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Was an increase in natural mortality responsible for the collapse of northern cod?

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Abstract

The collapse of the "northern cod" fishery off southern Labrador and the northern Grand Bank of Newfoundland from the largest cod fishery in the world was a social and economic disaster for the region. An analysis of traditional catch-at-age data in conjunction with research surveys, which assumed that research survey estimation errors of abundance by age year were independent, led assessment biologist to the conclusion that the collapse was caused by an increase in natural mortality. We construct a statistical model to test the hypothesis that the decline in abundance of cod was caused by a sudden increase in natural mortality in the winter of 1991. The results of the statistical analysis do not support this hypothesis. There is ambiguous evidence that natural mortality has increased since 1991; however, these results are found only in a model which has extraordinary patterns in the residuals. Our analysis suggests that even if natural mortality has been higher in recent years (as estimated using a model with correlated errors for research surveys), that overfishing was sufficiently high to cause a collapse of this population. We also demonstrate the usual assumption that estimation errors from research trawl surveys are independent is not valid, and can lead to invalid inference and unreasonable estimates of abundance.

Introduction

The cod fishery off southern Labrador and the northern Grand Bank of Newfoundland,

known as "northern cod" or Northwest Atlantic Fisheries Organization (NAFO) Division 2J3KL cod, was once the largest in the world (Garrod 1988). However, in 1992 the low numbers of cod in this region resulted in the Canadian Government imposing a two-year moratorium on its cod fishery (Lear and Parsons 1993). The estimated spawner biomass had decreased from 1.6 million t in 1962 to 22 thousand t in 1992 (Fig. 1). The immediate cause of the moratorium was a 500 thousand t drop of the estimate of harvestable biomass between 1991 and 1992. Although the causes of the decline in cod abundance are uncertain, one possibility is a sudden increase in natural mortality in the winter of 1991 (NAFO 1992). The hypothesis that natural mortality had increased was suggested by the observation that 1991 was unusually cold, and a large discrepancy in the predicted numbers at age and the estimated numbers at age from research trawl surveys. It was believed that an increase in natural mortality during 1991 could resolve this discrepancy. The purpose of this paper is to examine this hypothesis.

To test if natural mortality changed during 1991, we construct a statistical model with and without a parameter that represents the change in mortality and then estimate this parameter with estimates of abundance from research surveys. We then test the significance of the change in mortality. This requires that an appropriate model for the errors in the research surveys be used. Most fisheries models assume that the errors in the research survey estimates of numbers at age are independent among years and ages (Pope and Shepherd 1972, Gavaris 1988, Deriso et al. 1989, Pope and Stokes 1989, Shepherd and Nicholson 1991, Mohn and Cook 1993). However, this assumption was recently examined by Myers and Cadigan (1993) and found to be violated for most research surveys examined. They found from analyses of multiple surveys of the same population that there were generally positive correlations in the errors among ages for a given year. For example, if the number of three year olds were overestimated in 1990, the number of four year olds were usually overestimated as well. The correlations may occur because the efficiency of the sampling gear is not constant. For example, environmental conditions or differences in how the crew handles the fishing gear may affect the efficiency.

Methods

The adaptive framework for the estimation of population size presented by Gavaris (1988) is used. Population numbers at age are estimated in the last year for which survey data are available, and then the age composition for all years is constructed. The estimates

are obtained using maximum likelihood techniques and the assumption that the logarithm survey data are normally distributed. The methodology is constructed where possible to conform with that used in the cod stock assessment in 2J3KL (Bishop et al. 1993). For example, our population model is extended to age 13 and we use surveys from 1978 to 1992 for ages 3 - 12 and we removed the last year of data, 1992, to determine if we could replicate previous analyses.

Population model

Some notation is first developed. $N_{a,y}$ denotes the number of age a fish in the population at the beginning of year y . $C_{a,y}$ denotes the number of age a fish in year y that are caught by the fishery and $R_{a,y}$ denotes an index of abundance from a research survey. Lowercase letters are used to denote log transformations, e.g. $n_{a,y} = \log(N_{a,y})$ and $r_{a,y} = \log(R_{a,y})$. The number of ages and years modeled are A and Y , respectively.

The model for numbers at age is

$$N_{a,y} = C_{a,y}e^{m/2} + N_{a+1,y+1}e^m, \quad (1)$$

where m is the natural mortality rate which is usually assumed to be constant for all ages and years (Pope 1972). Numbers at age in the last year are estimated and are called the survivors. Let $F_{A,y}$ denote the fishing mortality rate and $Z_{A,y} = m + F_{A,y}$. Numbers at age A are approximated (Mohn and Cook, 1993) as:

$$N_{A,y} = \frac{C_{A,y}(1 + m/F_{A,y})}{1 - e^{-Z_{A,y}}}, \quad (2)$$

where $F_{A,y}$ is computed from fishing mortalities at younger ages in the same year. These fishing mortalities are:

$$F_{a,y} = \log\left(\frac{N_{a,y}}{N_{a+1,y+1}}\right) - m.$$

For the 2J3KL cod stock, we consider models where $m = 0.2$ for all years and ages except either the winter of 1991 or since the winter of 1991 where m is unknown and is estimated. We estimate the difference in the unknown mortality in the winter of 1991 (Δm_1) and since the winter of 1991 (Δm_2) from the nominal assumption of 0.2. That is, assuming that mortality varied just in the winter of 1991,

$$N_{a,1991} = C_{a,1991}e^{(m+\Delta m_1)/2} + N_{a+1,1992}e^{m+\Delta m_1/2}.$$

$N_{A,1991}$ is computed by replacing m with $m + \Delta m_1/2$ in (2). If mortality has changed since 1991 then (1) can be used with m replaced by $m + \Delta m_2$ for $y = 1991$ and 1992.

In the analysis of commercial catch-at-age data, the fishing mortality at the oldest age, $F_{A,y}$, is usually assumed to be a function of the fishing mortality at younger ages. We consider two assumptions. First, we examine a model in which older ages are assumed to be less vulnerable to fishing. For this model, $F_{A,y}$ is assumed to be one-half the average fishing mortalities for 7 to 9 year olds. Second, we examine a model that assumes that fishing mortality is approximately constant for the older ages in a given year, i.e. $F_{A,y}$ is equal to the average of fishing mortalities for ages 10 to 12 year olds. These alternative assumptions have been used for different assessments of this population. Our conclusions should be robust to either assumption for our results to be convincing.

Statistical model

Independent errors

The usual statistical model used in the analysis of commercial catch-at-age data assumes that the deviations in the log(survey estimates) from the population model are uncorrelated, normal random deviates with constant variance, i.e.

$$\begin{aligned} r_{a,y} &= q_a + n_{a,y} - tZ_{a,y} + \epsilon_{a,y}, \\ \epsilon_{a,y} &\underset{\sim}{\text{ind}} N(0, \sigma^2), \end{aligned} \quad (3)$$

where parameter t is the proportion of the year that has been completed when the survey took place, q_a is the catchability of the research surveys at age a and $\underset{\sim}{\text{ind}}$ denotes independently distributed over years and ages. Eq. (3) applies to some subset of the ages and years the population model covers. We use surveys during 1978-1992 for fish aged 3-12. In our case, we use surveys that usually take place in November, so we assume that $t = \frac{11}{12}$. The parameters we estimate are the $A - 1 = 12$ survivors in year Y and survey catchability coefficients (by age), Δm 's, and σ^2 . Note that $n_{a,y}$ is a nonlinear function of the survivors.

Let \mathbf{r} be the $(A - 1) \times Y$ matrix of log survey numbers and let \mathbf{r} be the $(A - 1)Y \times 1$ vector of log survey numbers formed by stacking the columns of \mathbf{r} . The variance-covariance matrix is $\Sigma = \text{cov}(\mathbf{r}) = \sigma^2 \mathbf{I}$, where \mathbf{I} is a $(A - 1)Y \times (A - 1)Y$ identity matrix.

Correlated errors

We consider an alternative statistical model which allows for correlated errors among ages in each year. We add a random effect (ξ) for years:

$$r_{a,y} = q_a + n_{a,y} - tZ_{a,y} + \epsilon_{a,y} + \xi_y, \quad (4)$$

$$\begin{aligned} \epsilon_{a,y} & \stackrel{\text{ind}}{\sim} N(0, \sigma^2), \\ \xi_y & \stackrel{\text{ind}}{\sim} N(0, \phi). \end{aligned} \tag{5}$$

This is a mixed effects model in which all errors are equally correlated within a year but are independent between years. The correlation is $\rho = \phi/(\sigma^2 + \phi)$. The variance-covariance matrix is $\Sigma = \text{cov}(\mathbf{r}) = \sigma^2 \mathbf{I} + \phi(\mathbf{I}_Y \otimes \mathbf{U}_{A-1})$, where \otimes denotes the Kronecker product (defined in the **Appendix**), \mathbf{I}_Y is a $Y \times Y$ identity matrix and \mathbf{U}_{A-1} is an $A - 1 \times A - 1$ matrix of 1's.

Linear models with random components, e.g. mixed models, are a well developed branch of statistics (Searle *et al.* 1992). Our models are applications of recent research with non-linear models.

Maximum likelihood estimates

Maximum likelihood (or equivalently least-squares) parameter estimates are obtained using the algorithms in Gumpertz and Pantula (1992). The derivatives of (3) with respect to the survivors, q_a 's and Δm 's are required in this algorithm and can be obtained from (1) and (2). Gumpertz and Pantula (1992) present a model with the same error structure as assumed here and call it a "onefold nested error structure"; we use a computational simplification they present for computing Σ^{-1} .

The likelihood ratio test is used to test statistical hypotheses. To test that a subset of p parameters is equal to some specified value, models with and without the parameter constraints are fit yielding reduced and full log-likelihoods. Two times the difference between the reduced and full log-likelihoods is asymptotically a chi square random variable with p degrees of freedom, (Cox and Hinkley 1974). For example, to test that $\phi = 0$ one computes twice the difference between the log-likelihoods obtained with independent and correlated errors and compares this value with the critical value for a $\chi^2_{(1-\alpha),1}$ where α is the level of the test. The loglikelihood (l) is given by

$$l = k - [\log(|\Sigma|) + (\mathbf{r} - \mathbf{E}(\mathbf{r}))' \Sigma^{-1} (\mathbf{r} - \mathbf{E}(\mathbf{r}))] / 2,$$

where k is a constant.

Cod at different ages are not caught with the same efficiency in the research surveys; this is why the survey catchability coefficients (q_a 's) are estimated for each age. There is unknown misreporting and aging errors in the catches that could lead to considerable error in their reporting. We do not address this problem here. We fit models with and without

Δm 's, with and without correlated errors, and using the two assumptions about the $F_{A,y}$, i.e. we examine 12 models in all. We also repeated the analyses using only data up to 1991 because this portion of the data was used to generate the original hypothesis.

Results

Results using data up to 1993

We analyze the commercial catch-at-age data and the fall research vessel indices of abundance at age (Table 1). Our method produces very similar results as those in Bishop et al. (1993) if the same estimation procedure is employed (i.e. no change in m in 1991 and independent errors; Table 2). If independent errors are assumed, the estimate of Δm_1 appears significant for both assumptions about $F_{13,y}$ (Fig. 2 and 3). Δm_1 in the winter of 1991 is estimated to be very large, and results in an increase in the estimates of survivors (Table 2). Inferences are very similar for the model with a change in natural mortality since 1991 (Δm_2 , Table 2). The estimated number of survivors for this model is very large, and the 1991 population size of ages 3-12 fish is the highest in the analysis - 3 times the population size in the mid 1980's. If errors are independent, these results suggest that the estimated increase in natural mortality is significant and consistent with a population that rapidly increased from 1987-1991 with a decline after 1991 (Fig. 4). The large increase in 1991 is completely unrealistic, and suggests that inferences made under the assumption of independent errors are unreliable.

There is very strong evidence that the errors are not independent (Fig. 2 and 3). The estimate of ϕ is highly significant whether Δm 's are included or not and regardless of which assumption is made about $F_{A,y}$. There is an enormous improvement in model fit under the assumption of correlated errors. The estimate of Δm_1 is not significantly different from 0 when model errors are assumed to be correlated within years. Δm_1 in the winter of 1991 does not have a significant effect on the estimates of survivors in this case. Under the assumption of correlated errors, the estimates of survivors decrease from that produced under the assumption of independent errors. Inferences are very similar for the model with Δm_2 (Table 2).

In each model fit under the assumption of correlated errors the population numbers estimated in recent years are less than those estimated under the assumption of independent errors (Table 2, Fig. 4). The population trajectories under the different model assumptions for the correlated errors are all very similar (Table 2, Fig. 4a). The major differences among these estimates is that the high fishing mortalities estimated in 1991 and 1992 are

reduced (Fig. 4b). In all cases examined under the assumption of correlated errors, fishing mortalities for 1989 and 1990 are high, e.g. above 1 for the older ages (Table 3).

The model fit is better under the assumption that $F_{13,y}$ is equal to the average of $F_{10,y}, F_{11,y}, F_{12,y}$ than the assumption that $F_{13,y}$ is equal to one half the average of $F_{7,y}, F_{8,y}, F_{9,y}$. This is evidence against the hypothesis that older fish are less vulnerable to the fishery.

The model errors in Eq. (4), i.e. the correlated errors model, should be standardized to compare with those in the independent errors model. This is done by multiplying the vector of differences between observed and predicted survey numbers by the square-root inverse of the estimated covariance matrix. We present the estimated model errors under the assumption that $F_{13,y}$ is equal to the average of $F_{10,y}, F_{11,y}, F_{12,y}$ (Fig. 5, Table 3). The patterns in the model errors are similar under the alternative assumption examined. Including correlated errors greatly reduces the year effects apparent in the model errors. Our approach has not eliminated all year effects, but it has clearly reduced them.

Our analysis demonstrates most of the variability in the research surveys is due to correlated errors, i.e. ϕ is much larger than σ^2 (Table 2). This result is clear from an examination of the residuals in the models which assume independent errors (Fig. 5).

We repeated the above analysis using only data up to 1991 because this portion of the data was used to claim that natural mortality had increased. Our result that there is no detectable increase in natural mortality remained the same.

Results using the 1993 data

We repeated the analysis using the 1993 survey and catch data. The catch data was very poorly estimated, and thus this analysis may not be as reliable as the previous analysis. The first difficulty that we had in the analysis was that there were no cod caught in 1993 over 8 years of age. We replaced the zero estimates of abundance for ages 9-12 in 1993 with half the minimum of the previously observed estimates. In our case we will replace the zero's with 0.005 (mean fish per tow).

The test of the hypothesis that natural mortality increased only in the first part of 1991 resulted in an estimate, under the correlated error model, of a decrease in natural mortality to 0.08 (S.E. = 0.53). This confirms the previous analyses that the joint analysis of the catch-at-age and the research surveys do not support this hypothesis. However, the fishing mortalities estimated are very high in both cases after 1990. The unstandardized residuals were also all positive after 1988 and negative before, except for 6 ages in 1986.

We next considered the correlated error model in which we estimated a change in mortal-

ity after the beginning of 1991. In this case the increase in natural mortality was significant, i.e. the increase was estimated to be 0.86 (S.E.=0.19). However, the extraordinary pattern in the residuals remain, all except 3 of the residuals after 1993 are positive. That is, an increase in natural mortality after 1991 does not "fix" the pattern in the residuals (Table 4). Again, the fishing mortality after 1987 is estimated to be very high, around one.

We considered that the residual pattern in the natural mortality may be caused by an increase in mortality before 1991. If natural mortality increased from 1988 on we estimated the increase to be 0.23 (S.E.=0.12), after 1989 the estimated increase was 0.26 (S. E. =0.14), and after 1990 the estimated increase was 0.41 (S. E. =0.17). However, in no case was the pattern in the residuals greatly reduced. In all cases the estimated fishing mortality was high after 1987.

We next considered the hypothesis that there had been a change in the efficiency of the research vessel trawls. We estimated the model for the years 1987-1993. The correlated error model again gave a much improved fit to the data. The estimated increase in natural mortality after 1991 was similar to that estimated above using all the data, 0.81 (S.E. = 0.32). However, this did not fit the pattern in the residuals, they were negative before 1990 and positive after in the correlated error model with or without an increase in natural mortality. In both cases the fishing mortality after 1987 was still very high, above one. In the case where mortality did not increase it was very high, i.e. it was above two after 1990.

Discussion

Changes in adult natural mortality?

Using the data up to 1993, there is no evidence from the joint analysis of the commercial catch at age data and the research surveys (with correlated errors) that natural mortality was the cause of the sudden change in the estimated abundance of northern cod. The pattern of residuals in the original catch-at-age analysis appear to be largely caused by correlated errors among ages in a given year. With the inclusion of the 1993 data, there is still no evidence for the original hypothesis, i.e. that natural mortality increased during the first monthes of 1991, and then returned to normal. However, we do estimate a nominally statistically significant increase for the period beginning in 1991 if we include the 1993 data. The reliability of this test must be questioned for three reasons. First, many statistical tests were performed in this analysis which will reduce the confidence we have when one particular hypothesis is significant. Second, the catch-at-age data for 1993 is very unreliable because

the catch was almost entirely recreational or outside Canadian jurisdiction, which were not adequately sampled. Third, estimates of the abundance from the research survey in 1993 were zero for ages above 9, which is not consistent with the lognormal model assumptions. Fourth, and more importantly, severe problems with model specification become clear when the 1993 data is included: the residuals change sign around 1987. This problem is not eliminated by estimating a higher natural mortality or by using only the last 7 years of data in the analysis. We must conclude that there is another problem with one of the model assumptions, e.g. the assumption that the catch-at-age is known exactly, and this means that it is very difficult to rely on any analysis using the 1993 data.

There is other evidence that natural mortality did not increase in the winter of 1991. Hutchings and Myers (in press) showed that the large amount of sea ice and the cold water temperatures of 1991 were not atypical of the period from 1800 to 1950. During this period the cod fishery of 200 thousand t a year was sustained; it is unlikely that such a large fishery would have been sustainable if natural mortality was substantially larger than it is now. Furthermore, Hutchings and Myers (in press) used a commercial catch-per-unit effort analyses to argue that the decline was actually not sudden, but was gradual, and the decline could have been caused by fishing mortality alone. This result was clear in the analysis of both the commercial trawl and gill net catch rates. Our analysis also does not suggest a sudden decline in abundance (Fig. 4a), the decline appears to have begun in 1985.

Is there evidence for other marine fish that the interannual variability in post larval mortality is significant? Although there is a long history of anecdotal evidence for severe climatic conditions causing mass mortalities of marine fish (Cushing 1982); there appears to have been only one study to attempt to quantify the importance of this variability in general. Myers and Cadigan (1993) tested the hypothesis that the interannual variability of the density-independent component of juvenile natural mortality is a major source of variability in abundance of marine demersal fish. They found that, except for North Sea sole, there was very little or no interannual variability in the density-independent component of juvenile mortality in the 14 stocks examined. Thus, we conclude, that there is no firm evidence that variable mortality in marine fish is generally important.

If natural mortality did or did not increase in the winter of 1991, our analysis suggests that fishing mortality was very high during the last few years of the fishery. Are such high fishing mortalities possible? One mechanism that may have led to high fishing mortalities is an increase in the concentration of the fish. We calculated the degree of concentration of the fish by using the standard method used in econometrics to study the distribution of

income among individuals (Dagum 1985). For each year from 1981 to 1992 we calculate a Lorenz curve for the fall research surveys in 2J3KL for the 76 strata used in the survey. To calculate concentration, consider the n strata used in the surveys. Let x_i be the estimated biomass of cod in strata i such that $x_1 \leq x_2 \leq \dots \leq x_n$. Let the area of strata i be a_i . The Lorenz curve is the polygon joining the points $(A_h/A_n, L_h/L_n)$, $h = 0, 1, \dots, n$ where $L_0 = 0$ and $L_h = \sum_{i=1}^h x_i$ is the total number of fish in the h strata with the fewest individuals, and $A_0 = 0$ and $A_h = \sum_{i=1}^h a_i$ is the total area of the h strata with the fewest individuals. Hence, the Lorenz curve has as its abscissa the cumulative area arrayed by increasing biomass, and as its ordinate the corresponding proportion of the total population of fish. If fish were equally distributed among strata, the Lorenz curve would be the identity function. As the the distribution of fish becomes more unequal, i.e. more concentrated, the Lorenz curve bends downwards and to the right within the unit square. The area between the identity function and the Lorenz curve is known as the Gini index, and is the most commonly accepted measure of the concentration.

The increase in concentration of cod over time in the research surveys is clear from the Lorenz curves (Fig. 6a) and the Gini index (Fig. 6b). This increase started well before 1991. It is unclear what has caused this increase, although overfishing is an obvious possibility. We do not claim that the estimates of fishing mortality in Table 3 are exact. They depend upon estimates which have large estimation error, and a model with misspecification (e.g. catches are not known exactly). However, an increase in fishing mortality would be possible without an increase in fishing effort if the fish were more concentrated, and thus more catchable (Clark 1990, Hillborn and Walters. 1992).

Even if natural mortality did increase in recent years as (Table 2, line 6), our analysis demonstrates that fishing mortality is still estimated to be very high, i.e. greater than 1 for older ages, in 1989 and 1990. Thus, it is not possible to blame the collapse of northern cod on an increase in natural mortality alone, even if it did occur.

Estimation with correlated error structure

The analysis is consistent with Myers and Cadigan (1993) results that the estimation errors in research surveys are generally positively correlated among ages for a given year. The source of this correlation is unknown; environmental conditions that would effect catchability, differences in how the crew handles the fishing gear, or sampling variability are obvious possible causes.

It is crucial that an appropriate error structure be used in the statistical analysis of catch

at age data. The assumption that the estimation errors in the research surveys are independent led to unsupportable inference of an increase in natural mortality during the winter of 1991. The assumption that the errors are independent and natural mortality is constant lead to greater estimates of abundance and less fishing mortality than the assumption of correlated errors (Table 2).

An alternative approach has been used in the Northeast Pacific (Fournier and Archibald 1982, Methot 1989) in which the estimation errors among ages in a given year was assumed to follow a multinomial distribution in which the variance was inflated by a predetermined, subjective factor and the estimation error for absolute abundance is assumed to follow a lognormal distribution. The important difference for statistical inference between these methods is that we estimate the relative sizes of the estimation error variance within and among years, instead of relying on a subjective "emphasis" factor (Methot 1989). As we have shown, the relative sizes of these error variances are crucial for inference, and should not be subjectively determined.

The correlation among ages for the research surveys has important consequences for the management of fish populations. First, assessment biologists should be humble about their knowledge of the stock status. The estimates for any one year simply are not sufficiently reliable to draw firm conclusions. Second, management strategies should be sufficiently conservative so that an overestimate of abundance in two consecutive years does not lead to overexploitation.

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Appendix

If \mathbf{A} is an $m \times n$ matrix and \mathbf{B} is an $r \times s$ matrix then the Kronecker product (direct product or tensor product) of \mathbf{A} and \mathbf{B} is

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{1,1}\mathbf{B} & a_{1,2}\mathbf{B} & \dots & a_{1,n}\mathbf{B} \\ a_{2,1}\mathbf{B} & a_{2,2}\mathbf{B} & \dots & a_{2,n}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1}\mathbf{B} & a_{m,2}\mathbf{B} & \dots & a_{m,n}\mathbf{B} \end{bmatrix}$$

$\mathbf{A} \otimes \mathbf{B}$ is a matrix of order $(mr \times ns)$.

References

- Bishop, C.A., Murphy, E.F., Davis, M.B., Baird, J.W., and G.A. Rose. 1993. An assessment of the cod stock in NAFO Divisions 2J+3KL. NAFO SRCR Doc. 93/86, Ser. No. N2271: 51 p.
- Clark, C. W. 1990. *Mathematical bioeconomics*. 2nd ed. New York: Wiley-Interscience.
- Cox, D. R., and D. V. Hinkley. 1974. *Theoretical Statistics*. Chapman and Hall, London, J.K. 210 p.
- Cushing, D.H. 1982. **Climate and Fisheries**. Academic Press, London, 373 p.
- Dagum, C. 1985. Lorenz curve. p. 156-161. *In* S. Kotz and N. L. Johnson [ed.] *Encyclopedia of statistical sciences*. Vol 5. Wiley and Sons, New York.
- Deriso, R. B., T. J. Quinn II, and P. R. Neal. 1985. Catch-age analysis with auxiliary information. *Can. J. Fish. Aquat. Sci.* 42: 815-824.
- Fournier, D. and C. Archibald. 1982. A general theory for analyzing catch at age data. *Can. J. Fish. Aquat. Sci.* 39: 1195-1207.
- Garrod, D. J. 1988. North Atlantic cod: fisheries and management to 1986, p. 185-218. *In* J. A. Gulland (Ed.) *Fish population dynamics: the implications for fishery management*. John Wiley and Sons, New York, NY. 422 p.
- Gavaris, S. 1988. An adaptive framework for the estimation of population size. *CAFSAC Res. Doc.* 88/29: 12 p.
- Gumpertz, M. L., and S. G. Pantula. 1992. Nonlinear regression with variance components. *J. Amer. Stat. Assoc.* 87: 201-209.
- Hillborn, R. and C. J. Walters. 1992. *Quantitative fisheries stock assessment: choice, dynamics and uncertainty*. New York, Chapman and Hall, 570 p.
- Hutchings, J. A. and R. A. Myers. in press. What can be learned from the collapse of a renewable resource? Atlantic cod, *Gadus morhua*, of Newfoundland and Labrador. *Can. J. Fish. Aquat. Sci.*
- Lear, W.H., and L.S. Parsons. 1993. History and management of the fishery for northern cod in NAFO Divisions 2J, 3K and 3L, p. 55-89. *In* L.S. Parsons and W.H. Lear [eds.] *Perspectives on Canadian marine fisheries management*. *Can. Bull. Fish. Aquat. Sci.* 226.

- R. D. Methot. 1989. Synthetic estimates of historic abundance and mortality for Northern Anchovy. *Am. Fish. Soc. Symp.* 6: 66-82.
- Mohn, R. K., and R. Cook. 1993. Introduction to sequential population analysis. NAFO SCS NO 17. 110 p.
- Myers, R. A. and N. G. Cadigan. 1993. Is juvenile natural mortality in marine demersal fish variable? *Can. J. Fish. Aquat. Sci.* 50: 1576-1590.
- NAFO 1992 Report of the Special Meeting of Scientific Council, June 1992. NAFO SCS. Doc. 92/20 Ser NO N2112.
- Pope, J. G. 1972. An investigation of the accuracy of virtual population analysis. *ICNAF Res. Bull.* 9, 65-74.
- Pope, J. G. and T. K. Stokes. 1972. Use of multiplicative models for separable virtual population analysis (VPA), integrated analysis, and the general VPA tuning problem. *Am. Fish. Soc. Symp.* 6: 92-101.
- Pope, J. G. and J. G. Shepherd. 1972. A simple method for the consistent interpretation of catch-at-age data. *J. Cons. int. Explor. Mer.* 40: 176-184.
- Searle, S. R., G. Casella, and C. E. McCulloch. 1992. Variance components. John Wiley and Sons, New York, NY. 501 p.
- Shepherd, J. G., and M. D. Nicholson. 1991. Multiplicative modelling of catch-at-age, and its application to catch forecasts. *J. Cons. int. Explor. Mer.* 47: 284-294.

Table 1a. Fall research survey abundance estimates for 2J3KL cod (average catch in numbers per tow).

Age	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
3	5.39	1.94	2.48	5.12	5.87	12.22	10.79	7.27	4.77	2.04	3.93	8.98	10.93	3.35	1.78
4	11.51	11.78	3.83	2.74	5.92	10.62	15.23	12.35	20.70	4.03	3.20	8.30	12.95	13.97	2.30
5	13.95	16.79	13.23	3.26	3.83	10.83	11.34	10.01	31.29	13.23	5.29	6.20	8.61	9.00	2.72
6	5.51	10.53	13.31	9.67	2.79	3.87	9.59	7.28	21.28	11.61	10.57	6.52	5.64	3.31	1.42
7	1.62	2.27	4.99	8.79	5.82	2.43	2.30	4.24	10.14	4.38	10.13	8.23	3.90	1.10	0.35
8	0.63	0.92	1.19	3.66	5.31	5.33	1.37	0.92	5.26	2.67	2.58	4.84	3.98	0.50	0.04
9	0.47	0.31	0.37	0.74	2.59	2.93	2.09	0.78	1.37	1.38	1.55	1.62	1.68	0.35	0.02
10	0.33	0.26	0.23	0.23	0.57	1.42	1.30	0.67	0.58	0.34	0.79	0.98	0.55	0.16	0.01
11	0.12	0.19	0.11	0.10	0.16	0.36	0.54	0.41	0.68	0.17	0.15	0.43	0.23	0.04	0.00
12	0.09	0.06	0.16	0.11	0.09	0.14	0.28	0.15	0.42	0.19	0.11	0.16	0.12	0.02	0.01

Table 1b. Catches for 2J3KL cod in thousands of fish.

Age	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
3	1323	1152	2554	2185	1702	2585	782	650	831	2329	2779	1696	7693	3111	430
4	17556	12361	12025	7172	31286	13616	14871	14824	15219	9217	14651	17639	40557	31654	3860
5	39206	37439	28814	13191	19003	42602	31760	36614	44168	32340	20184	21150	36410	53805	14535
6	20319	29202	30016	24800	14397	19028	38624	33922	45869	49061	47917	25212	22695	29553	12211
7	7711	10982	18017	22014	25435	12044	12503	28006	26025	28469	45725	38708	16390	9064	4526
8	3078	3460	4830	11848	16930	14701	7246	7050	14722	19505	18608	28499	17940	6164	1372
9	1530	1300	1217	3175	11936	8934	8910	3836	3104	5818	9026	8696	9156	4745	376
10	1083	757	520	779	1923	6341	4227	5162	2000	1346	4337	3640	2865	1696	199
11	437	560	232	309	338	1018	2536	2905	1977	676	774	1695	1084	641	104
12	219	183	229	195	156	248	451	1681	1101	873	422	572	478	250	18
13	105	116	56	125	90	90	146	254	574	391	366	244	103	88	9

Table 2. Model estimates for 2J3KL cod under the twelve combinations of assumptions considered. Estimates constrained to be zero are denoted by "-". Numbers are in millions at the beginning of the year. Standard errors are in parentheses. The parameters are: the variance of the random year effects, ϕ , the correlation of the estimation errors among ages in a year, ρ , the change in natural mortality, Δm , and the residual variance of the estimation errors, σ^2 .

$F_{A,y}$ assumption	ϕ	ρ	Δm	σ^2	Log likelihood	1992 Numbers at age 3	1992 Numbers at ages 3-12
	-	-	-	0.27	-113.43	77 (41)	332 (68)
	-	-	2.41* (0.26)	0.18	-83.10	101 (44)	347 (62)
average of	-	-	1.48** (0.09)	0.11	-47.97	446 (162)	922 (184)
$F_{10,y}, F_{11,y}$ and $F_{12,y}$	0.25	0.85	-	0.04	-10.21	71 (24)	204 (41)
	0.22	0.83	0.31* (0.43)	0.05	-9.97	72 (25)	207 (42)
	0.19	0.81	0.39** (0.22)	0.05	-8.95	85 (29)	240 (51)
	-	-	-	0.31	-124.87	77 (44)	336 (74)
1/2 the	-	-	2.62* (0.28)	0.20	-92.55	105 (49)	355 (69)
average of	-	-	1.56** (0.09)	0.12	-56.49	501 (192)	1004 (215)
$F_{7,y}, F_{8,y}$ and $F_{9,y}$	0.31	0.86	-	0.05	-18.88	57 (21)	173 (36)
	0.34	0.87	-0.37* (0.53)	0.05	-18.60	55 (20)	169 (36)
	0.29	0.85	0.12** (0.29)	0.05	-18.79	61 (24)	183 (44)

Note: * corresponds to Δm_1 and ** corresponds to Δm_2 .

Table 3a. Estimated population numbers in thousands for 2J3KL cod from model with correlated errors, $\Delta m_1 = 0$, and $F_{13,y}$ is equal to the average of $F_{10,y}, F_{11,y}, F_{12,y}$

Age	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
3	160609	359772	320673	350102	432419	338689	157865	129797	162171	189539	136529	54453	70711
4	123306	129184	292580	261005	284300	353327	276707	128497	104162	130260	153647	104820	41768
5	189194	90073	99278	211235	201372	219309	275867	212778	96865	72024	90687	89098	57177
6	135896	128827	61810	64087	134397	136132	146425	185896	144945	61043	39831	41303	24263
7	83837	84103	83035	37579	35253	75086	80762	78379	107806	75314	27165	12075	7075
8	17383	52338	48939	44968	19869	17549	36135	42574	38412	46891	26637	7411	1685
9	4324	9862	32130	24749	23515	9711	7989	16264	17207	14611	12604	5576	490
10	1673	2439	5201	15506	12179	11190	4480	3732	8051	5921	4094	2034	272
11	928	900	1292	2518	6957	6146	4491	1858	1838	2667	1554	760	131
12	699	550	457	752	1141	3402	2404	1888	910	804	650	292	42
13	187	365	274	233	391	526	1264	972	756	363	141	100	13
+	718036	858412	945667	1012734	1151793	1171068	994388	802634	683122	599437	493540	317922	203626

Table 3b. Estimated fishing mortalities for 2J3KL cod.

Age	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
3	0.02	0.01	0.01	0.01	0.00	0.00	0.01	0.02	0.02	0.01	0.06	0.07	0.01
4	0.11	0.06	0.13	0.06	0.06	0.05	0.06	0.08	0.17	0.16	0.34	0.41	0.11
5	0.18	0.18	0.24	0.25	0.19	0.20	0.19	0.18	0.26	0.39	0.59	1.10	0.33
6	0.28	0.24	0.30	0.40	0.38	0.32	0.42	0.34	0.45	0.61	0.99	1.56	0.81
7	0.27	0.34	0.41	0.44	0.50	0.53	0.44	0.51	0.63	0.84	1.10	1.77	1.23
8	0.37	0.29	0.48	0.45	0.52	0.59	0.60	0.71	0.77	1.11	1.36	2.52	2.30
9	0.37	0.44	0.53	0.51	0.54	0.57	0.56	0.50	0.87	1.07	1.62	2.82	1.89
10	0.42	0.44	0.53	0.60	0.48	0.71	0.68	0.51	0.90	1.14	1.48	2.54	1.66
11	0.32	0.48	0.34	0.59	0.52	0.74	0.67	0.51	0.63	1.21	1.47	2.69	2.10
12	0.45	0.50	0.47	0.45	0.57	0.79	0.71	0.72	0.72	1.54	1.67	2.94	0.64
13	0.40	0.47	0.45	0.55	0.52	0.75	0.68	0.58	0.75	1.30	1.54	2.73	1.46

Table 3c. Standardized log residuals for 2J3KL cod.

Age	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
3	-0.67	2.13	-1.14	-0.23	-0.20	-1.01	0.45	-0.37	0.99	0.16	-0.79	-0.21	-1.13	-0.67	0.57
4	0.45	1.12	-1.52	0.98	0.17	-1.48	0.86	0.86	-0.39	0.22	0.54	-0.02	1.38	-0.57	-0.62
5	-0.23	0.05	-0.21	-0.32	-1.40	-0.85	1.65	-0.21	1.54	0.60	-0.90	-1.06	0.63	-0.98	1.10
6	0.52	-1.51	1.22	1.89	2.12	-0.26	-2.54	1.66	0.79	0.34	-0.18	0.83	0.88	0.97	-0.33
7	-1.80	0.16	0.99	0.96	-1.39	-0.58	0.53	0.38	-0.49	0.08	-0.68	1.47	0.41	-2.20	-1.11
8	0.01	-0.22	-0.15	1.93	0.72	-0.72	0.16	-0.96	1.21	-1.13	0.73	0.47	0.50	0.78	-0.28
9	-1.64	0.35	0.52	-0.91	-1.22	0.98	-0.12	-0.83	-1.38	0.83	0.49	-1.60	-0.23	-0.82	-0.28
10	1.12	-0.43	-0.96	1.43	-3.76	0.21	-0.22	1.57	0.47	0.51	0.22	0.99	0.20	-0.37	-0.89
11	1.09	-0.09	0.48	0.35	0.60	-0.51	0.39	-2.34	-0.68	-0.41	-0.18	0.86	-0.41	2.35	-0.03
12	0.50	-0.15	0.00	0.09	-1.10	-1.00	1.29	-0.26	0.74	-0.87	-0.29	-0.70	-0.79	1.91	-0.44

Residual matrix

ERRORM	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	-0.50	-0.83	-0.64	-0.73	-0.48	0.16	-0.18	-0.35	-0.06	-0.78	-0.51	0.02	0.66	1.19	1.25	1.78
4	-0.27	-0.15	-0.54	-0.97	-0.95	-0.32	-0.05	-0.49	0.26	-0.64	-0.69	-0.14	0.09	1.59	1.45	1.81
5	-0.20	-0.02	-0.20	-0.87	-0.75	-0.45	-0.42	-0.62	0.28	-0.36	-0.49	-0.07	-0.10	0.95	1.59	1.72
6	-0.13	-0.39	-0.16	-0.46	-0.91	-0.53	-0.38	-0.73	0.36	-0.57	-0.35	0.04	0.28	0.61	1.60	1.71
7	-0.20	-0.50	-0.80	-0.17	-0.51	-0.57	-0.50	-0.62	0.09	-0.66	-0.05	0.19	0.35	0.60	1.59	1.77
8	-0.34	-0.17	-0.73	-0.78	-0.16	-0.10	-0.58	-0.80	0.24	-0.52	-0.42	0.25	0.51	0.91	0.93	1.77
9	-0.26	-0.51	-0.70	-0.77	-0.62	-0.25	-0.51	-0.58	0.16	-0.59	-0.23	0.08	0.41	0.94	1.58	1.84
10	-0.40	-0.18	-0.26	-0.63	-0.39	-0.50	-0.46	-0.83	-0.09	-0.60	-0.17	0.46	0.25	0.76	1.42	1.62
11	-0.20	-0.22	-0.51	-0.43	-0.45	-0.08	-0.76	-0.71	0.05	-0.61	-0.63	0.55	0.33	0.76	1.11	1.78
12	-0.51	-0.49	-0.16	-0.25	-0.29	-0.37	0.02	-1.50	-0.21	-0.75	-0.58	0.60	0.46	0.81	1.08	2.17

Standardized Residual matrix

RESIDM	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
3	-0.44	-0.13	-1.35	-2.01	0.34	0.94	0.33	-3.55	-0.13	-0.72	-0.25	0.87	0.29	-1.29	-0.06	-0.57
4	1.26	0.87	1.12	0.83	1.17	-0.31	1.82	0.13	-0.69	-0.35	0.22	2.09	-0.87	0.18	1.50	-0.84
5	0.27	-0.71	0.45	0.21	1.18	-0.09	-0.02	0.62	0.63	-0.78	-1.98	-0.16	0.61	1.67	0.48	-0.52
6	-0.02	0.90	-0.76	-0.30	-0.51	-0.55	0.48	-0.41	1.21	-0.00	0.20	0.60	-0.45	-0.61	-0.01	-0.99
7	-0.92	1.26	0.75	1.59	0.72	-0.55	1.48	-1.17	-0.62	-0.23	0.61	-0.45	0.09	-1.28	-0.48	0.15
8	0.80	-0.23	-0.66	1.11	-0.22	-0.68	-1.16	1.52	-2.26	0.87	1.81	0.66	-0.10	0.58	-0.41	0.27
9	-0.01	-0.71	1.39	0.85	-0.22	1.45	0.18	0.38	1.19	-0.42	0.86	0.50	-0.54	-1.17	-0.83	0.44
10	1.46	-2.01	0.54	-1.01	-1.01	1.93	0.35	0.42	-0.97	0.66	0.56	0.02	-1.23	-1.39	-0.64	1.26
11	0.45	0.03	-0.14	-0.46	0.56	0.58	-2.07	-0.26	-0.65	-0.77	-0.32	-0.70	-1.06	-0.11	-0.76	1.87
12	1.33	-0.96	-2.09	-0.43	-0.87	0.65	-1.56	-1.43	1.57	-0.43	0.17	1.72	-2.23	0.13	-0.41	2.47

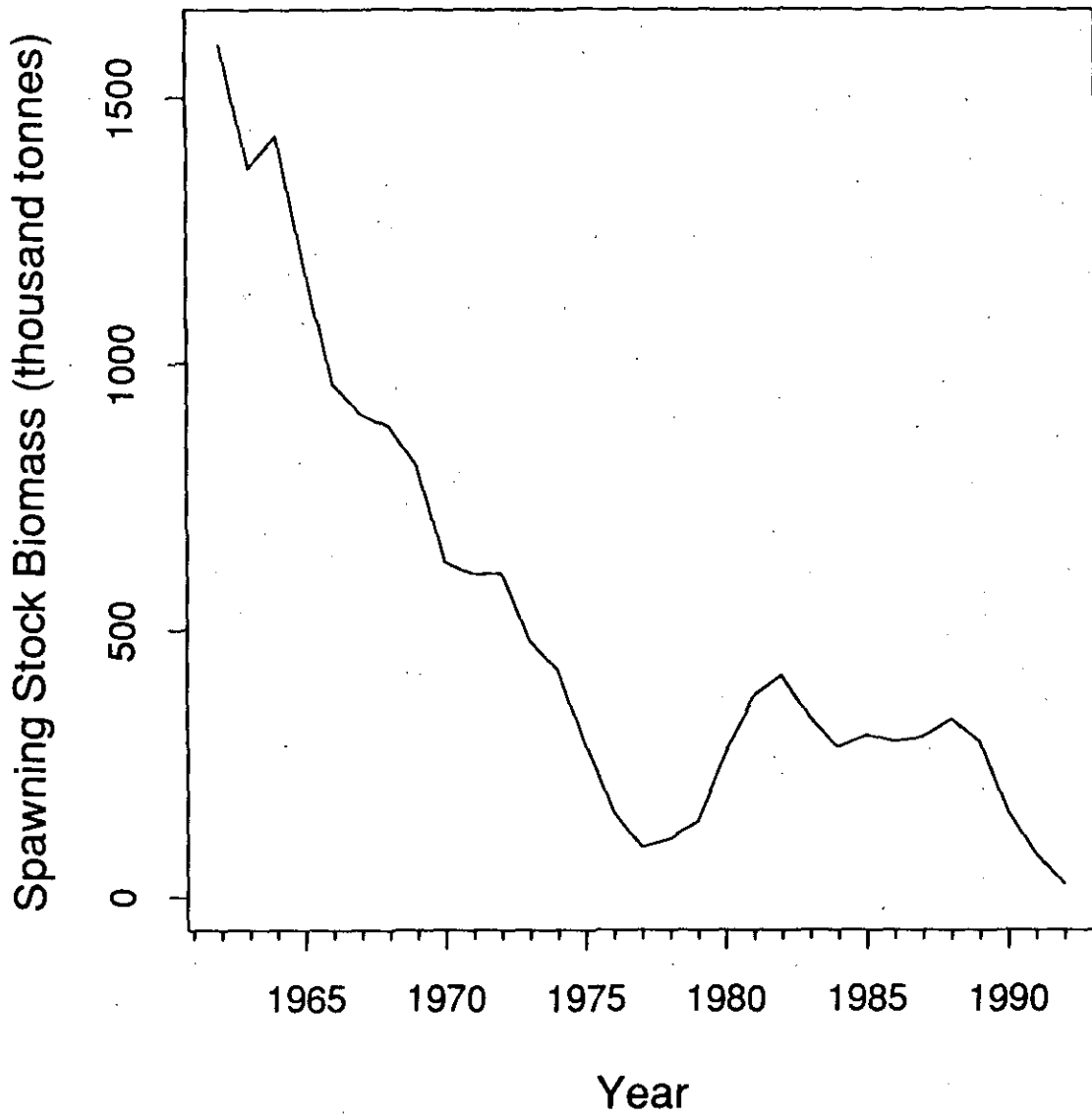


Fig. 1. Estimates of spawning stock biomass (age 7 and older) of cod in NAFO Div. 2J3KL from Bishop et al. (1993).

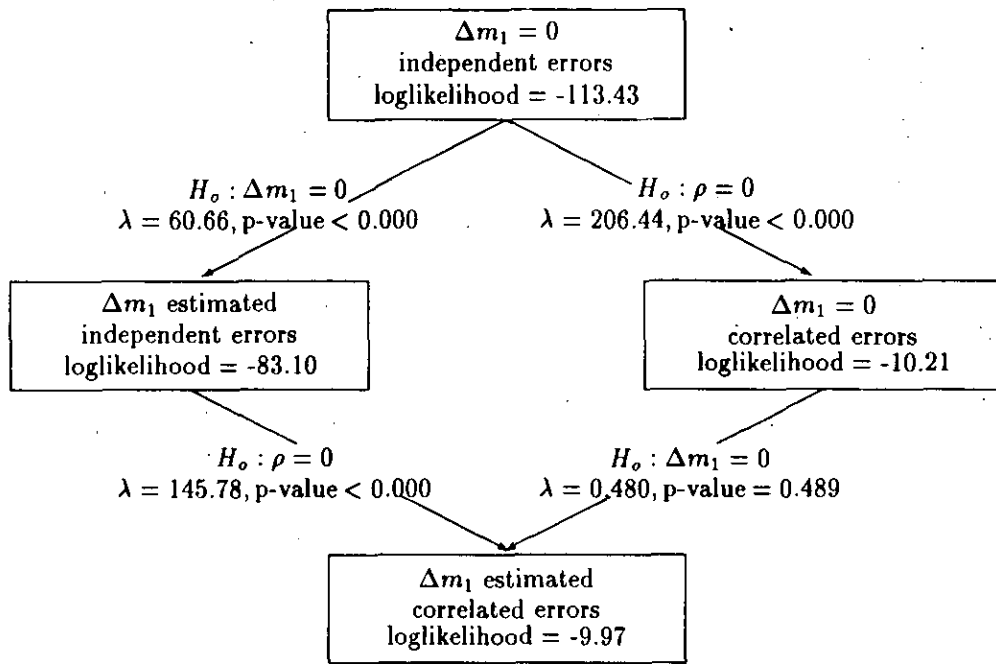


Fig. 2. Likelihood ratio tests. The degrees of freedom equals 1 for all tests. The alternative hypothesis is that the parameter, i.e. ρ or Δm_1 , is not equal zero. $F_{13,y}$ is equal to the average of $F_{10,y}$, $F_{11,y}$, $F_{12,y}$.

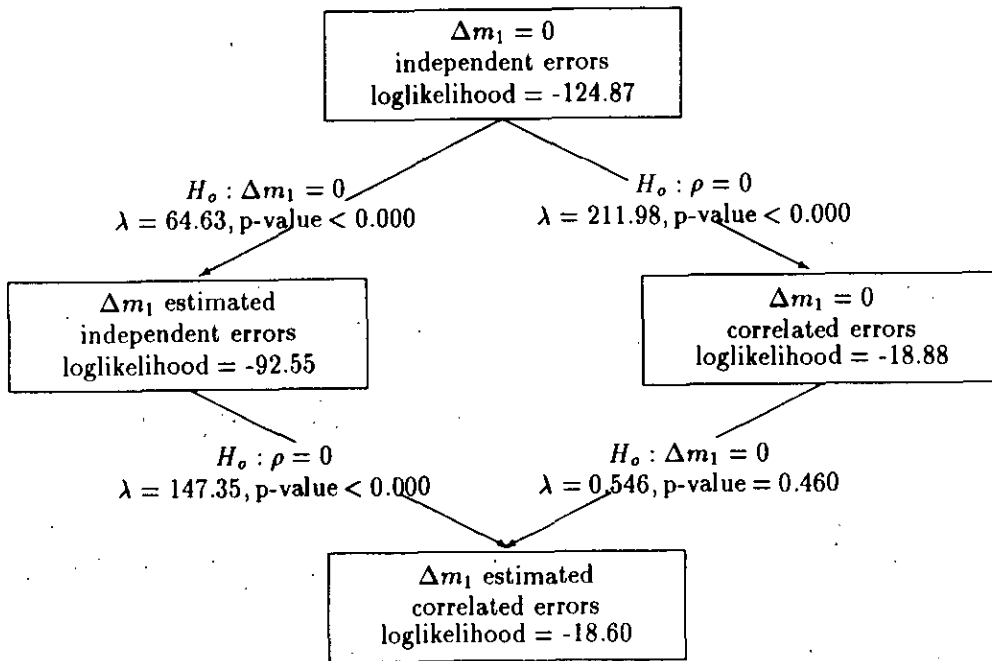


Fig. 3. Likelihood ratio tests. The degrees of freedom equals 1 for all tests. The alternative hypothesis is that the parameter, i.e. ρ or Δm_1 , is not equal zero. $F_{13,y}$ is one-half the average of $F_{7,y}$, $F_{8,y}$, $F_{9,y}$.

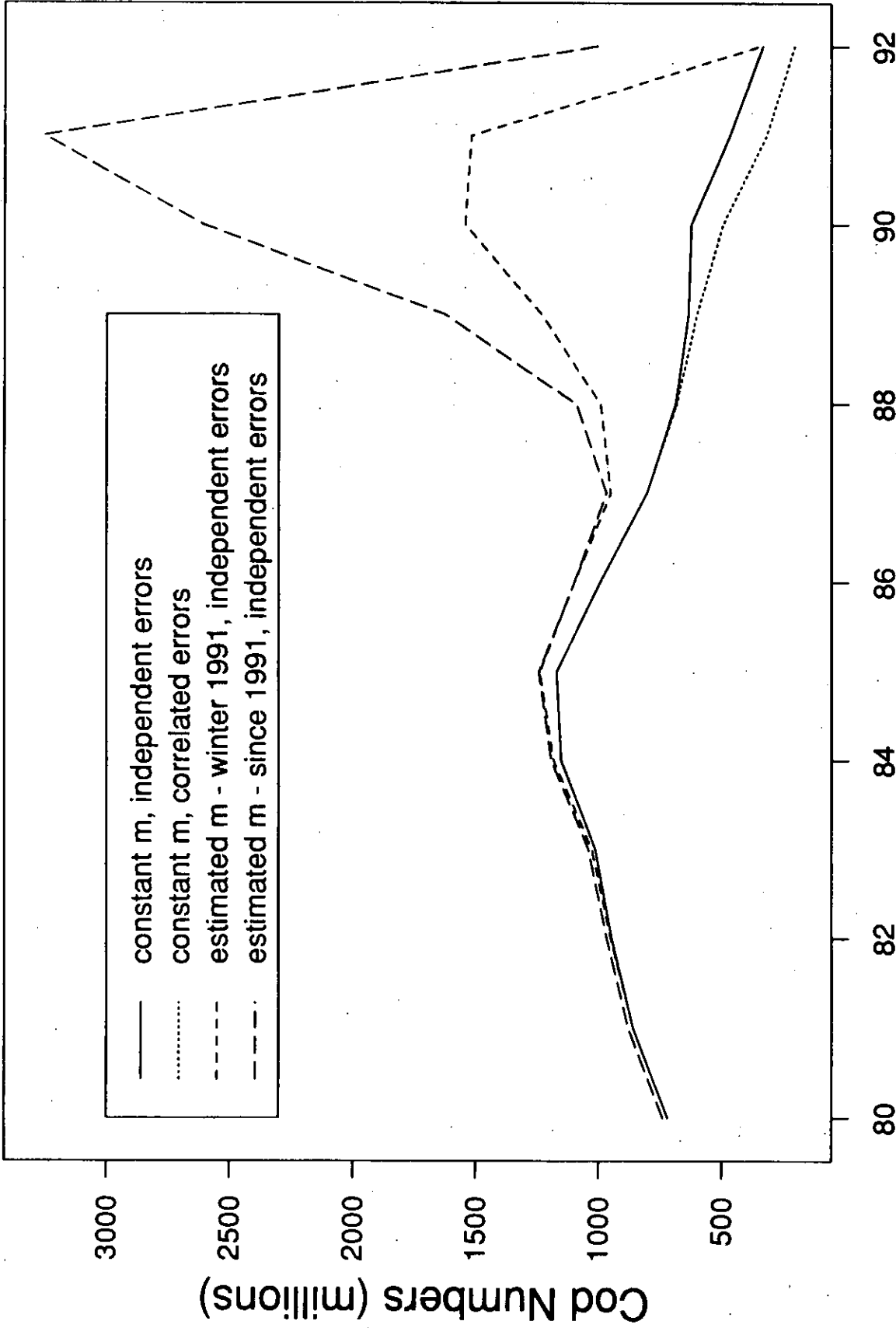


Fig. 4a. Estimated numbers of cod (ages 3-12) under the following assumptions: (1) constant natural mortality, independent errors (—), (2) constant natural mortality, correlated errors (···), (3) natural mortality estimated in winter of 1991, independent errors (---), (4) natural mortality estimated since 1991, independent errors (- - -), and (5) natural mortality estimated since 1991, correlated errors (— — —). In each of the above cases we assumed that $F_{13,y}$ is equal to the average of $F_{10,y}, F_{11,y}, F_{12,y}$.

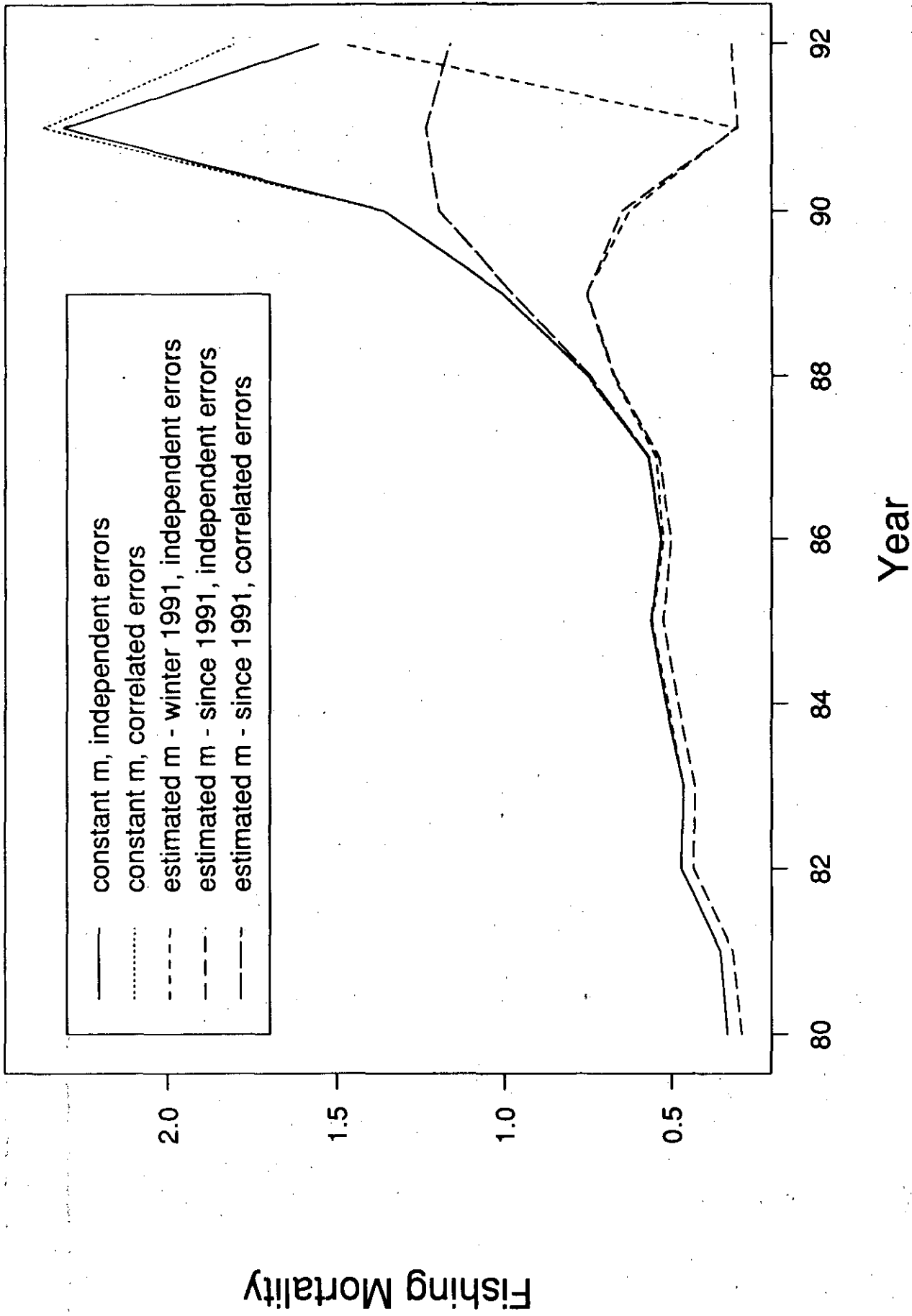


Fig. 4b. Estimated fishing mortality of cod (ages 7-9) under the assumptions listed in

Fig. 4a.

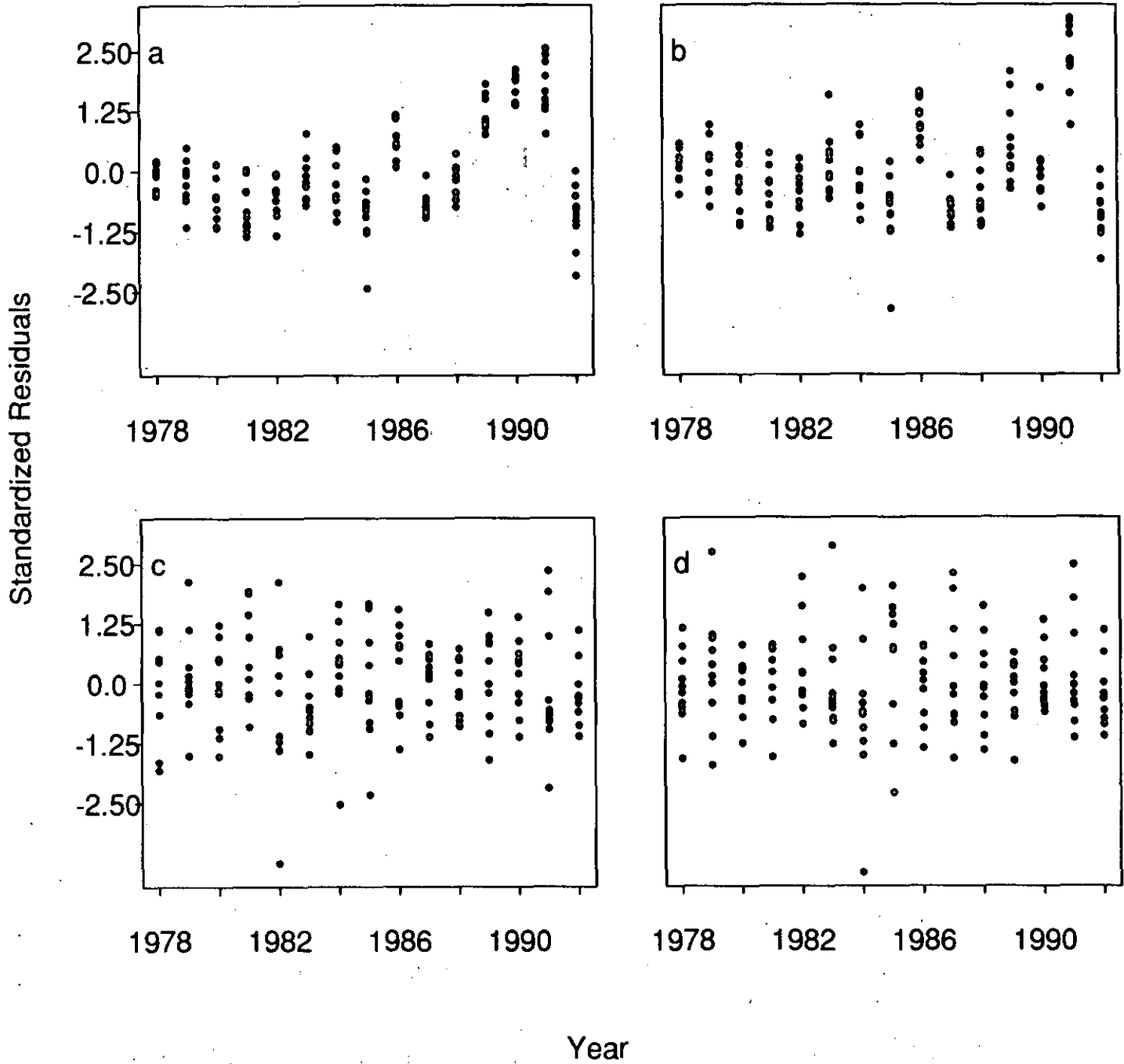
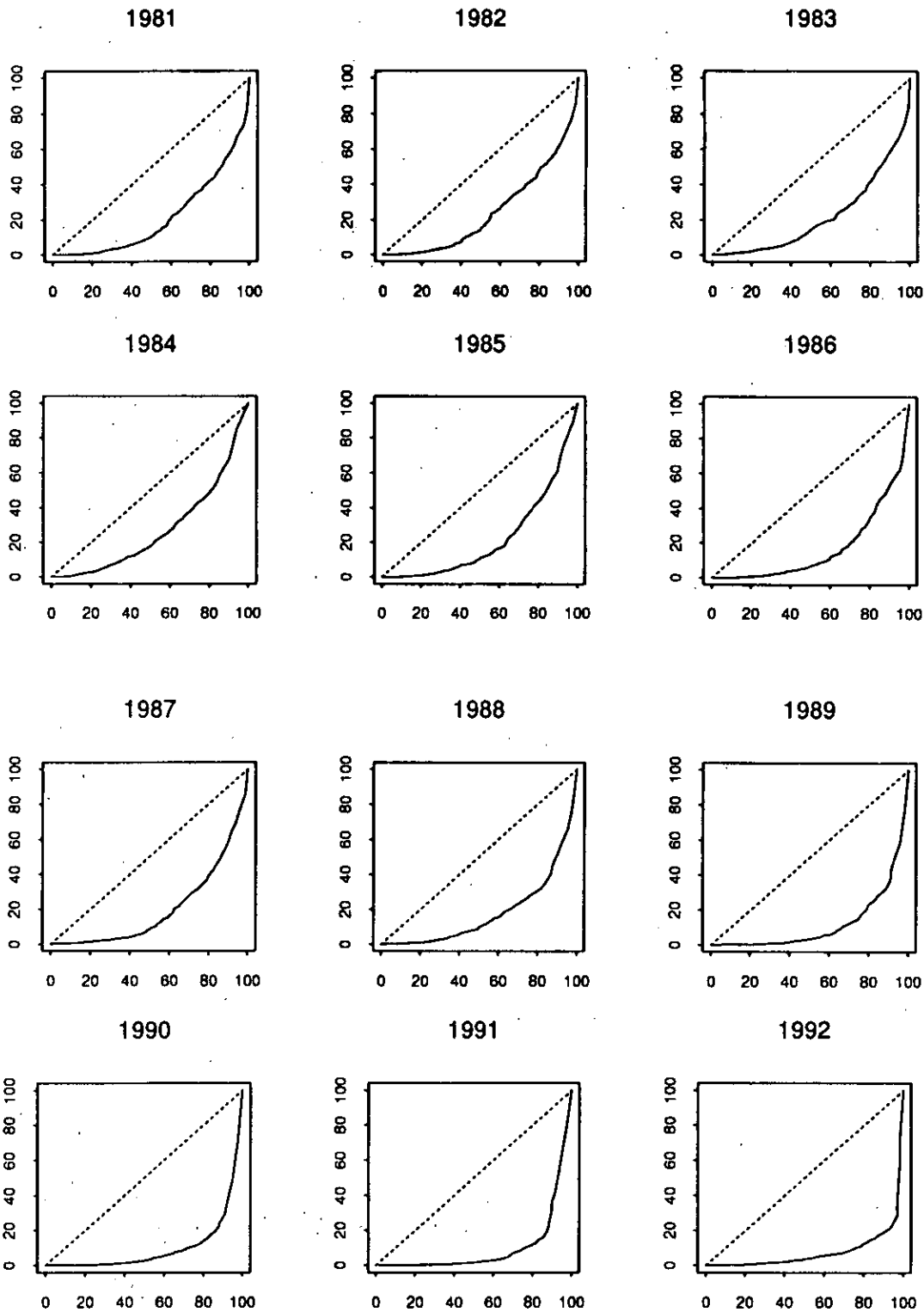


Fig. 5. Model errors under the assumption that $F_{13,y}$ is equal to the average of $F_{10,y}, F_{11,y}, F_{12,y}$. a. independent errors and constant m assumed. b. independent errors assumed and Δm_1 estimated. c. correlated errors and constant m assumed. d. correlated errors assumed and Δm_1 estimated.

Cumulative % Biomass



Cumulative % Area

Fig. 6a. Lorenz diagrams for NAFO Div. 2J3KL cod. The Lorenz curve bends downwards and to the right within the unit square as cod become more concentrated. If fish were equally distributed among strata, the Lorenz curve would be the identity function (dotted line).

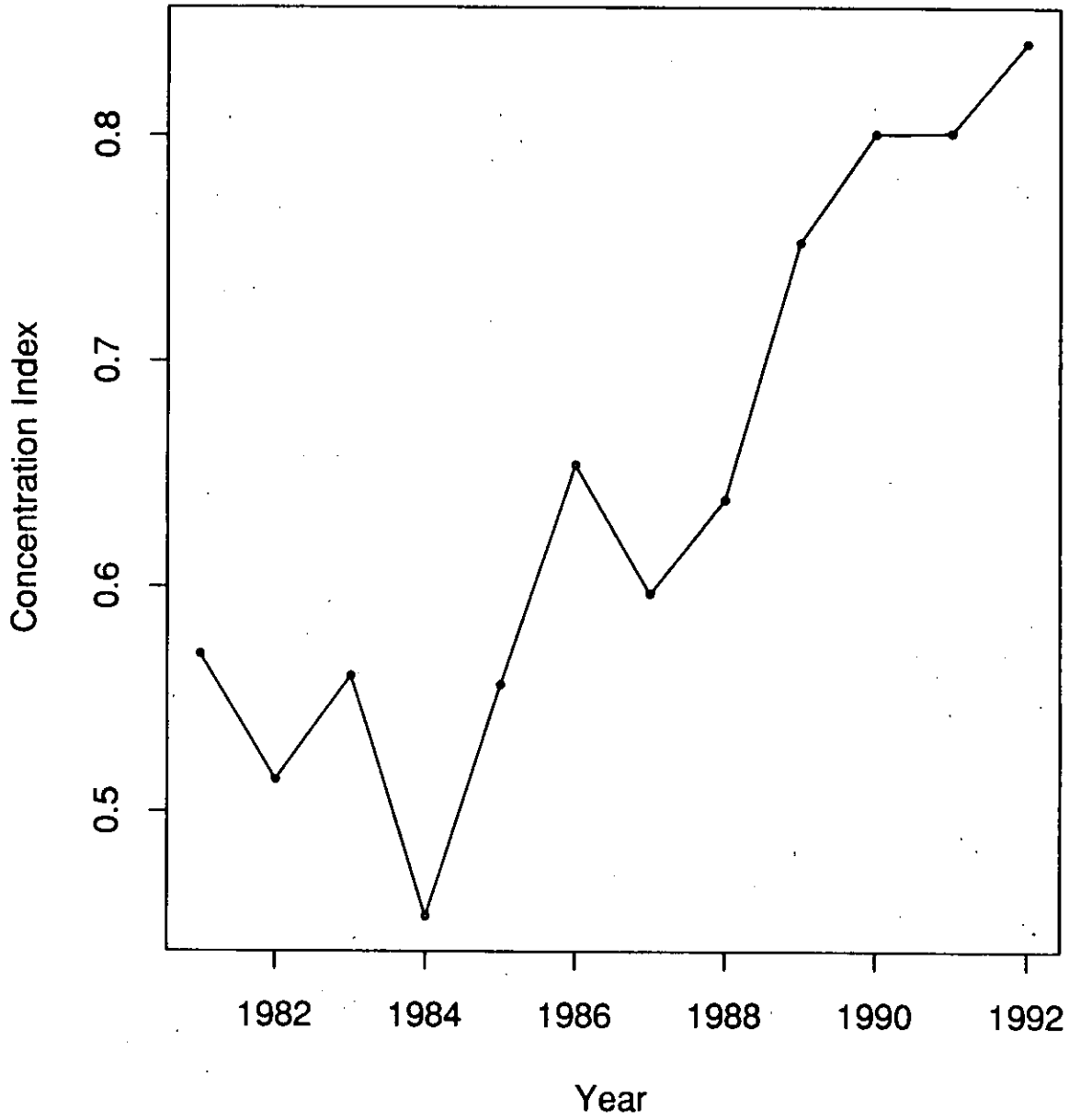


Fig. 6b. Concentration index or Gini index for 5a.