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	-0.48	-0.81	-0.61	-0.69	-0.44	0.21	-0.14	-0.29	0.06	-0.58	-0.13	0.61	1.49	1.25	0.54	0.00
4	-0.18	-0.07	-0.44	-0.87	-0.85	-0.22	0.06	-0.38	0.40	-0.45	-0.39	0.36	0.91	1.96	0.62	-0.45
5	-0.05	0.13	-0.06	-0.71	-0.59	-0.29	-0.25	-0.45	0.45	-0.16	-0.21	0.36	0.69	1.62	0.86	-1.35
6	-0.01	-0.27	-0.04	-0.35	-0.79	-0.41	-0.25	-0.59	0.51	-0.41	-0.16	0.37	1.01	1.22	1.75	-1.58
7	-0.09	-0.43	-0.73	-0.10	-0.44	-0.48	-0.42	-0.52	0.20	-0.54	0.08	0.43	0.94	1.15	1.71	-0.76
8	-0.25	-0.01	-0.63	-0.67	-0.07	-0.02	-0.48	-0.68	0.37	-0.37	-0.24	0.50	1.10	1.41	0.72	-0.69
9	-0.11	-0.37	-0.43	-0.61	-0.45	-0.10	-0.38	-0.41	0.35	-0.37	0.03	0.43	1.08	1.46	0.36	-0.47
10	-0.36	-0.02	-0.11	-0.28	-0.21	-0.32	-0.29	-0.73	0.10	-0.38	0.11	0.86	0.98	1.03	0.10	-0.48
11	-0.12	-0.17	-0.24	-0.20	0.07	0.17	-0.49	-0.50	0.14	-0.33	-0.31	0.98	1.19	1.47	-1.29	-0.35
12	-0.05	-0.54	-0.19	0.02	-0.07	0.31	0.25	-1.29	-0.09	-0.81	-0.34	0.75	1.02	1.39	0.83	-1.17

Standardized Residual matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	-0.67	-1.13	-0.85	-0.97	-0.62	0.29	-0.19	-0.40	0.08	-0.81	-0.18	0.85	2.08	1.75	0.76	0.00
4	-0.25	-0.09	-0.62	-1.21	-1.19	-0.30	0.08	-0.53	0.56	-0.64	-0.55	0.50	1.27	2.74	0.87	-0.63
5	-0.07	0.18	-0.08	-1.00	-0.82	-0.40	-0.35	-0.62	0.64	-0.22	-0.30	0.51	0.96	2.26	1.20	-1.90
6	-0.01	-0.38	-0.06	-0.49	-1.11	-0.57	-0.34	-0.82	0.71	-0.58	-0.22	0.52	1.41	1.71	2.45	-2.21
7	-0.12	-0.60	-1.02	-0.14	-0.62	-0.68	-0.58	-0.72	0.28	-0.76	0.12	0.60	1.32	1.61	2.39	-1.07
8	-0.35	-0.01	-0.88	-0.94	-0.09	-0.03	-0.67	-0.95	0.52	-0.51	-0.34	0.70	1.55	1.98	1.01	-0.97
9	-0.15	-0.52	-0.61	-0.85	-0.63	-0.14	-0.53	-0.57	0.49	-0.52	0.04	0.59	1.51	2.05	0.51	-0.66
10	-0.50	-0.02	-0.15	-0.39	-0.30	-0.45	-0.41	-1.02	0.13	-0.53	0.15	1.20	1.37	1.44	0.14	-0.68
11	-0.17	-0.24	-0.34	-0.28	0.10	0.24	-0.69	-0.70	0.20	-0.47	-0.43	1.38	1.66	2.06	-1.80	-0.49
12	-0.07	-0.76	-0.26	0.03	-0.09	0.43	0.35	-1.81	-0.12	-1.13	-0.48	1.05	1.43	1.94	1.15	-1.64

Table 2. 2J3KL cod; Dependent lognormal errors. Model bounds: ages 3 to 12, years 1978 to 1993.
Survey bounds: ages 3 to 12, years 1978 to 1993. Term F = alpha_y* average F for ages 11 to 11.

Age	Survivors	SE's	Catchabilities	SE's
3	8275.30	3006.23	-10.07	0.21
4	10125.93	1851.44	-9.45	0.20
5	5046.58	574.46	-8.95	0.20
6	2726.85	253.85	-8.59	0.20
7	856.39	36.57	-8.46	0.20
8	180.56	8.04	-8.33	0.20
9	30.48	3.22	-8.16	0.21
10	8.50	2.62	-8.12	0.21
11	7.77	2.37	-8.21	0.21
12	6.65	1.77	-7.83	0.21

	Var est.	Loglik
phi	0.5816	-55.21868
sigma	0.0755	

Numbers at age matrix

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	161269	359968	320641	349976	432380	338517	157653	129440	158047	171826	80163	16177	12843	8275
4	124123	129725	292740	260979	284197	353295	276566	128324	103869	126883	139145	58671	10430	10126
5	192324	90743	99720	211366	201351	219225	275840	212662	96723	71784	87923	77224	19394	5047
6	137963	131389	62358	64449	134504	136115	146356	185874	144851	60927	39635	39040	14541	2727
7	85455	85795	85133	38028	35549	75174	80747	78322	107788	75237	27070	11915	5223	856
8	17886	53662	50324	46686	20237	17792	36207	42562	38365	46876	26574	7333	1554	181
9	4194	10273	33214	25883	24921	10012	8188	16322	17198	14574	12592	5524	426	30
10	1821	2333	5538	16393	13107	12342	4726	3895	8099	5913	4063	2025	229	9
11	1043	1021	1205	2794	7684	6907	5434	2060	1971	2707	1548	734	123	8
12	1030	644	556	681	1367	3997	3026	2660	1075	913	683	286	21	7
+	727108	865553	951429	1017235	1155297	1173375	994744	802122	677986	577640	419395	218930	64784	27265

Fishing mortality matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	0.00	0.01	0.02	0.01	0.01	0.01	0.00	0.00	0.01	0.02	0.02	0.01	0.11	0.24	0.04	0.13
4	0.07	0.06	0.11	0.06	0.13	0.06	0.06	0.05	0.06	0.08	0.17	0.17	0.39	0.91	0.53	0.79
5	0.23	0.22	0.18	0.18	0.24	0.25	0.19	0.20	0.19	0.18	0.26	0.39	0.61	1.47	1.76	1.32
6	0.43	0.27	0.28	0.23	0.29	0.39	0.38	0.32	0.43	0.34	0.46	0.61	1.00	1.81	2.63	1.54
7	0.58	0.44	0.27	0.33	0.40	0.43	0.49	0.53	0.44	0.51	0.63	0.84	1.11	1.84	3.16	2.34
8	0.57	0.56	0.35	0.28	0.46	0.43	0.50	0.58	0.60	0.71	0.77	1.11	1.37	2.65	3.73	2.30
9	0.50	0.50	0.39	0.42	0.51	0.48	0.50	0.55	0.54	0.50	0.87	1.08	1.63	2.98	3.71	1.43
10	0.42	0.50	0.38	0.46	0.48	0.56	0.44	0.62	0.63	0.48	0.90	1.14	1.51	2.60	3.19	0.00
11	0.50	0.40	0.28	0.41	0.37	0.52	0.45	0.63	0.51	0.45	0.57	1.18	1.49	3.34	2.72	0.00
12	0.50	0.40	0.28	0.41	0.37	0.52	0.45	0.63	0.51	0.45	0.57	1.18	1.49	3.34	2.72	0.00

Residual matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	-0.68	-1.01	-0.81	-0.90	-0.65	0.00	-0.34	-0.49	-0.14	-0.78	-0.33	0.41	1.46	1.99	1.41	0.85
4	-0.38	-0.27	-0.65	-1.07	-1.06	-0.42	-0.14	-0.58	0.19	-0.66	-0.60	0.15	0.71	2.12	1.70	0.95
5	-0.27	-0.10	-0.28	-0.94	-0.81	-0.51	-0.47	-0.67	0.23	-0.38	-0.44	0.14	0.47	1.43	1.88	0.74
6	-0.19	-0.45	-0.22	-0.53	-0.98	-0.59	-0.43	-0.77	0.32	-0.60	-0.34	0.19	0.83	1.05	1.95	0.76
7	-0.22	-0.56	-0.86	-0.23	-0.58	-0.62	-0.55	-0.65	0.07	-0.67	-0.05	0.30	0.82	1.04	1.94	0.82
8	-0.40	-0.16	-0.78	-0.83	-0.22	-0.17	-0.63	-0.83	0.22	-0.52	-0.39	0.35	0.96	1.34	1.24	0.82
9	-0.31	-0.58	-0.64	-0.81	-0.65	-0.30	-0.58	-0.61	0.14	-0.58	-0.18	0.23	0.91	1.41	1.61	0.94
10	-0.58	-0.24	-0.33	-0.50	-0.44	-0.54	-0.51	-0.95	-0.13	-0.60	-0.11	0.64	0.78	1.24	1.41	0.86
11	-0.39	-0.44	-0.51	-0.47	-0.20	-0.10	-0.76	-0.77	-0.13	-0.60	-0.57	0.72	0.94	1.63	0.59	1.04
12	-0.36	-0.85	-0.49	-0.29	-0.37	-0.00	-0.06	-1.60	-0.39	-1.12	-0.65	0.45	0.73	1.51	2.67	0.82

Standardized Residual matrix

	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	0.44	1.18	-0.28	1.10	-0.07	0.88	-0.26	-0.14	0.52	0.45	-0.50	-0.31	0.01	-0.47	-0.29	0.05
4	-0.34	0.26	-0.89	0.29	-0.02	-0.55	-0.09	-0.08	0.61	-1.61	-0.97	-0.30	-0.81	0.34	-0.20	-1.03
5	-1.32	-0.15	-0.28	-0.96	0.36	0.75	0.38	-1.25	0.67	3.65	-0.19	0.85	-0.53	0.88	0.56	-0.48
6	1.86	-1.66	-0.71	-0.31	-0.60	-0.69	-0.37	-1.87	0.24	0.33	-0.12	-1.89	-0.27	0.70	0.34	1.92
7	1.01	0.57	0.39	0.97	-1.01	-0.08	-0.97	0.26	1.63	-0.11	0.92	-2.72	-0.43	-0.21	0.47	-0.49
8	-0.66	0.01	0.18	1.05	0.41	2.42	1.43	-0.31	-0.58	-0.13	0.30	0.51	0.28	-0.55	-0.73	-0.42
9	-0.61	0.78	1.70	1.32	-0.63	0.19	-0.53	-1.75	-0.79	0.29	0.69	-0.60	0.45	-2.32	-0.86	-0.61
10	0.00	0.83	-0.06	-0.82	0.08	0.29	-0.18	0.96	0.53	0.51	-1.12	0.67	0.96	0.51	1.12	-0.85
11	-0.36	1.35	-0.33	0.54	0.64	-0.22	-3.22	0.52	0.99	1.12	-0.17	0.98	1.41	-1.27	-0.58	-0.77
12	-0.23	-0.71	-3.35	-0.25	-2.32	-1.01	1.65	-0.87	-0.06	1.61	-0.03	0.85	-0.69	1.26	2.14	1.12