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On Stability of Cod Stock Estimates in NAFO Area 2J3KL

by

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1. Introduction

At NAFO meeting in 1993 the document was presented (Gasjukov P. 1993), where the ADAPT - method algorithm with regularization was suggested to stabilized commercial fish stock estimates. The algorithm impact was illustrated with an example, based on the data on the silver hake from NAFO subareas 4XVW. To continue the investigation we suppose to test the fitness of the above method of some fishery objects stocks in the North-West Atlantic, which are estimated by means of the ADAPT - method (Gavaris S., 1988). These fishery objects include cod from the NAFO areas 2J3KL.

The estimation of that species stock is performed based on the data since 1978 (Bishop C.A. *et al.*, 1993). In 1991 the trawling survey data show a dramatic decrease of the stock, pursued also in 1992. The above - mentioned stock state alteration is incompatible to the previous estimates and seems to be of a leap like pattern. To explain it, the supposition was made, that the dramatic reduction of cod abundance in the areas 2J3KL was caused by a sudden increase of natural mortality, occurred in winter of 1991 (Bishop C.A. *et al.*,);

All facts indicated prevent a mechanical utilization of estimates stabilization method, presented by Gasjukov (1993) and require to perform some adaptation of the latter. Besides, to obtain stable estimates of the cod stock for the latest years, some additional researches had been required to consider the above - mentioned variations of the stock size.

2. Data used

The version, utilized by Canadian scientists (Bishop C.A. *et al.*,) in 1993, was taken as a background to formulate the adaptive framework for evaluation of cod abundance and other population parameters in the NAFO areas 2J3KL. The choice defines initial data for calculations, including the following:

- catch by age - groups and years of fishery in 1978-1992;
- average fish weight by age - groups and years of fishery in 1978-1992;
- abundance indices by age - groups and years of fishery, based on the data of the fall trawling survey in 1978-1992.

The age - groups range is 3 - 13. All data are presented in the work by Bishop C.A. *et al.* (1993).

3. Adaptive framework and parameters definition

Abundance values of 4 - 12 age - groups in the terminal year are the unknown parameters being defined. In addition to the cohort analysis equation by Pope J. (1972) and the catch equation by Baranov (Ricker W., 1975), the following equations are used:

- abundance of the age - group 3 in the terminal year is the long - term average of the Virtual Population Analysis (VPA) for the calculation period, excluding the terminal year;
- coefficients of fishery mortality **F** of the oldest age group by the years of fishery are available

$$F_{13,y} = 0.5 \cdot \frac{F_{7,y} + F_{8,y} + F_{9,y}}{3}$$

(Bishop C.A. *et al.*, 1993).

An objective function of the adaptive framework is determined as

$$SS = \sum_{a=3}^{13} \sum_{y=78}^{T} \left[obs\ ln\ I_{ay} - calc\ ln\ I_{ay} \right]^2 + \alpha \cdot \| N_a - N^{\star} \|, \qquad (I)$$

where obs - observed values;

$calc$ - calculated values;

a - age group index;

T - terminal year;

N_a - vector of unknown abundance values of 4 - 12 age - groups in the terminal year;

N^* - test element - vector with components, equal to the long - term average abundance values of age - groups 4 - 12;

$\| \|$ - symbol of the Euclidean norm of vector ;

α - regularization parameter.

Calculated value in the objective function are estimated by means of the regression equation, assuming that errors are of multiplicative nature

$$I_{ay} = q_a \cdot N_{ay} \cdot \xi^{RV} , \qquad (2)$$

where q_a - appropriate factor of proportionality (catchability rate);

ξ^{RV} - random error;

N_{ay} - estimated abundance of age - group a in year y, obtained by means of VPA.

The latter equation of the objective function (without a stabilizing term), differs from the objective function, utilized by Bishop (Bishop C.A. *et al.*, I993). The difference is that in equation (1) factor of proportionality is estimated by (2) , while in the above - mentioned paper those factors are included into the number of unknown parameters, estimated by minimization of the objective function. Thus, the minimization of (1) results in about twice reduce of the unknown values number , while the results of calculations are similar.

Special methods are suggested to estimate the regularization parameter efficiency (Gasjukov P., 1993). In the paper, taking in account the requirement to estimate parameters in the tome interval between 1978 and 1990, a cross validation method is selected for the purpose, though it is highly time - consuming. In the above - mentioned author's work and next one (Gasjukov P.,) the term "moving check" was used for that method . It is a literal translation of the corresponding Russian term into English.

The method of cross validation to estimate the regularization parameter in (1) is as follows (Efron B., 1988), (Gasjukov P., 1991).

Let us consider the entire sample of abundance indices, by means of which the objective function (1) is estimated as a set of elements

$$U = \left\{ I_{ay},\ a = 1,\dots,a_k;\ y = 1,\dots T \right\}$$

where a_k – the oldest age group. Let assume some value of the regularization parameter α. If one element, e.g. I_{ay} is excluded from the U set, the objective function SS is replaced by a mew one SS_{ay}, minimization of which provides estimation of vectors of the unknown parameters $N_a^{(a,y)}$, obtain the abundance values $N_{ay}^{(a,y)}$ and other value, factors of proportionality $q_a^{(a,y)}$ and, hence, allows to calculate $calc\ I_{ay}$, by means of the equation (2) for all $a=1,\dots,a_k$; $y=1,\dots,T$.

Let define the value of $T_{ay}(\alpha)$

$$T_{ay}(\alpha) = obs\ ln\ I_{ay} - calc\ ln\ I_{ay}, \qquad (3)$$

where a and y – indices of element, excluded from U set.

Then

$$T(\alpha) = \frac{1}{a_k \cdot T} \sum \sum T_{ay}(\alpha) \qquad (4)$$

and the regularization parameter is

$\underset{\alpha}{arg\ min}\ T(\alpha)$. The advantages of the cross validation method utilized to estimate the regularization parameter, as compared to the method, used in paper of Gasjukov (1993a, 1993b) are as follows:

- the method may be used for small samples;
- the method provides an independent estimate of the regularization parameter for each calculation version.

However, as was mentioned before, the method is highly time-consuming.

4. Results of calculations for 1978 -1990

The retrospective analysis is one of the methods to investigate the stock estimate stability. The retrospective analysis of cod stock estimates in the NAFO areas 2J3KL, obtained by means of the ADAPT - method was performed in the paper by Baird J.W. *et al.* (Baird J.W. *et al.*,).

In the present part of work the retrospective analysis with the ADAPT - method and regularization was carried out from 1985 to 1990. The results of calculations, i.e. total biomass from age 4+ and older (B4+), total biomass from age 5+ and older (B5+), and average fishing mortality rate of age - groups 7 - 9, represented by F(13) in compliance with the rule of calculation the fishing mortality rate of the oldest age group, are shown in tables 1 - 3 and figure 1. The latter tables present also relative deviations of appropriate values in every terminal year, as compared to values, obtained in those years on the base of complete information (during 1978 - 1990). Table 4 gives the regularization parameter values in every year. All calculations are performed at a priori assumed test element N^*, components of which equal to long-term values of appropriate age - groups.

The results on the regularization parameter show the following:

1.The regularization parameter is not a constant value from year to year.

2.The regularization parameter nay equals 0 in some cases.

The supposed explanation of these facts is the relation between the test element and the unknown abundances vector in specific terminal year. As is shown by (1), the stabilizing term contribution in the objective function increases and the accuracy of the regularization parameter value decreases as the vector of terminal year abundances approaches the test element.

It is interesting to note the trend in function $T(\alpha)$, which is a mean - root - square error of cross validation. To illustrate it, the function is shown in the Figure 2 in the version, where 1986 is the terminal year. There is a zone of the function low sensitivity to the a parameter variations, i.e. the left part of the curve. Further , as α increases, the function values decrease and the function approaches its minimum, which corresponds to the regularization parameter optimal value (equal to the assumed test element value). Afterwards the function starts to increase.

The calculation results, presented in Tables 1 - 3 and Figure 1, show that biomass value s B4+ and B5+ in retrospective analysis are compatible to the appropriate values, obtained upon the complete information for the period from 1978 to 19909. Thus, the maximum biomass deviation at age 4+ and older as compared to the estimate based on the complete information, never exceeds 17%, while the similar estimate at age 5+ and older is up to 12%. In addition four estimates from five have the relative error below 10%.

The same good agreement occurs between the average fishing mortality rates of age - groups 7 - 9 (in tables and figures those values are represented by the fishing mortality of age - group 13). From figure 1 it can be seen that the fishing mortality estimates in 1985 significantly differ from that obtained in the base of the complete information. Actually difference between those estimates amounts less than 0.1.

Thus, in the period of 1978 - 1990 the cod stock estimates in NAFO areas 2J3KL, obtained by means of ADAPT - method with regularization, are stable.

5.Results of calculations for 1978 - 1992

Figure 3 shows the biomass values B4+ and B5+, and average fishing mortality rates for calculation versions, where 1990, 1991 and 1992 are respectively selected as the terminal year. Despite the fact that the ADAPT - method with regularization was used, the agreement between the estimates was not observed. Thus, the biomass estimates B4+ in 1990 (1990 is terminal year) deviates from the similar estimates for the terminal year of 1991 in 40%, and from those for the terminal year of 1992 - in 97%.

The same significant deviations were observed also in other calculated values.

Figure 4 shows the similar results, obtained for the terminal years of 1991 and 1992, assuming that abundance indices in the terminal year were excluded from calculations (masked). Comparison of data, presented in the Figures 3 and 4, shows that for the same values of catch by age -groups , consideration of abundance indices causes a significant variation of the results.

All mentioned above evidences that something happened to the cod population of NAFO areas 2J3KL in 1991, which results in sharp decrease of its abundance in the above year and next one, and stimulates a significant decrease of the abundance indices, obtained in inventory trawling surveys.

Therefore, a spasmodic variation of population parameters occurs when proceeding from the terminal year of 1990 to the terminal year of 1991, which results in poor stability and agreement of estimates, obtained in those years.

6.Some hypotheses on the cod stock state variability in 1991

At least three hypotheses may be propose to explain the reasons of the sharp decrease of abundance indices, observed in the inventory trawling survey:

1.The alteration of the trawling survey scheme in 1991, all previous estimates require to be brought in agreement with the new ones.

2.In 1991 the spasmodic variability of the natural mortality results in the mass fish mortality.

3.In 1991 the fishery removal of cod exceeded the value reported, which resulted in the species abundance decrease.

The first hypothesis was propounded based in information from (Anon, 1993). The second one was presented in the NAFO documents (Baird *et al.*, 1992). The third hypothesis is theoretically accessible, however, it seems improbable due to the international nature of fishery. In addition, the latter hypothesis is similar to the second one, as concerned to the consequences, when the abundance indices obtained in the inventory trawling surveys were used as the values observed. Therefore, the third hypothesis will not by considered further.

Testing of the first and second hypotheses is based in the following assumption: if the hypothesis is valid, it should provide estimates with good agreement by years of fishery, i.e. stable estimates from the retrospective analysis point of view.

To test the first hypothesis, let assume that factor M_{RV} should be used to provide on agreement between the abundance indices, obtained in the trawling surveys in 1991 and 1992, and those in 1978 - 1990:

$$I'_{ay} = M_{RV} \cdot I_{ay} \text{ , } y=1991,1992.$$

Hence testing of the hypothesis is performed as following:

Assuming 1991 as the terminal year, the value of M_{RV} factor is selected , which provides the minimum standard deviation of $B5+$ biomass values in 1985 - 1990, obtained by means of the ADAPT - method with regularization and terminal year of 1990. and similar values, obtained with the terminal year of 1991, i.e. :

$$M_{RV} = \arg\min_{M_{RV}} \sqrt{\sum_{y=85}^{90} \left(B5+_{y}^{90} - B5+_{y}^{91} \right)^{2}} . \qquad (5)$$

2.The calculations by means of the ADAPT - method with regularization and the terminal year of 1992 is performed using the available value of M_{RV}. If no agreement of the estimates for the stability criterion is observed, the hypothesis is rejected. Otherwise, it may be supposed, that the hypothesis is in agreement to the observations.

To test the second hypothesis, let assume, that the spasmodic variability of the natural mortality occurred in 1991:

$$M_{91} = M + dm_{91},$$

where dm - the appropriate increase.Then,the second hypothesis testing is as follows:

1.Assuming 1991 as the terminal year, the value of dm is selected, which provides the minimum standard deviation of $B5+^{90}$ biomass values in 1985 - 1990 ,obtained by means of the ADAPT - method with regularization and the terminal year of 1990, and similar values, obtained with the terminal year of 1991, i.e.

$$dm = \arg\min_{dm} \sqrt{\sum_{y=85}^{90} \left(B5+_{y}^{90} - B5+_{y}^{91} \right)^{2}} . \qquad (6)$$

2.The calculations by means of the ADAPT - method with regularization and the terminal year of 1992 is performed using the dm value obtained. If no agreement of the estimates for the stability criterion is observed, the hypothesis is rejected. Otherwise, it may be supposed, that the hypothesis is in agreement to the observations.

As is seen from the verification schemes for both hypotheses, data of 1991 are used to estimate the correction values in the model, and data of 1992 - to verify the hypotheses validity. The calculation scheme of testing is very time - consuming as it suppose the solution of hierarchical optimization function. At the first level the unknown abundance values in the terminal year are estimated for the given regularization factor and correction parameter (M_{RV} or dm). At the second level the regularization parameter is estimated by means of cross-validation for the given correction parameter. And finally, at the third level the correction parameter is estimated. The author had to deny searching minimum (5) or (6) at the third level due to the lack of highly efficient computer, and restricted to assessment of the values, which showed standard deviations never exceeding the respective values, obtained by means of the retrospective analysis.

The results of the first hypothesis testing reveal the following. The acceptable value of M_{RV} factor is 3.5. Then, the regularization parameter in the terminal year of 1991 equals to 2.697E-2**2, and the standard deviation - 38.4 thousand tones. In 1992 the regularization parameter equals to 2.903E-9**2, and standard deviation is 119.0 thousand tones.

The respective values (B4+, B5+ and F(I3)) in 1990, 1991 and 1992 are given in tables 5 - 7 and Figure 5. The results show that while the above - mentioned value of M_{RV} provides a satisfactory agreement of estimates by years obtained for the terminal year of 1991, utilization of the same factor value to correct in addition the abundance indices of 1992 does not provide the estimates agreement to accessible level. The most significant deviations are obtained in 1990, amounting to -253.5 thousand tones (61%) for biomass B4+ and -192.7 thousand tones for B5+ (53). Significant values of deviations do not evidence the good agreement hypothesis seems to by rejected.

The results of the second hypothesis test reveal the following. The increment of the cod natural mortality in 1991 dm=2.0 provides estimates (for the terminal year of 1991) rather good agreed to the retrospective ones. In addition the regularization parameter equals to 2.56E-8**2, and B5+ standard deviation is -30.3 thousand tones. For the terminal year of 1992 the regularization parameter equals to 9.54E-10**2 and B5+ standard deviation is - 37.7 thousand tones. Besides in 1990, the deviation of B4+ value from the stable estimate for the terminal year of 1990. equals to 92.9 thousand tones (11%), and that of B5+ value amounts to -13.6 thousand tones (3%). Those results evidence that for the second hypothesis the utilization of dm=2.0 provides the estimates satisfactorily agreed by the years of fishery.

The complete results of the retrospective analysis for the period from 1985 to 1992 are given in Tables 8 - 10 and Figure 6. The relative error of the estimates suppose the latter are stable. Thus, the relative error does not exceed 5% in 1985, 1986, 1988, 1989 and 1990, increases to 15% in 1987 and approaches 22% in 1991. It seems, that in the latter case the agreement of the estimates may be better if we are able to consider uneven variability of the natural mortality for various age - groups, occurred in 1991.

7.Conclusion

The ADAPT - method with regularization was used to estimate cod stocks in NAFO area 2J3KL. The method used cross validation to estimate the regularization parameter. All calculations are performed at a priori assumed test element. In the period from 1978 to 1990 the retrospective analysis results show that stock estimates are stable from year to year.

The agreement of estimates by the years of fishery is destroyed from 1990 to 1991. Two hypothesis were considered to explain the latter. As appears, the assumption, that natural mortality of cod in NAFO areas 2J3KL had changed spasmodically, agreed with the observations. Consideration of the natural mortality (0.2 + 2.0) in ADAPT - method with regularization provided the stable estimates, agreed by the years of fishery.

It seems useful in future calculations replace the natural mortality of cod in NAFO areas 2J3KL , utilized by NAFO, to the following values:

M = 0.2 in 1978 - 1990 , 1992 and nest years;

M = 2.2 in 1991.

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Table 1 Retrospective values of cod biomass B4+ in NAFO areas 2J3KL (1978 - 1990)

Year of assess	TERMINAL YEAR					
	85	86	87	88	89	90
78	522.5	538.9	534.1	534.0	536.3	535.3
79	643.5	670.1	660.3	660.5	663.0	661.6
80	710.7	747.6	734.2	734.6	738.6	736.9
81	715.2	765.8	748.1	748.7	754.8	752.4
82	767.6	834.2	820.4	824.5	834.2	829.5
83	776.2	855.9	842.4	854.9	870.3	862.4
84	902.0	955.8	941.0	923.2	951.5	940.3
85	980.9	990.8	1013.7	980.2	1000.1	991.6
86		939.9	1029.1	956.9	962.6	978.0
87			948.8	853.1	854.1	890.2
88				714.3	723.0	780.9
89					584.8	706.7
90						731.3
Relative error	0.0I	0.04	0.07	0.09	0.I7	-

Table 2 Retrospective values of cod biomass B5+ in NAFO areas 2J3KL (1978 - 1990)

Year of assess	TERMINAL YEAR					
	85	86	87	88	89	90
78	362.3	375.6	373.5	374.3	376.0	375.3
79	509.6	530.4	524.4	524.3	527.2	526.0
80	639.0	672.3	659.7	660.1	663.0	661.3
81	639.1	680.4	664.9	665.2	669.7	667.8
82	588.1	641.2	622.2	622.8	629.3	626.8
83	610.1	681.9	669.0	674.0	685.0	679.6
84	625.8	711.1	698.9	714.1	731.5	722.5
85	706.4	755.0	741.6	721.2	750.6	739.1
86		779.6	806.5	768.5	787.4	779.4
87			855.5	770.8	771.9	793.1
88				662.5	659.2	702.8
89					507.8	576.7
90						508.8
Relative error	0.04	0.00	0.08	0.06	0.I2	-

Table 3. Retrospective values of cod fishing mortality $\mathbf{F}(13)$ in NAFO areas 2J3KL (1978 - 1990)

Year of assess	TERMINAL YEAR					
	85	86	87	88	89	90
78	0.255	0.248	0.251	0.251	0.250	0.250
79	0.227	0.219	0.222	0.221	0.220	0.220
80	0.157	0.146	0.148	0.148	0.146	0.147
81	0.169	0.158	0.160	0.160	0.158	0.159
82	0.237	0.210	0.219	0.219	0.217	0.218
83	0.226	0.203	0.216	0.217	0.214	0.215
84	0.288	0.228	0.246	0.244	0.238	0.239
85	0.333	0.266	0.274	0.268	0.250	0.255
86		0.246	0.270	0.254	0.232	0.241
87			0.232	0.270	0.242	0.255
88				0.268	0.295	0.307
89					0.319	0.327
90						0.235
Relative error	0.3I	0.02	-0.09	-0.I3	-0.02	-

Table 4. Regularization factor α by years

year	85	86	87	88	89	90
α	0.0	4.080E-8	0.0	8.094E-9	8.179E-8	7.330E-9

Table 5 Retrospective values of cod biomass B4+ in NAFO areas 2J3KL for the first hypothesis

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	522.5	538.9	534.1	534.0	536.3	535.3	534.2	533.1
79	643.5	670.1	660.3	660.5	663.0	661.6	659.9	658.1
80	710.7	747.6	734.2	734.6	738.6	736.9	734.2	731.5
81	715.2	765.8	748.1	748.7	754.8	752.4	748.9	745.1
82	767.6	834.2	820.4	824.5	834.2	829.5	824.0	818.3
83	776.2	855.9	842.4	854.9	870.3	862.4	853.3	844.7
84	902.0	955.8	941.0	923.2	951.5	940.3	926.3	914.1
85	980.9	990.8	1013.7	980.2	1000.1	991.6	968.7	944.3
86		939.9	1029.1	956.9	962.6	978.0	939.0	894.8
87			948.8	853.1	854.1	890.2	843.3	778.7
88				714.3	723.0	780.9	734.9	643.9
89					584.8	706.7	665.6	527.4
90						731.3	683.3	496.9
91							710.1	562.6
92								580.4

Table 6 Retrospective values of cod biomass B5+ in NAFO areas 2J3KL for the first hypothesis

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	362.3	375.6	373.5	374.3	376.0	375.3	374.5	373.7
79	509.6	530.4	524.4	524.3	527.2	526.0	524.6	523.2
80	639.0	672.3	659.7	660.1	663.0	661.3	659.1	656.9
81	639.1	680.4	664.9	665.2	669.7	667.8	664.7	661.7
82	588.1	641.2	622.2	622.8	629.3	626.8	623.1	619.1
83	610.1	681.9	669.0	674.0	685.0	679.6	673.5	667.2
84	625.8	711.1	698.9	714.1	731.5	722.5	712.2	702.7
85	706.4	755.0	741.6	721.2	750.6	739.1	724.8	712.4
86		779.6	806.5	768.5	787.4	779.4	756.0	730.9
87			855.5	770.8	771.9	793.1	750.7	702.0
88				662.5	659.2	702.8	653.1	582.5
89					507.8	576.7	529.1	426.6
90						508.8	469.5	317.5
91							531.9	331.9
92								442.6

Table 7. Retrospective values of cod fishing mortality
F(13) in NAFO areas 2J3KL for the first hypothesis

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	0.255	0.248	0.251	0.251	0.250	0.250	0.251	0.251
79	0.227	0.219	0.222	0.221	0.220	0.220	0.221	0.222
80	0.157	0.146	0.148	0.148	0.146	0.147	0.147	0.148
81	0.169	0.158	0.160	0.160	0.158	0.159	0.160	0.161
82	0.237	0.210	0.219	0.219	0.217	0.218	0.220	0.221
83	0.226	0.203	0.216	0.217	0.214	0.215	0.218	0.220
84	0.288	0.228	0.246	0.244	0.238	0.239	0.243	0.248
85	0.333	0.266	0.274	0.268	0.250	0.255	0.262	0.271
86		0.246	0.270	0.254	0.232	0.241	0.251	0.262
87			0.232	0.270	0.242	0.255	0.270	0.284
88				0.268	0.295	0.307	0.338	0.372
89					0.319	0.327	0.386	0.487
90						0.235	0.317	0.644
91							0.180	0.704
92								0.126

Table 8 Retrospective values of cod biomass B4+ in
NAFO areas 2J3KL (1978 - 1992)
(natural mortality is 0.2 + 2.0 in 1991)

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	522.5	538.9	534.1	534.0	536.3	535.3	534.3	534.0
79	643.5	670.1	660.3	660.5	663.0	661.6	66	659.6
80	710.7	747.6	734.2	734.6	738.6	736.9	734.4	733.8
81	715.2	765.8	748.1	748.7	754.8	752.4	749.2	748.3
82	767.6	834.2	820.4	824.5	834.2	829.5	824.5	822.4
83	776.2	855.9	842.4	854.9	870.3	862.4	853.9	851.1
84	902.0	955.8	941.0	923.2	951.5	940.3	927.7	925.5
85	980.9	990.8	1013.7	980.2	1000.1	991.6	970.9	964.9
86		939.9	1029.1	956.9	962.6	978.0	942.8	932.5
87			948.8	853.1	854.1	890.2	849.7	838.3
88				714.3	723.0	780.9	745.0	736.4
89					584.8	706.7	683.2	708.8
90						731.3	688.7	824.2
91							690.9	1050.1
92								122.0
Relat.err.	0.02	0.0I	0.I3	-0.03	-0.I7	-0.II	-	-

Table 9 Retrospective values of cod biomass B5+ in NAFO areas 2J3KL (1978 - 1992) (natural mortality is 0.2 + 2.0 in 1991)

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	362.3	375.6	373.5	374.3	376.0	375.3	374.6	374.3
79	509.6	530.4	524.4	524.3	527.2	526.0	524.7	524.3
80	639.0	672.3	659.7	660.1	663.0	661.3	659.3	658.7
81	639.1	680.4	664.9	665.2	669.7	667.8	665.0	664.3
82	588.1	641.2	622.2	622.8	629.3	626.8	623.5	622.6
83	610.1	681.9	669.0	674.0	685.0	679.6	674.0	671.7
84	625.8	711.1	698.9	714.1	731.5	722.5	713.0	709.7
85	706.4	755.0	741.6	721.2	750.6	739.1	726.3	724.2
86		779.6	806.5	768.5	787.4	779.4	758.3	752.2
87			855.5	770.8	771.9	793.1	754.8	743.6
88				662.5	659.2	702.8	660.1	648.0
89					507.8	576.7	540.7	532.4
90						508.8	489.3	522.4
91							536.4	694.9
92								91.2
relative error	0.03	0.04	0.I5	0.02	0.05	0.22	-	-

Table 10. Retrospective values of cod fishing mortality F(13) in NAFO areas 2J3KL (1978-1992) (natural mortality is 0.2 + 2.0 in 1991)

YEAR of assess	TERMINAL YEAR							
	85	86	87	88	89	90	91	92
78	0.255	0.248	0.251	0.251	0.250	0.250	0.251	0.251
79	0.227	0.219	0.222	0.221	0.220	0.220	0.221	0.221
80	0.157	0.146	0.148	0.148	0.146	0.147	0.147	0.147
81	0.169	0.158	0.160	0.160	0.158	0.159	0.160	0.160
82	0.237	0.210	0.219	0.219	0.217	0.218	0.219	0.220
83	0.226	0.203	0.216	0.217	0.214	0.215	0.217	0.218
84	0.288	0.228	0.246	0.244	0.238	0.239	0.243	0.244
85	0.333	0.266	0.274	0.268	0.250	0.255	0.261	0.264
86		0.246	0.270	0.254	0.232	0.241	0.250	0.254
87			0.232	0.270	0.242	0.255	0.269	0.272
88				0.268	0.295	0.307	0.335	0.339
89					0.319	0.327	0.377	0.388
90						0.235	0.304	0.340
91							0.404	0.777
92								0.757
Relative error	0.26	-0.03	-0.I5	-0.2I	-0.I8	-0.3I	-	-

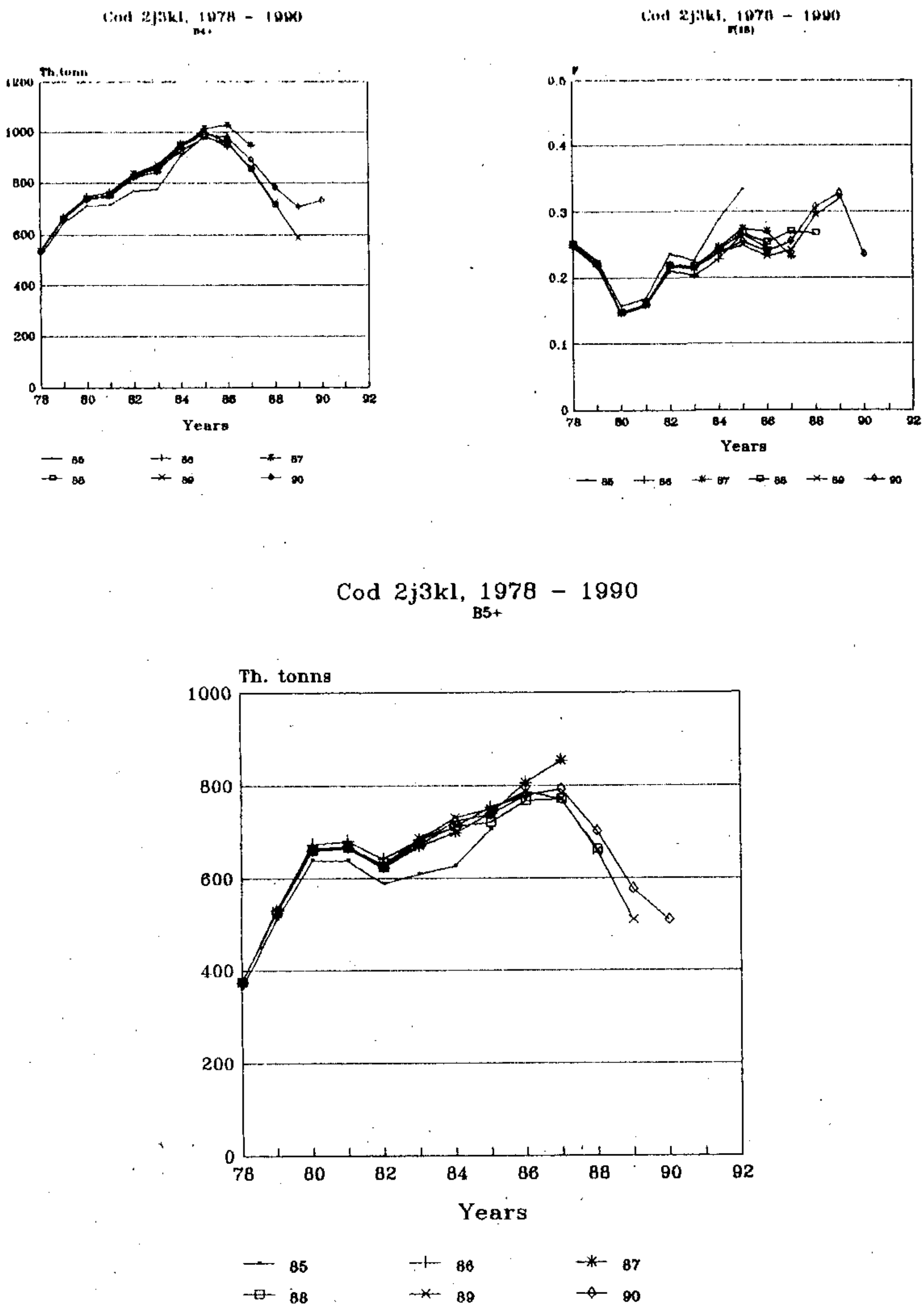


Figure 1.Results of cod retrospective analysis
in NAFO areas 2J3KL (1978-1990)

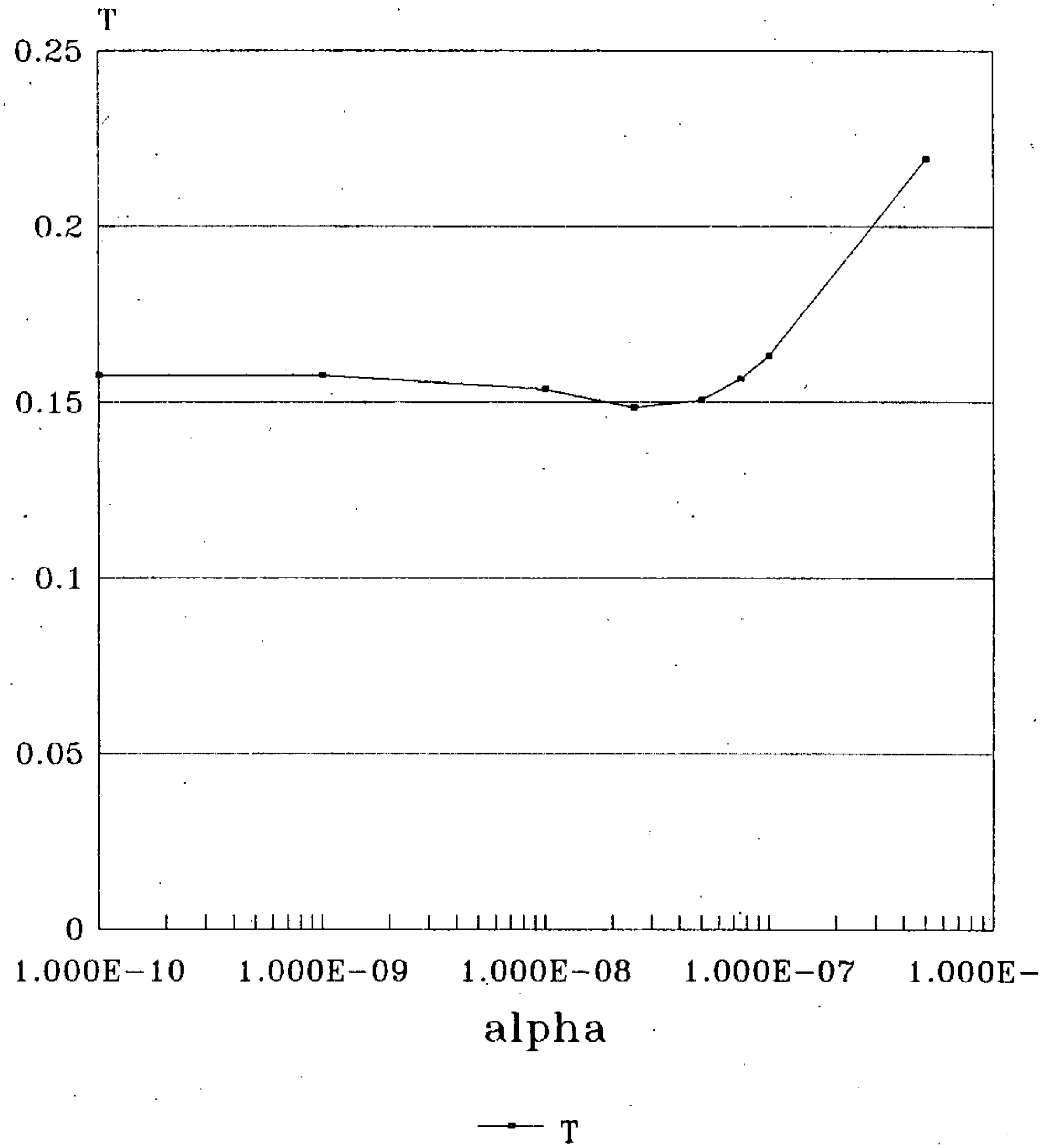


Figure 2. Function $T(\alpha)$ in 1986

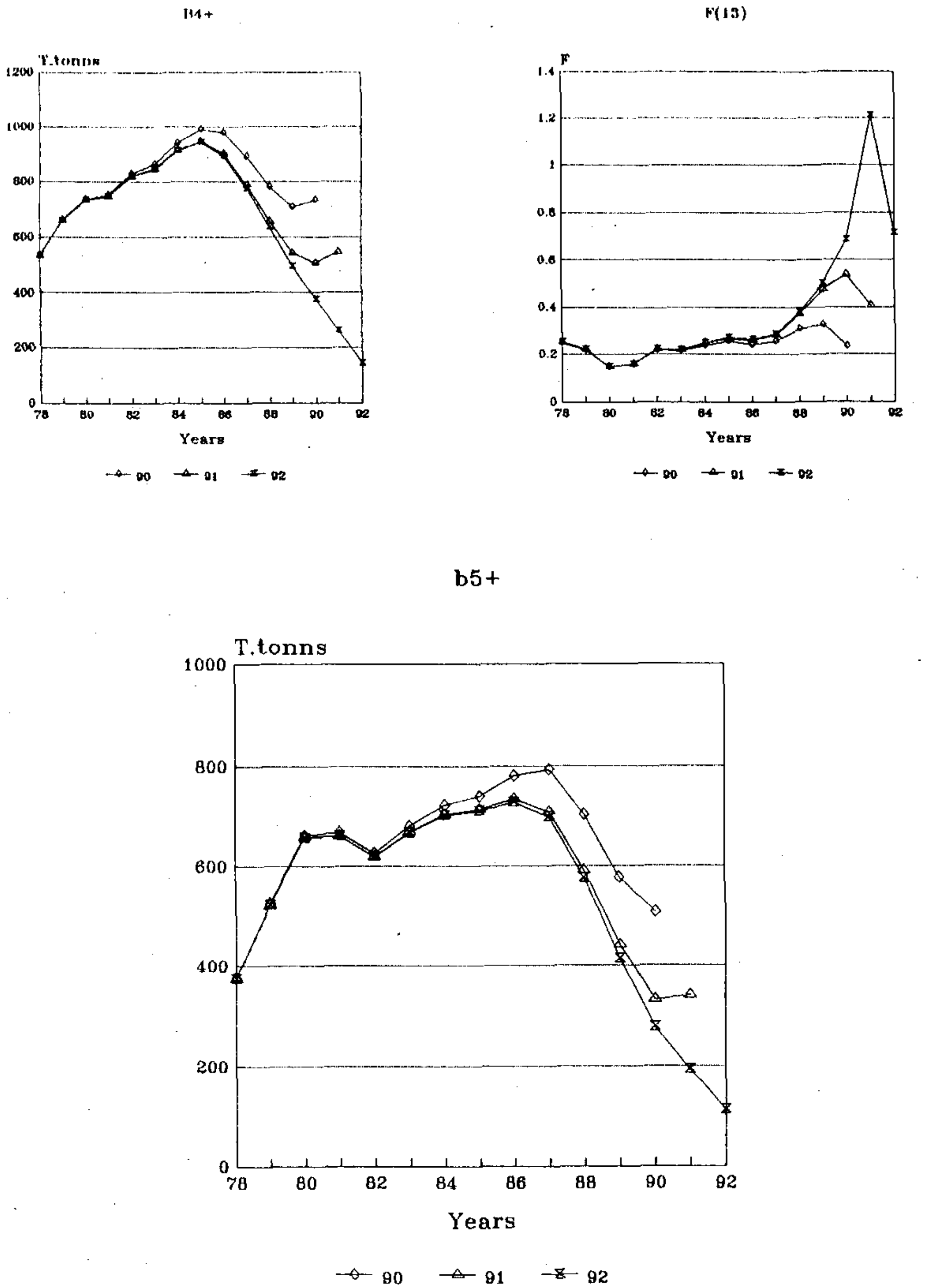
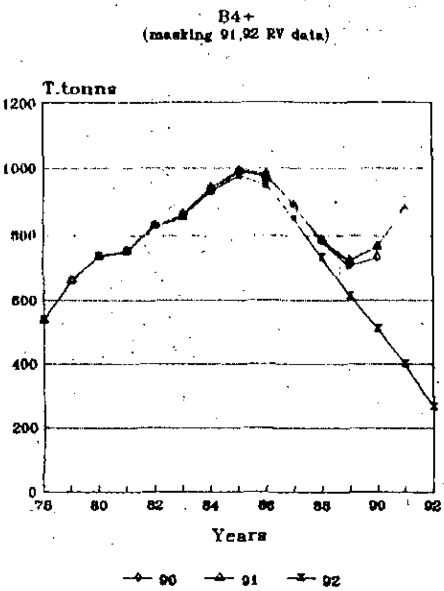
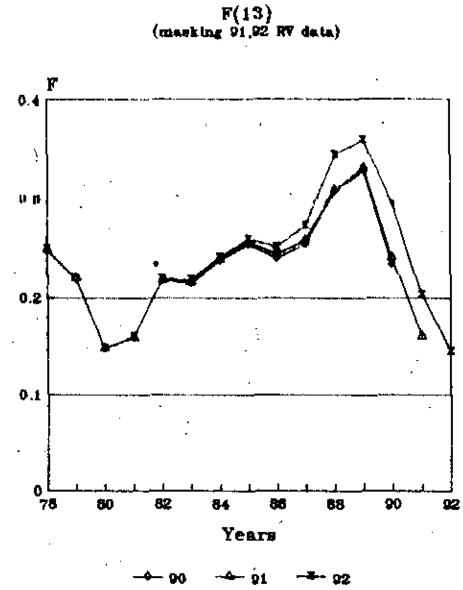


Figure 3. Results of the retrospective analysis in 1990-1992 (abundance indices of 1991 and 1992 are used)





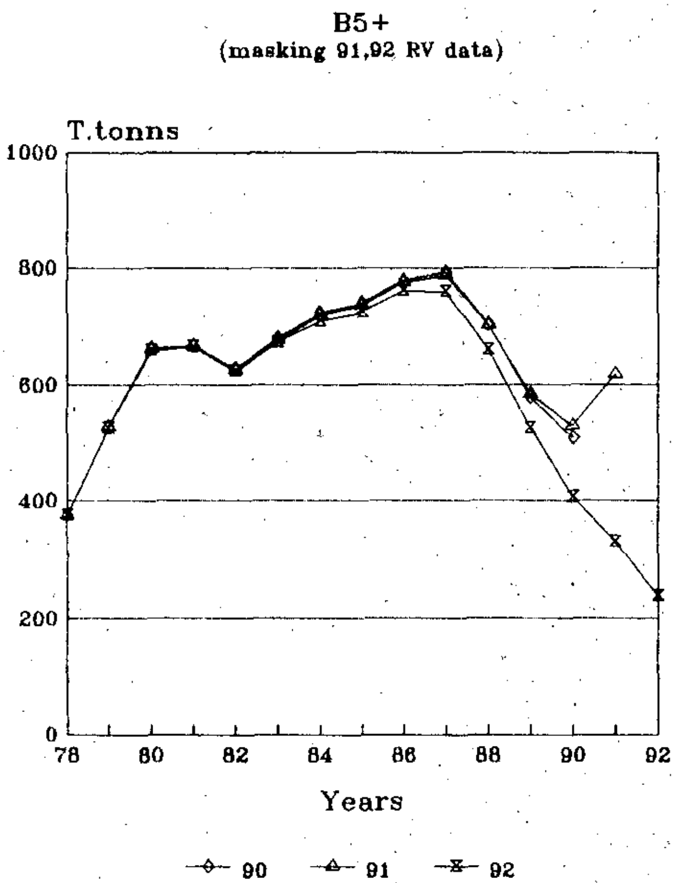


Figure 4. Results of the retrospective analysis in 1990-1992 (abundance indices of 1991 and 1992 are masked)

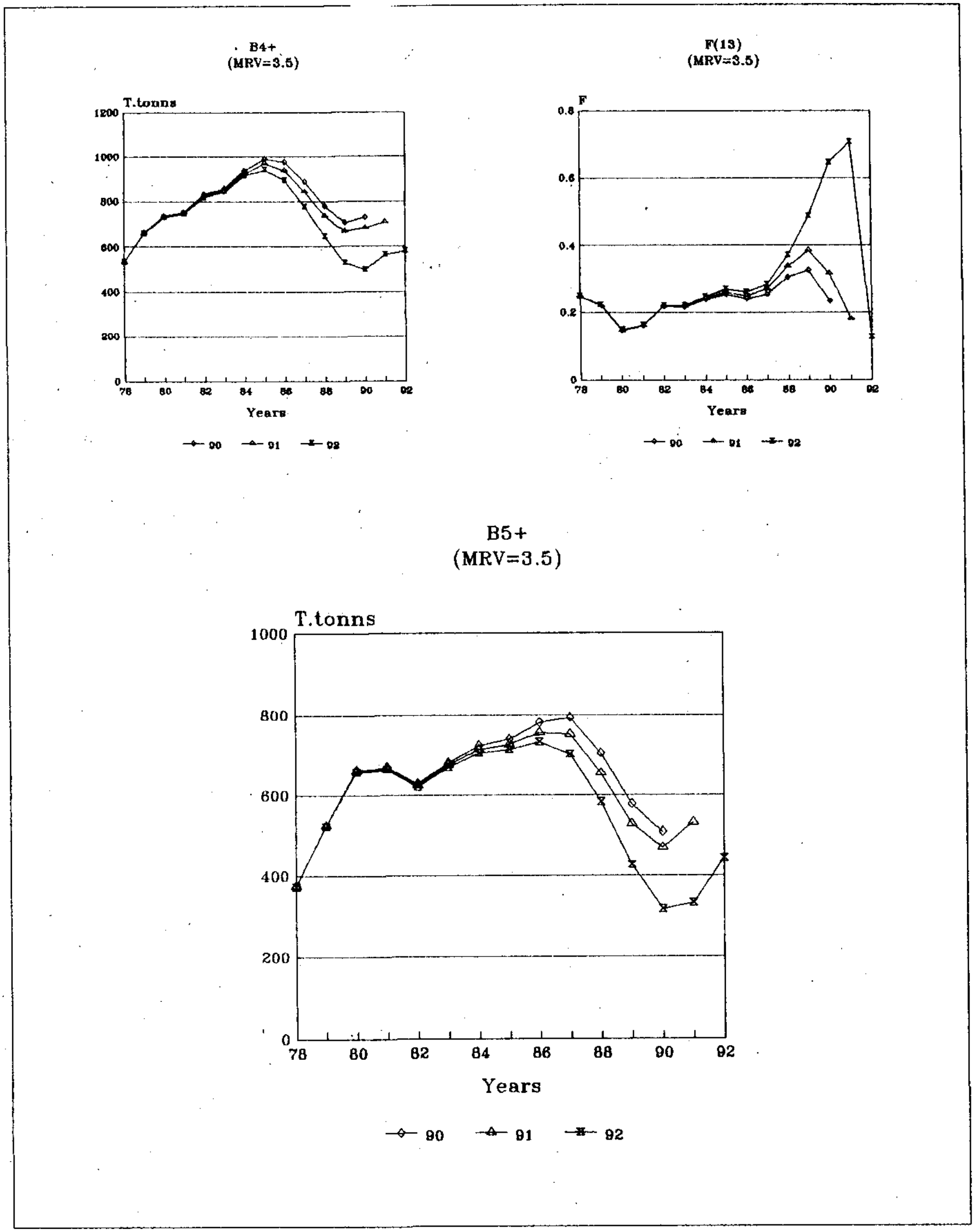
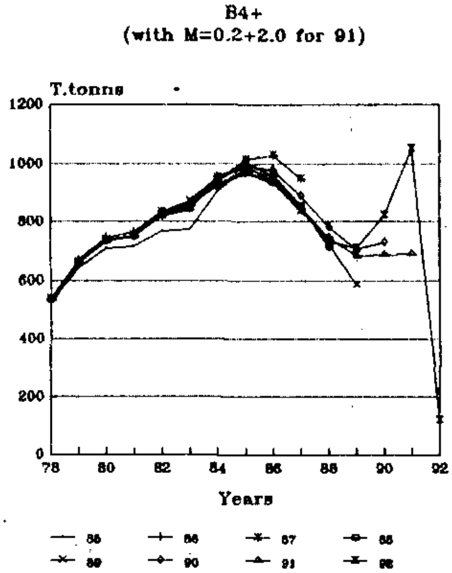
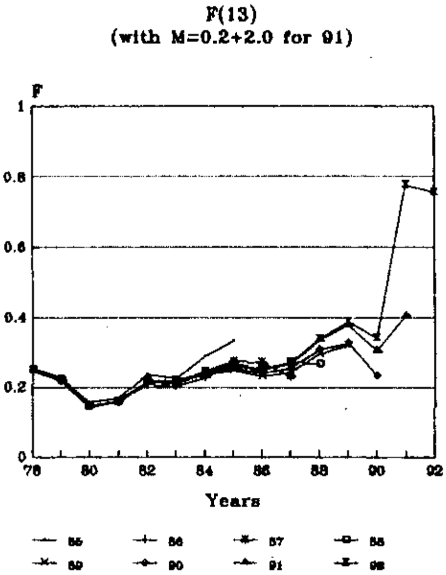


Figure 5. Results of the retrospective analysis of cod in NAFO areas 2J3KL for the first hypothesis





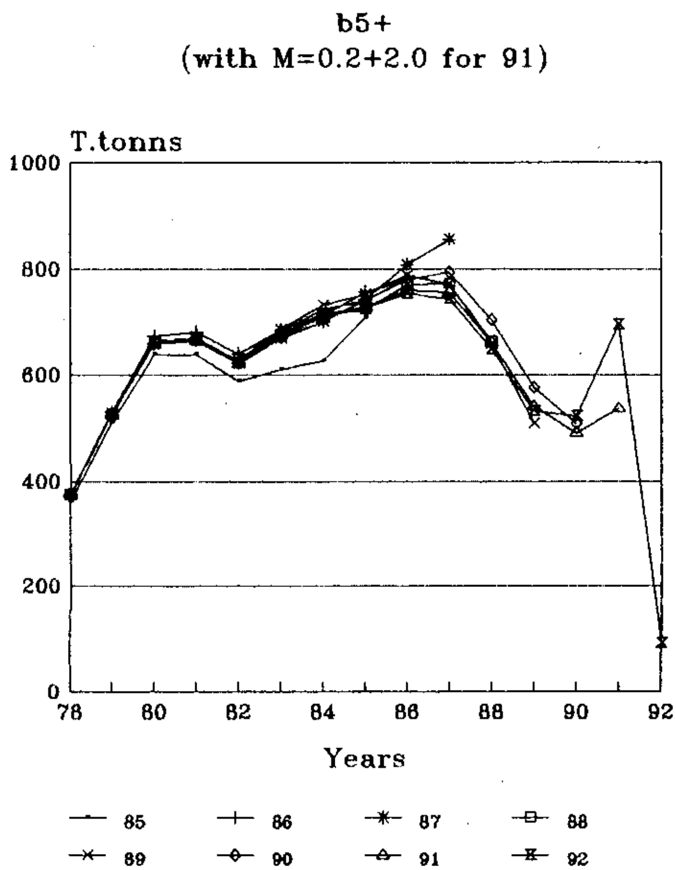


Figure 6. Results of the retrospective analysis of cod in NAFO areas 2J3KL (natural mortality is 0.2 + 2.0 in 1991)