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Greenland Halibut SCAA Robustness Tests

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Abstract

This paper provides details of the full set of SCAA (Statistical Catch At Age) operating models intended for use in the MSE process for Greenland halibut. It first summarises the Reference Case operating model and the six variants thereof to serve as robustness tests, and then provides values of key associated parameter values and plots of estimated trends for various components of the biomass. Two Appendices list the data used for conditioning (fitting) these models and the complete mathematical specifications of the models and the (penalised) likelihood maximised in the fitting process.

Introduction

The Management Strategy Evaluation (MSE) process for Greenland halibut is to take account of two sets of Operating Models (OM) – the one set based on assessments conducted using XSA, and the other on SCAA-based assessments.

The following are the SCAA Reference Case (RC) and robustness test OMs for this process:

1) Reference Case: Case 2 of Butterworth and Rademeyer (2010): Beverton-Holt, *h*=0.9, *M*=0.2, exponential decrease in selectivity for ages 11+;

2) RC with flat commercial selectivity (estimated) for ages 11+;

3) RC with flat commercial selectivity (fixed similar to XSA value) for ages 11+;

4) RC with *M*=0.1;

5) RC with *M*=0.2 for ages 0-10, linear increase to *M*=0.4 for age 14; and constant thereafter;

6) RC with *h* = 0.6 in the assessment, to simulate a stock that has a large maximum recruitment which has been severely recruitment-overfished;

7) RC with a modified Ricker stock-recruitment relationship: $R_y = \alpha B_y^{sp} \exp\left(-\beta \left(B_y^{sp}\right)^{\gamma}\right)$.

Data and Methodology

The data used to fit these models within an SCAA framework is given in Appendix A. Note the fecundity-at-age matrix in Table A.4 is not the most recent version of this matrix as advised by Healey (pers. commn). The results following are based on this earlier matrix. Re-runs of these SCAA cases using these updated data will be used for the final MSE analyses. However the differences in their results from the plots and results given in the Tables and Figures below are minimal.

The SCAA methodology applied is detailed in Appendix B.

Results and Discussion

The results of the SCAA variants explored are listed in Table 1, with corresponding biomass trajectories plotted in Fig. 1 (which also includes the baseline XSA results for ease of comparison) and stock-recruitment relationships shown in Fig. 2. Table 2 contains the estimated numbers-at-age matrix for the Reference Case, and Table 3 lists the estimates required should another wish to project trajectories forward for any of these OMs.

The biomass trajectories in Fig. 1 show that the various robustness tests generally span the range between the SCAA and XSA Reference Cases, with differences in scale for the 10+ biomass arising primarily from the different ways in which the commercial selectivity for ages 11+ is set.

Table 1: Results of fits of SCAA Reference Case and the intended robustness test operating models (see text for details) to the commercial catch and survey data. Values fixed on input rather than estimated are shown in **bold**. Quantities shown in parenthesis are Hessian-based CVs.

	1) Reference Case			2) flat commercial selectivitiy for ages 11+ estimated			3) flat commercial selectivity for ages 11+ fixed to XSA value			4) M = 0.1			5) M_{10-}=0.2, M_{14+}=0.4, linear in between			6) h=0.6			7) Modified Ricker		
'-lnL:overall	-630.8			-625.3			-610.3			-631.3			-629.3			-629.5			-631.0		
'-lnL:Survey	-29.9			-30.2			-30.7			-27.0			-30.3			-32.3			-30.2		
'-lnL:CAA	-222.8			-220.6			-203.1			-221.8			-223.2			-221.8			-222.1		
'-lnL:CAAsurv	-462.8			-463.6			-463.4			-464.8			-461.7			-459.8			-462.5		
'-lnL:RecRes	17.6			17.6			23.1			15.9			18.7			17.4			17.0		
'-lnL:SelPen	67.0			71.5			63.9			66.3			67.2			67.1			66.7		
h	**0.90**			**0.90**			**0.90**			**0.90**			**0.90**			**0.60**			0.80	γ =	0.015
M	**0.20**			**0.20**			**0.20**			**0.10**			**0.2-0.4**			**0.20**			**0.20**		
θ	0.31			0.18			0.04			0.28			0.35			0.59			0.31		
ϕ	0.28			0.34			0.46			0.29			0.27			0.13			0.26		
ρ - surveys	0.60			0.57			0.50			0.65			0.57			0.47			0.58		
ρ_{CAAage}	0.28	0.35	0.35	0.28	0.35	0.35	0.50	0.29	0.34	0.50	0.28	0.35	0.49	0.29	0.36	0.50	0.29	0.36	0.49	0.28	0.35
ρ_{CAAyr}	-0.32	-0.49	-0.49	-0.32	-0.49	-0.49	-0.69	-0.32	-0.49	-0.69	-0.32	-0.49	-0.69	-0.32	-0.49	-0.69	-0.32	-0.48	-0.69	-0.32	-0.49
K^{sp}	340	(0.06)		328	(0.06)		425	(0.06)		1056	(0.06)		181	(0.06)		420	(0.12)		374	(0.34)	
B^{sp}_{2008}	37	(0.42)		25	(0.58)		9	(0.27)		169	(0.33)		22	(0.46)		65	(0.45)		37	(2.33)	
B^{5-9}_{2008}	128			114			94			123			129			118			125		
B^{10+}_{2008}	53			38			16			198			39			85			53		
$MSYL^{sp}$	0.18	(0.14)		0.19	(0.18)		0.20	(0.07)		0.20	(0.15)		0.17	(0.13)		0.33	(0.15)		0.16	(0.54)	
B^{sp}_{MSY}	60	(0.17)		61	(0.19)		83	(0.09)		206	(0.19)		31	(0.16)		138	(0.20)		60	(0.25)	
MSY	27	(0.05)		27	(0.06)		38	(0.06)		28	(0.06)		28	(0.05)		21	(0.12)		26	(0.42)	
σ_{comCAA}	0.07			0.07			0.08			0.07			0.07			0.07			0.07		
Survey	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$	q's x10^6	σ_{surv}	$\sigma_{survCAA}$
CanFall	421	0.17	0.02	454	0.16	0.02	508	0.15	0.02	640	0.22	0.02	416	0.17	0.02	435	0.15	0.02	426	0.17	0.02
EU	218744	0.28	0.05	237401	0.28	0.05	264633	0.29	0.05	243516	0.28	0.05	209096	0.27	0.05	198912	0.26	0.05	218826	0.28	0.05
CanSpr	22	0.41	0.05	23	0.40	0.05	25	0.41	0.05	29	0.42	0.05	22	0.41	0.05	22	0.40	0.05	22	0.41	0.05
σ_R_out	0.21			0.21			0.24			0.20			0.22			0.21			0.21		

Table 2: Numbers-at-age (in 10^6) matrix for the SCAA Reference Case.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1975	218.780	179.122	146.652	120.063	98.266	80.242	65.425	50.969	35.640	22.103	13.030	8.072	5.893	4.444	3.428	2.689	2.134	1.708	1.375	1.112	4.841
1976	206.542	179.122	146.652	120.064	98.275	80.300	65.499	51.680	37.308	24.041	14.417	8.783	6.089	4.548	3.486	2.721	2.152	1.718	1.381	1.116	4.850
1977	154.541	169.102	146.652	120.065	98.281	80.336	65.583	52.093	38.670	26.176	16.433	10.110	6.732	4.753	3.597	2.783	2.187	1.739	1.393	1.123	4.865
1978	182.844	126.528	138.449	120.066	98.287	80.374	64.423	48.598	34.276	25.208	19.017	12.579	7.889	5.323	3.794	2.891	2.248	1.773	1.413	1.135	4.888
1979	218.502	149.700	103.592	113.350	98.284	80.355	64.105	46.448	29.964	20.870	17.715	14.297	9.691	6.181	4.221	3.035	2.328	1.818	1.439	1.150	4.913
1980	183.625	178.895	122.564	84.813	92.792	80.394	64.600	48.006	30.824	19.109	14.081	11.978	11.262	7.716	4.958	3.405	2.458	1.891	1.480	1.173	4.952
1981	180.897	150.340	146.466	100.345	69.428	75.885	64.344	47.289	30.063	18.312	12.229	9.037	9.346	8.905	6.159	3.985	2.751	1.993	1.537	1.205	5.000
1982	198.683	148.106	123.087	119.914	82.141	56.766	61.653	49.528	31.253	16.939	11.166	8.441	6.983	7.338	7.072	4.932	3.211	2.226	1.618	1.251	5.063
1983	176.497	162.668	121.258	100.774	98.163	67.179	46.196	48.105	34.411	19.323	11.089	8.015	6.605	5.533	5.866	5.691	3.987	2.605	1.810	1.318	5.154
1984	196.814	144.503	133.181	99.276	82.496	80.293	54.470	35.498	31.815	21.554	13.203	8.311	6.306	5.254	4.435	4.730	4.607	3.238	2.120	1.476	5.286
1985	218.509	161.138	118.309	109.038	81.270	67.472	65.041	41.587	22.929	19.327	14.441	9.806	6.513	5.001	4.203	3.571	3.825	3.739	2.634	1.728	5.521
1986	194.399	178.900	131.928	96.862	89.264	66.491	54.065	49.518	28.919	15.266	14.327	11.369	7.795	5.219	4.031	3.403	2.899	3.113	3.048	2.150	5.923
1987	167.516	159.161	146.471	108.012	79.297	73.035	53.348	41.346	34.796	19.516	11.387	11.306	9.054	6.255	4.211	3.265	2.764	2.360	2.538	2.488	6.598
1988	147.101	137.150	130.309	119.918	88.419	64.846	59.549	40.741	23.564	19.929	13.735	8.661	8.800	7.146	4.986	3.381	2.636	2.240	1.918	2.067	7.415
1989	143.340	120.436	112.289	106.687	98.170	72.335	52.948	46.614	26.737	15.507	14.820	10.724	6.863	7.036	5.750	4.031	2.743	2.144	1.825	1.565	7.747
1990	110.615	117.357	98.605	91.933	87.335	80.281	58.958	41.195	32.040	18.686	11.372	11.073	8.355	5.420	5.611	4.619	3.255	2.223	1.742	1.486	7.601
1991	104.170	90.564	96.083	80.727	75.241	71.306	65.036	42.621	20.983	17.190	11.541	7.394	8.031	6.270	4.168	4.390	3.659	2.602	1.789	1.408	7.387
1992	125.654	85.287	74.147	78.661	66.056	61.334	55.255	45.570	22.677	9.038	7.955	6.600	4.965	5.712	4.642	3.173	3.411	2.884	2.072	1.435	7.120
1993	181.129	102.876	69.827	60.701	64.359	53.807	47.069	37.603	22.016	8.265	3.633	4.214	4.267	3.441	4.152	3.489	2.443	2.671	2.286	1.657	6.912
1994	238.618	148.295	84.227	57.164	49.665	52.424	29.887	22.458	15.708	8.703	3.560	1.776	2.722	2.955	2.500	3.120	2.685	1.913	2.117	1.828	6.923
1995	213.727	195.363	121.412	68.953	46.767	40.426	27.210	12.848	8.136	5.290	3.282	1.578	1.105	1.838	2.110	1.856	2.380	2.089	1.509	1.687	7.055
1996	141.007	174.984	159.949	99.397	56.425	38.138	31.758	19.352	7.432	4.898	3.355	2.242	1.081	0.797	1.374	1.618	1.449	1.883	1.668	1.213	7.084
1997	139.009	115.446	143.264	130.945	81.331	45.983	29.747	22.008	10.406	4.201	2.954	2.215	1.485	0.762	0.586	1.041	1.253	1.139	1.497	1.337	6.713
1998	163.421	113.811	94.519	117.287	107.155	66.331	35.485	20.190	10.997	5.347	2.452	1.868	1.524	1.074	0.571	0.450	0.814	0.992	0.910	1.204	6.525
1999	176.303	133.798	93.180	77.381	95.981	87.412	51.374	24.383	10.474	5.851	3.196	1.578	1.300	1.111	0.809	0.440	0.353	0.646	0.793	0.733	6.269
2000	197.866	144.344	109.543	76.283	63.317	78.230	69.601	35.655	11.116	5.272	3.300	2.100	1.053	0.920	0.820	0.615	0.341	0.278	0.514	0.636	5.667
2001	208.975	161.999	118.178	89.678	62.412	51.560	61.935	46.556	13.648	4.889	2.703	2.061	1.336	0.721	0.664	0.613	0.471	0.267	0.220	0.410	5.091
2002	171.204	171.094	132.632	96.749	73.385	50.897	40.449	40.763	16.620	6.310	3.174	1.789	1.413	0.964	0.540	0.509	0.479	0.373	0.213	0.177	4.459
2003	128.975	140.170	140.079	108.583	79.173	59.857	40.009	26.921	15.420	7.958	4.147	2.125	1.238	1.026	0.724	0.415	0.399	0.379	0.298	0.171	3.761
2004	135.183	105.596	114.760	114.678	88.850	64.540	44.496	23.327	9.479	6.237	4.594	2.837	1.427	0.881	0.760	0.551	0.323	0.314	0.302	0.239	3.185
2005	103.976	110.678	86.454	93.952	93.846	72.479	48.756	27.622	9.956	4.463	3.842	3.242	1.973	1.040	0.663	0.586	0.432	0.256	0.251	0.243	2.778
2006	158.453	85.128	90.615	70.779	76.894	76.622	57.629	34.820	13.367	4.443	2.527	2.616	2.351	1.480	0.800	0.519	0.464	0.345	0.206	0.203	2.456
2007	132.129	129.730	69.697	74.186	57.930	62.797	61.087	41.702	17.906	6.415	2.617	1.752	1.919	1.779	1.145	0.628	0.412	0.372	0.278	0.167	2.164
2008	146.386	108.178	106.213	57.061	60.725	47.348	50.847	46.001	23.109	9.808	4.153	1.807	1.333	1.490	1.401	0.911	0.504	0.333	0.301	0.226	1.900
2009	148.662	119.851	88.568	86.958	46.709	49.650	38.427	38.992	28.047	13.973	6.702	2.980	1.397	1.046	1.183	1.122	0.734	0.408	0.270	0.245	1.734

Table 3: Quantities needed for projections for the SCAA Reference Case.

h	0.900			
K^{sp}	339.94			
σ_R	0.210			
Survey biomass indices:				
	q's x10^6	σ_{surv}	λ_{2008}	ρ
CanFall	421.005	0.170	-0.017	0.597
EU	218744	0.281	-0.514	0.597
CanSpr	22.109	0.406	0.061	0.597
Selectivities				
age	CanFall	EU	CaSpr	Comm
0	0.285	0.050	0.068	0.000
1	0.533	0.063	0.159	0.000
2	1.000	0.079	0.375	0.000
3	0.696	0.126	0.623	0.001
4	0.769	0.178	0.728	0.009
5	0.664	0.491	1.000	0.015
6	0.515	0.965	0.658	0.173
7	0.463	1.000	0.370	0.521
8	0.279	0.800	0.127	0.866
9	0.116	0.369	0.044	1.000
10	0.060	0.283	0.015	0.869
11	0.041	0.155	0.005	0.387
12	0.027	0.085	0.002	0.282
13	0.018	0.047	0.001	0.206
14	0.012	0.026	0.000	0.150
15	0.008	0.014	0.000	0.110
16	0.006	0.008	0.000	0.080
17	0.004	0.004	0.000	0.059
18	0.003	0.002	0.000	0.043
19	0.002	0.001	0.000	0.031
20+	0.001	0.001	0.000	0.023

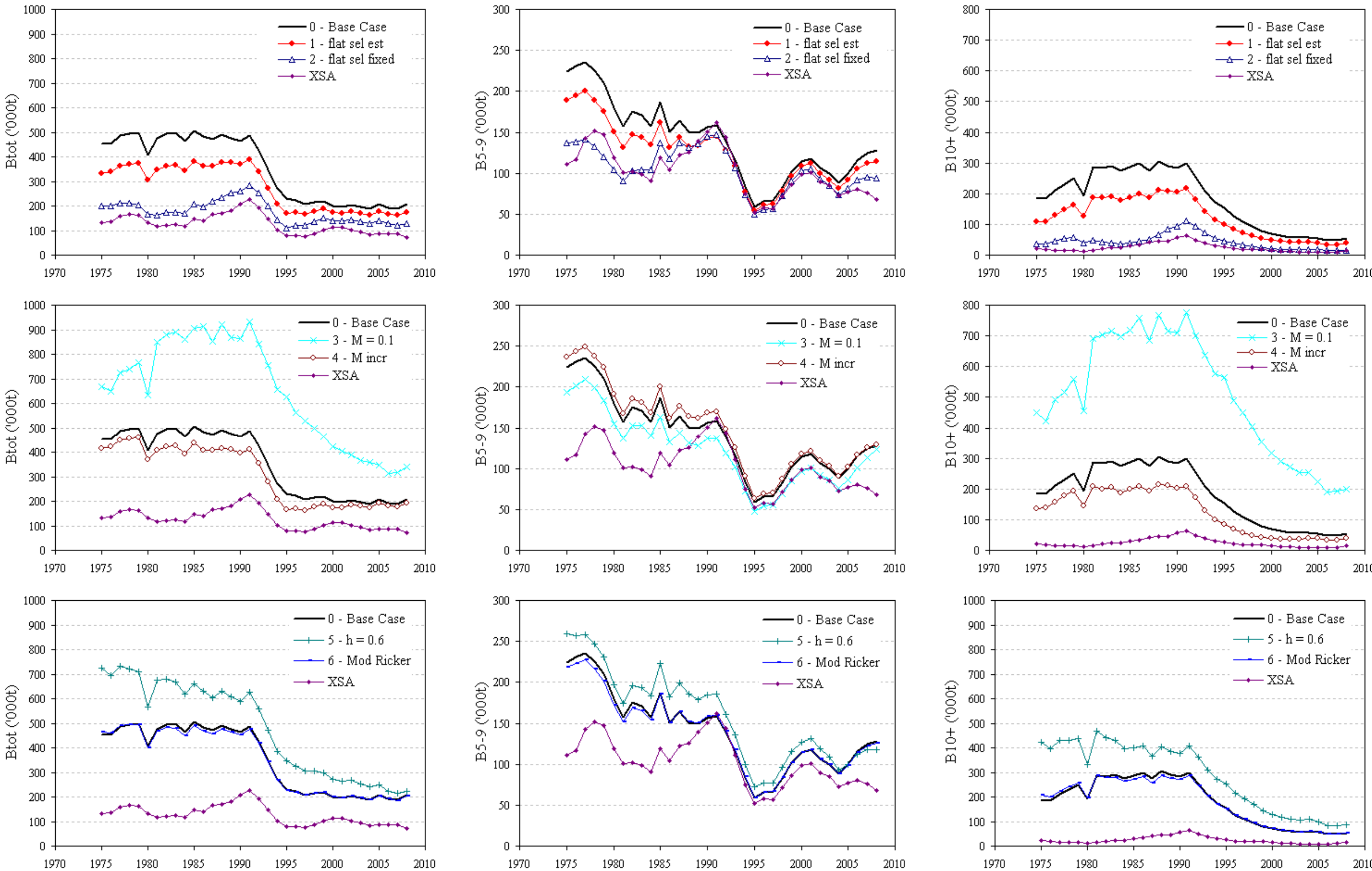

Fig. 1: Biomass trajectories for a series of SCAA variants and the 2008 XSA (Healy and Mahe, 2009).

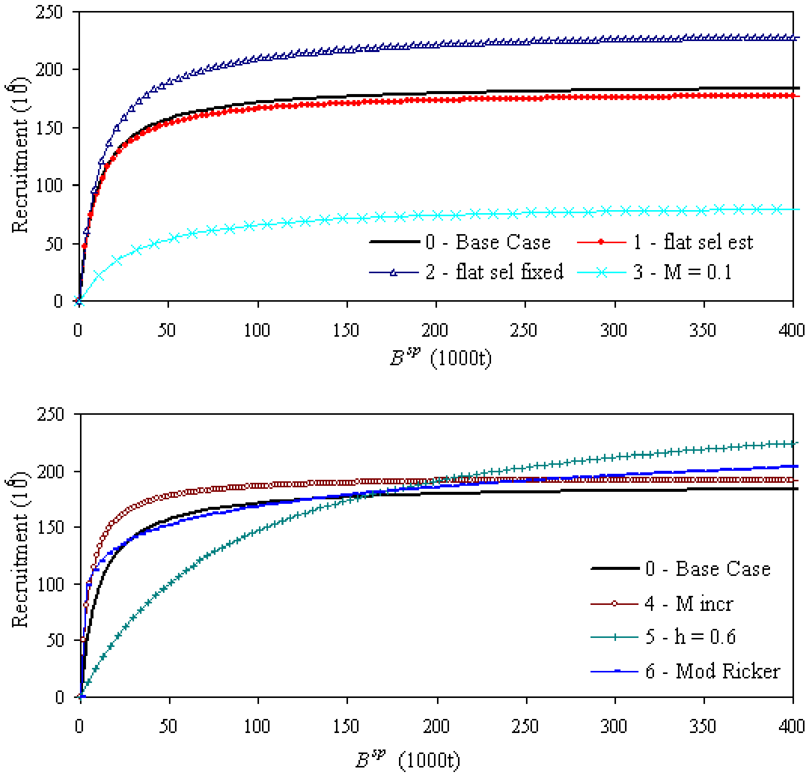

Fig. 2: Stock-recruitment relationships for a series of SCAA variants.

APPENDIX A – Data

Table A1: Landings (tons) for Greenland Halibut in Sub-area 2 and Div. 3KLMNO (Healey and Mahé, 2008).

Year	Landings (t)	Year	Landings (t)
1960	938	1984	26711
1961	741	1985	20347
1962	588	1986	17976
1963	1621	1987	32442
1964	4252	1988	19215
1965	10069	1989	20034
1966	19276	1990	47454
1967	26525	1991	65008
1968	32392	1992	63193
1969	37275	1993	62455
1970	36889	1994	51029
1971	24834	1995	15272
1972	30038	1996	18840
1973	29105	1997	19858
1974	27588	1998	19946
1975	28814	1999	24226
1976	24611	2000	34177
1977	32048	2001	38232
1978	39070	2002	34062
1979	34104	2003	35151
1980	32867	2004	25486
1981	30754	2005	23255
1982	26278	2006	23531
1983	27861	2007	22747

Table A2. Catch at age matrix (000s) for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healey and Mahé, 2008).

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0	0	0	0	334	2819	5750	4956	3961	1688	702	135	279	288
1976	0	0	0	0	17	610	3231	5413	3769	2205	829	260	101	53
1977	0	0	0	0	534	5012	10798	7346	2933	1013	220	130	116	84
1978	0	0	0	0	2982	8415	8970	7576	2865	1438	723	367	222	258
1979	0	0	0	0	2386	8727	12824	6136	1169	481	287	149	143	284
1980	0	0	0	0	209	2086	9150	9679	5398	3828	1013	128	53	27
1981	0	0	0	0	863	4517	9806	11451	4307	890	256	142	43	69
1982	0	0	0	0	269	2299	6319	5763	3542	1684	596	256	163	191
1983	0	0	0	0	701	3557	9800	7514	2295	692	209	76	106	175
1984	0	0	0	0	902	2324	5844	7682	4087	1259	407	143	106	183
1985	0	0	0	0	1983	5309	5913	3500	1380	512	159	99	87	86
1986	0	0	0	0	280	2240	6411	5091	1469	471	244	140	70	117
1987	0	0	0	0	137	1902	11004	8935	2835	853	384	281	225	349
1988	0	0	0	0	296	3186	8136	4380	1288	465	201	105	107	129
1989	0	0	0	0	181	1988	7480	4273	1482	767	438	267	145	71
1990	0	0	0	95	1102	6758	12632	7557	4072	2692	1204	885	434	318
1991	0	0	0	220	2862	7756	13152	10796	7145	3721	1865	1216	558	422
1992	0	0	0	1064	4180	10922	20639	12205	4332	1762	1012	738	395	335
1993	0	0	0	1010	9570	15928	17716	11918	4642	1836	1055	964	401	182
1994	0	0	0	5395	16500	15815	11142	6739	3081	1103	811	422	320	215
1995	0	0	0	323	1352	2342	3201	2130	1183	540	345	273	251	201
1996	0	0	0	190	1659	5197	6387	1914	956	504	436	233	143	89
1997	0	0	0	335	1903	4169	7544	3215	1139	606	420	246	137	89
1998	0	0	0	552	3575	5407	5787	3653	1435	541	377	161	92	51
1999	0	0	0	297	2149	5625	8611	3793	1659	623	343	306	145	151
2000	0	0	0	271	2029	12583	21175	3299	973	528	368	203	129	104
2001	0	0	0	448	2239	12163	22122	5154	1010	495	439	203	156	75
2002	0	0	0	479	1662	7239	17581	6607	1244	659	360	224	126	81
2003	0	0	0	1279	4491	10723	16764	6385	1614	516	290	144	76	85
2004	0	0	0	897	4062	8236	10542	4126	1307	529	289	184	87	75
2005	0	0	0	534	1652	5999	10313	3996	1410	444	244	114	64	46
2006	0	0	0	216	1869	6450	12144	4902	1089	372	136	47	32	40
2007	0	0	0	88	570	3732	11912	5414	1230	472	163	80	41	29

Table A3. Catch weights-at-age (kg) matrix for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healy and Mahé, 2008).

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.764
1976	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.144
1977	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.992
1978	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.894
1979	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	6.077
1980	0.000	0.000	0.126	0.244	0.514	0.659	0.869	1.050	1.150	1.260	1.570	2.710	3.120	5.053
1981	0.000	0.000	0.126	0.244	0.392	0.598	0.789	0.985	1.240	1.700	2.460	3.510	4.790	7.426
1982	0.000	0.000	0.126	0.244	0.525	0.684	0.891	1.130	1.400	1.790	2.380	3.470	4.510	7.359
1983	0.000	0.000	0.126	0.244	0.412	0.629	0.861	1.180	1.650	2.230	3.010	3.960	5.060	7.061
1984	0.000	0.000	0.126	0.244	0.377	0.583	0.826	1.100	1.460	1.940	2.630	3.490	4.490	7.016
1985	0.000	0.000	0.126	0.244	0.568	0.749	0.941	1.240	1.690	2.240	2.950	3.710	4.850	7.010
1986	0.000	0.000	0.126	0.244	0.350	0.584	0.811	1.100	1.580	2.120	2.890	3.890	4.950	7.345
1987	0.000	0.000	0.126	0.244	0.364	0.589	0.836	1.160	1.590	2.130	2.820	3.600	4.630	6.454
1988	0.000	0.000	0.126	0.244	0.363	0.569	0.805	1.163	1.661	2.216	3.007	3.925	5.091	7.164
1989	0.000	0.000	0.126	0.244	0.400	0.561	0.767	1.082	1.657	2.237	2.997	3.862	4.919	6.370
1990	0.000	0.000	0.090	0.181	0.338	0.546	0.766	1.119	1.608	2.173	2.854	3.731	4.691	6.391
1991	0.000	0.000	0.126	0.244	0.383	0.592	0.831	1.228	1.811	2.461	3.309	4.142	5.333	7.081
1992	0.000	0.000	0.175	0.289	0.430	0.577	0.793	1.234	1.816	2.462	3.122	3.972	5.099	6.648
1993	0.000	0.000	0.134	0.232	0.368	0.547	0.809	1.207	1.728	2.309	2.999	3.965	4.816	6.489
1994	0.000	0.000	0.080	0.196	0.330	0.514	0.788	1.179	1.701	2.268	2.990	3.766	4.882	6.348
1995	0.000	0.000	0.080	0.288	0.363	0.531	0.808	1.202	1.759	2.446	3.122	3.813	4.893	6.790
1996	0.000	0.000	0.161	0.242	0.360	0.541	0.832	1.272	1.801	2.478	3.148	3.856	4.953	6.312
1997	0.000	0.000	0.120	0.206	0.336	0.489	0.771	1.159	1.727	2.355	3.053	3.953	5.108	6.317
1998	0.000	0.000	0.119	0.228	0.373	0.543	0.810	1.203	1.754	2.351	3.095	4.010	5.132	6.124
1999	0.000	0.000	0.176	0.253	0.358	0.533	0.825	1.253	1.675	2.287	2.888	3.509	4.456	5.789
2000	0.000	0.000	0.000	0.254	0.346	0.524	0.787	1.192	1.774	2.279	2.895	3.645	4.486	5.531
2001	0.000	0.000	0.000	0.249	0.376	0.570	0.830	1.168	1.794	2.367	2.950	3.715	4.585	5.458
2002	0.000	0.000	0.217	0.251	0.369	0.557	0.841	1.193	1.760	2.277	2.896	3.579	4.407	5.477
2003	0.000	0.000	0.188	0.247	0.389	0.564	0.822	1.199	1.651	2.166	2.700	3.404	4.377	5.409
2004	0.000	0.000	0.180	0.249	0.376	0.535	0.808	1.196	1.629	2.146	2.732	3.538	4.381	5.698
2005	0.000	0.000	0.252	0.301	0.396	0.564	0.849	1.247	1.691	2.177	2.705	3.464	4.264	5.224
2006	0.000	0.000	0.129	0.267	0.405	0.605	0.815	1.092	1.495	1.874	2.396	3.139	3.747	4.701
2007	0.000	0.000	0.000	0.276	0.389	0.581	0.833	1.137	1.500	1.948	2.607	3.057	3.869	4.954

Table A4: Proportion mature-at-age for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healy pers. comm.). Note in the assessment, the maturity-at-age in 2008 and pre-1975 is taken as the average over the 1975-2007 period.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.04	0.04	0.03	0.12	0.21	0.34	0.50	0.77
1976	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.06	0.07	0.06	0.21	0.34	0.50	0.72
1977	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.04	0.11	0.12	0.14	0.34	0.50	0.79
1978	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.03	0.08	0.18	0.20	0.29	0.50	0.78
1979	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.06	0.06	0.16	0.28	0.31	0.50	0.80
1980	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.11	0.12	0.28	0.41	0.45	0.76
1981	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.07	0.18	0.23	0.45	0.55	0.76
1982	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.06	0.13	0.28	0.40	0.63	0.77
1983	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.07	0.12	0.24	0.40	0.59	0.80
1984	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.11	0.21	0.38	0.54	0.84
1985	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.19	0.35	0.56	0.78
1986	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.13	0.30	0.51	0.79
1987	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.10	0.22	0.43	0.77
1988	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.04	0.15	0.17	0.34	0.71
1989	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.02	0.09	0.33	0.29	0.57
1990	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.16	0.08	0.21	0.58	0.52
1991	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.15	0.97	0.25	0.41	0.74
1992	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.38	1.00	0.56	0.73
1993	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.10	0.11	0.68	1.00	0.84
1994	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.06	0.26	0.25	0.88	0.99
1995	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.17	0.53	0.47	0.98
1996	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.08	0.36	0.78	0.80
1997	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.10	0.20	0.61	0.91
1998	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.10	0.21	0.43	0.86
1999	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.09	0.21	0.41	0.80
2000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.12	0.21	0.41	0.73
2001	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.18	0.53	0.41	0.69
2002	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.07	0.36	0.53	0.90	0.71
2003	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.17	0.82	0.85	0.93
2004	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.35	0.97	0.97
2005	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.56	0.58	1.00
2006	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.56	0.80	0.86
2007	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.49	0.79	0.94

Table A5: Survey data (mean numbers per tow) of Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO. Decimalized year reflects the timing of each survey series (e.g. EU Summer survey). (Healey and Mahé, 2008)
Note: 1978-1994 2J3K survey data are direct abundance (in '000s) and have not been converted to Campelen equivalents as have the rest of the data.

2J3K Canadian Fall, 1978-1994

	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1978.9	2538	25686	54708	55914	57650	45141	28923	13379	6983	5112	4237	2541	1611	1184
1979.9	2805	22523	28846	25799	35886	38805	18843	7378	3316	3179	2102	1843	1520	1834
1980.9	2994	8911	15315	22680	35995	42154	27942	9511	4207	3229	3601	2393	1551	1419
1981.9	7563	22486	30875	21226	34277	38654	26647	11458	5281	2824	2255	1030	579	450
1982.9	2137	5991	23971	31204	31061	29062	32070	32617	13535	5375	2801	1790	1276	2517
1983.9	1004	5905	19036	31465	40182	34742	38908	31538	11559	3040	2049	1497	1089	1100
1984.9	1452	7148	21435	36094	72180	38931	30683	21712	10222	4132	1869	1216	964	1665
1985.9	7460	18147	20024	36224	44886	37715	22359	12761	6293	3498	1592	1218	517	1337
1986.9	13005	22185	32997	55685	45213	57886	45327	12676	3306	1430	960	961	441	686
1987.9	1491	8685	47694	35752	35854	33486	33956	20722	7621	2156	1065	642	504	461
1988.9	4025	12436	28404	50345	58938	39603	29733	9257	2525	809	542	309	267	480
1989.9	3407	10414	35816	69334	77935	56524	32108	9627	2884	675	558	161	56	173
1990.9	547	5347	14506	68019	65410	48199	28837	6828	1839	718	488	267	160	191
1991.9	5814	6726	11369	37832	38273	27416	9020	2155	475	231	104	61	14	7
1992.9	1684	14858	26664	34313	23316	17109	8406	962	95	48	13	0	0	0
1993.9	7510	62818	97955	46098	18385	6912	2520	739	63	0	0	13	0	0
1994.9	14541	30412	42221	43669	31165	7237	3136	947	114	38	7	0	4	0

2J3K Canadian Fall, 1995-2007

	1	2	3	4	5	6	7	8	9	10	11	12	13+
1996.9	98.68	47.82	32.01	9.54	6.28	2.47	0.84	0.19	0.18	0.04	0.02	0.01	0.02
1997.9	28.05	58.62	43.61	21.13	10.37	5.01	2.00	0.64	0.20	0.06	0.03	0.02	0.01
1998.9	23.35	25.07	31.19	21.87	10.86	4.45	2.07	0.57	0.13	0.06	0.03	0.02	0.01
1999.9	15.99	34.42	24.07	28.28	20.04	10.53	3.81	0.70	0.14	0.07	0.02	0.01	0.03
2000.9	38.57	21.94	16.43	13.20	13.76	7.21	2.16	0.50	0.06	0.03	0.02	0.00	0.00
2001.9	43.90	22.72	17.00	14.07	9.77	7.59	3.40	0.69	0.11	0.02	0.01	0.00	0.01
2002.9	40.67	24.08	12.50	9.68	6.03	1.97	0.72	0.19	0.04	0.01	0.00	0.00	0.00
2003.9	45.70	26.67	11.69	9.49	6.39	2.27	0.89	0.27	0.04	0.02	0.01	0.01	0.00
2004.9	32.49	32.93	13.89	12.31	9.21	2.68	1.20	0.36	0.08	0.03	0.01	0.00	0.01
2005.9	16.06	16.15	8.56	13.84	10.98	6.85	3.96	0.66	0.12	0.03	0.03	0.01	0.01
2006.9	32.34	17.98	8.50	17.60	13.03	9.11	4.18	1.15	0.18	0.03	0.02	0.01	0.00
2007.9	32.61	14.51	12.81	18.77	9.57	10.35	6.17	2.14	0.34	0.08	0.04	0.02	0.01

EU Summer, 1995-2007

	1	2	3	4	5	6	7	8	9	10	11	12+
1995.6	12.41	2.54	2.23	1.91	2.66	5.10	3.77	2.12	1.31	0.26	0.07	0.02
1996.6	5.84	7.97	2.42	3.04	4.20	5.82	2.49	1.62	0.42	0.09	0.03	0.04
1997.6	3.33	3.78	6.00	6.50	7.11	8.46	4.99	2.15	0.66	0.22	0.03	0.02
1998.6	2.74	2.13	7.69	11.00	12.33	11.30	7.84	2.62	0.75	0.20	0.03	0.01
1999.6	1.06	0.70	3.01	10.47	13.41	12.58	5.55	1.82	0.35	0.10	0.01	0.00
2000.6	3.75	0.29	0.60	2.17	7.09	14.10	5.40	2.32	0.45	0.11	0.05	0.00
2001.6	8.03	1.43	1.81	0.99	2.79	7.79	6.63	3.21	0.18	0.05	0.01	0.00
2002.6	4.08	2.94	2.80	1.67	3.79	5.59	5.73	1.28	0.13	0.06	0.02	0.01
2003.6	2.20	1.00	0.61	1.51	2.48	2.94	1.93	0.47	0.13	0.10	0.02	0.01
2004.6	2.19	3.29	4.37	1.97	6.97	7.80	2.54	0.64	0.29	0.13	0.08	0.05
2005.6	0.54	0.81	3.18	2.50	6.89	7.59	2.92	0.61	0.11	0.12	0.06	0.02
2006.6	0.68	0.40	0.65	1.17	5.98	7.46	3.31	0.77	0.22	0.18	0.13	0.06
2007.6	0.42	0.09	0.57	0.34	3.44	7.37	5.76	1.51	0.31	0.21	0.08	0.05

3LNO Canadian Spring, 1996-2007

	1	2	3	4	5	6	7	8+
1996.4	1.62	4.24	4.60	2.18	0.83	0.28	0.06	0.00
1997.4	1.16	3.92	5.16	3.23	1.46	0.51	0.10	0.01
1998.4	0.22	0.81	3.85	6.19	4.96	1.24	0.33	0.07
1999.4	0.29	0.55	1.15	1.98	3.39	1.09	0.24	0.05
2000.4	0.79	1.07	1.07	1.51	1.95	2.04	0.56	0.03
2001.4	0.57	0.71	0.74	0.68	0.80	0.72	0.28	0.02
2002.4	0.64	0.57	0.60	0.58	0.61	0.21	0.05	0.01
2003.4	0.93	2.14	1.66	1.57	1.06	0.21	0.05	0.01
2004.4	0.66	0.57	1.18	1.18	1.16	0.26	0.04	0.02
2005.4	0.35	0.31	1.09	0.95	1.37	0.82	0.21	0.03
2006.4	Survey not completed							
2007.4	1.60	0.52	0.80	0.40	1.41	1.49	1.12	0.18

Table A6: Survey data in terms of weight for ages combined: 2J3K Fall and 3LNO Spr (Healey, 2008), EU survey (Vázquez and González-Troncoso, 2008).

Year	2J3K Fall Mean weight (kg)/tow	EU survey Biomass (tons)	3LNO - Spr Mean weight (kg)/tow
1978	38.4		
1979	28.1		
1980	30.0		
1981	32.1		
1982	35.6		
1983	36.9		
1984	37.2		
1985	27.5		
1986	35.4		
1987	25.5		
1988	23.6	6926	
1989	25.4	4472	
1990	21.2	5799	
1991	11.5	8169	
1992	8.2	8728	
1993	15.3	6529	
1994	10.8	8037	
1995	14.1	10875	
1996	21.6	11594	1.43
1997	24.8	16098	2.10
1998	23.8	24229	3.50
1999	32.5	21207	2.33
2000	23.9	16959	2.30
2001	22.7	13872	1.13
2002	14.1	12100	0.53
2003	15.3	6214	1.13
2004	17.5	12292	0.87
2005	20.3	11698	1.23
2006	25.7	11706	
2007	29.1	13040	2.17

Appendix B - The SCAA Model

The model used for these assessments is an Age-Structured Production Model (ASPM). Models of this type fall within the more general class of Statistical Catch-at-Age Analyses. The approach used in an ASPM assessment involves constructing an age-structured model of the population dynamics and fitting it to the available abundance indices by maximising the likelihood function. The model equations and the general specifications of the model are described below, followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is used to minimize the total negative log-likelihood function (the package AD Model Builder™, Otter Research, Ltd is used for this purpose).

B.1. Population dynamics

B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,1} = R_{y+1} \qquad \text{(B1)}$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M_a/2} - C_{y,a}\right) e^{-M_a/2} \qquad \text{for } 1 \le a \le m-2 \qquad \text{(B2)}$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M_{m-1}/2} - C_{y,m-1}\right) e^{-M_{m-1}/2} + \left(N_{y,m}\, e^{-M_m/2} - C_{y,m}\right) e^{-M_m/2} \qquad \text{(B3)}$$

where

$N_{y,a}$ is the number of fish of age *a* at the start of year *y* (which refers to a calendar year),

R_y is the recruitment (number of 0-year-old fish) at the start of year *y*,

M_a denotes the natural mortality rate for fish of age *a*,

$C_{y,a}$ is the predicted number of fish of age *a* caught in year *y*, and

m is the maximum age considered (taken to be a plus-group).

These equations reflect Pope's form of the catch equation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov form (Baranov, 1918) (for which catches are incorporated under the assumption of steady continuous fishing mortality). Pope's form has been used in order to simplify computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulations will be minimal.

B.1.2. Recruitment

The number of recruits at the start of year *y* is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by a Beverton-Holt stock-recruitment relationship (Beverton and Holt, 1957), parameterised in terms of the "steepness" of the stock-recruitment relationship, *h*, and the pre-exploitation equilibrium spawning biomass, K^{sp}, and recruitment, R_0 and allowing for annual fluctuation about the deterministic relationship:

$$R_y = \frac{4hR_0 B_y^{sp}}{K^{sp}(1-h) + (5h-1)B_y^{sp}}\, e^{(\varsigma_y - \sigma_R^2/2)} \qquad \text{(B4)}$$

where

ς_y reflects fluctuation about the expected recruitment for year *y*, which is assumed to be normally distributed with standard deviation σ_R (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

B_y^{sp} is the spawning biomass at the start of year *y*, computed as:

$$B_y^{sp} = \sum_{a=1}^{m} f_{y,a} w_{y,a}^{strt} N_{y,a} \qquad \text{(B5)}$$

where

$w_{y,a}^{strt}$ is the mass of fish of age *a* during spawning, and

$f_{y,a}$ is the proportion of fish of age *a* that are mature.

In the fitting procedure, K^{sp} is estimated while *h* has thus far been fixed at 0.9 for reasons elaborated in the main text.

B.1.3. Total catch and catches-at-age

The catch by mass in year *y* is given by:

$$C_y = \sum_{a=1}^{m} w_{y,a}^{mid} C_{y,a} = \sum_{a=1}^{m} w_{y,a}^{mid} N_{y,a} e^{-M_a/2} S_{y,a} F_y \qquad \text{(B6)}$$

where

$w_{y,a}^{mid}$ denotes the mass of fish of age *a* landed in year *y*,

$C_{y,a}$ is the catch-at-age, i.e. the number of fish of age *a*, caught in year *y*,

$S_{y,a}$ is the commercial selectivity (i.e. combination of availability and vulnerability to fishing gear) at age *a* for year *y*; when $S_{y,a}$ = 1, the age-class *a* is said to be fully selected, and

F_y is the proportion of a fully selected age class that is fished.

The model estimate of the mid-year exploitable ("available") component of biomass is calculated by converting the numbers-at-age into mid-year mass-at-age (using the individual weights of the landed fish) and applying natural and fishing mortality for half the year:

$$B_y^{ex} = \sum_{a=1}^{m} w_{y,a}^{mid} S_{y,a} N_{y,a} e^{-M_a/2} (1 - S_{y,a} F_y / 2) \qquad \text{(B7)}$$

whereas for survey estimates of biomass in spring:

$$B_y^{surv,spring} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a/4} \left(1 - S_{y,a} F_y / 4\right) \qquad \text{(B8)}$$

Summer:

$$B_y^{surv,summer} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a/2} \left(1 - S_{y,a} F_y /2\right) \quad \text{(B9)}$$

and fall:

$$B_y^{surv,fall} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a 3/4} \left(1 - S_{y,a} F_y 3/4\right) \quad \text{(B10)}$$

where

S_a^{surv} is the survey selectivity for age a (which is sometimes generalised to be year-dependent).

B.1.4. Initial conditions

For the first year (y_0) considered in the model therefore, the stock is assumed to be at a fraction (θ) of its pre-exploitation biomass, i.e.:

$$B_{y_0}^{sp} = \theta \cdot K^{sp} \quad \text{(B11)}$$

with the starting age structure:

$$N_{y_0,a} = R_{start} N_{start,a} \qquad \text{for } 1 \le a \le m \quad \text{(B12)}$$

where

$$N_{start,1} = 1 \quad \text{(B13)}$$

$$N_{start,a} = N_{start,a-1} e^{-M_{a-1}} (1 - \phi S_{a-1}) \qquad \text{for } 2 \le a \le m-1 \quad \text{(B14)}$$

$$N_{start,m} = N_{start,m-1} e^{-M_{m-1}} (1 - \phi S_{m-1}) \Big/ (1 - e^{-M_m} (1 - \phi S_m)) \quad \text{(B15)}$$

where ϕ characterises the average fishing proportion over the years immediately preceding y_0.

Unless indicated otherwise though, the stock is assumed to be at pristine equilibrium in 1960, i.e. θ=1 and ϕ=0 for the results reported here.

B.2. The (penalised) likelihood function

The model can be fit to (a subset of) CPUE and survey abundance indices, and commercial and survey catch-at-age data to estimate model parameters (which may include residuals about the stock-recruitment function, the fishing selectivities, the annual catches or natural mortality, facilitated through the incorporation of penalty functions described below). Contributions by each of these to the negative of the (penalised) log-likelihood (- $\ell n L$) are as follows.

B.2.1 CPUE relative abundance data

The likelihood is calculated assuming that an observed CPUE index for a particular fishing fleet is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp\left(\varepsilon_y^i\right) \quad \text{or} \quad \varepsilon_y^i = \ell n\left(I_y^i\right) - \ell n\left(\hat{I}_y^i\right) \quad \text{(B16)}$$

where

I_y^i is the CPUE index for year y and series i,

$\hat{I}_y^i = \hat{q}^i \hat{B}_y^{ex}$ is the corresponding model estimate, where $\widehat{B}_y^{ex}$ is the model estimate of exploitable resource biomass, given by equation (B7)[1],

$\hat{q}^i$ is the constant of proportionality (catchability) for CPUE series *i*, and

ε_y^i from $N\left(0, \left(\sigma_y^i\right)^2\right)$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell\text{n}\, L^{CPUE} = \sum_i \sum_y \left[\ell\text{n}\left(\sigma_y^i\right) + \left(\varepsilon_y^i\right)^2 / 2\left(\sigma_y^i\right)^2\right] \qquad \text{(B17)}$$

where

σ_y^i is the standard deviation of the residuals for the logarithm of index *i* in year *y*.

Homoscedasticity of residuals is assumed, so that $\sigma_y^i = \sigma^i$ is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n_i \sum_y \left(\ell\text{n}(I_y^i) - \ell\text{n}(q^i \widehat{B}_y^{ex})\right)^2} \qquad \text{(B18)}$$

where

n_i is the number of data points for CPUE index *i*.

The catchability coefficient q^i for CPUE index *i* is estimated by its maximum likelihood value:

$$\ell n\, \hat{q}^i = 1/n_i \sum_y \left(\ln I_y^i - \ln \hat{B}_y^{ex}\right) \qquad \text{(B19)}$$

B.2.2. Survey abundance data

In general, data from the surveys are treated as relative abundance indices in exactly the same manner to the CPUE series above, with survey selectivity function S_a^{surv} replacing the commercial selectivity $S_{y,a}$. Account is also taken of the time of year when the survey is held. For these analyses, selectivities are estimated as detailed in section B.4.2 below.

To allow for serial correlation between the survey residuals, the ε_y^i input to equation B17 is given by:

$$\varepsilon_y^i = \lambda_y^i - \rho\lambda_{y-1}^i \qquad \text{(B20)}$$

where

$$\lambda_y^i = \ell\text{n}\left(I_y^i\right) - \ell\text{n}\left(\hat{I}_y^i\right)$$

[1] Ideally $\widehat{B}_y^{ex}$ should be fleet specific, corresponding to the selectivity for the fleet linked to CPUE index *i*. However, this requires the total annual catch and catch-at-age data to be provided on a fleet-disaggregated basis, and these data are not immediately available in this form.

ρ is the serial correlation coefficient, which is estimated (or set to zero in the case of the Baseline assessment B1). Note that ρ could be series dependent, but analyses for the data set available indicated that estimation of series-specific values was not justified in AIC terms. The standard deviation of the ε_y^i is termed σ_{surv} in Table 1.

B.2.3. Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an "adjusted" lognormal error distribution is given by:

$$-\ell\text{n}\, L^{CAA} = \sum_y \sum_a \left[\ell n\left(\sigma_{com} / \sqrt{p_{y,a}} \right) + p_{y,a} \left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a} \right)^2 / 2\left(\sigma_{com}\right)^2 \right] \qquad \text{(B21)}$$

where

$p_{y,a} = C_{y,a} / \sum_{a'} C_{y,a'}$ is the observed proportion of fish caught in year *y* that are of age *a*,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a'} \hat{C}_{y,a'}$ is the model-predicted proportion of fish caught in year *y* that are of age *a*,

where

$$\hat{C}_{y,a} = N_{y,a}\, e^{-M_a/2}\, S_{y,a}\, F_y \qquad \text{(B22)}$$

and

σ_{com} is the standard deviation associated with the catch-at-age data (termed "σ_{comCAA}" in Table 1), which is estimated in the fitting procedure by:

$$\hat{\sigma}_{com} = \sqrt{\sum_y \sum_a p_{y,a} \left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a} \right)^2 / \sum_y \sum_a 1} \qquad \text{(B23)}$$

The log-normal error distribution underlying equation (B21) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation (B21), for which the summation over age *a* is taken from age a_{minus} (considered as a minus group) to a_{plus} (a plus group).

B.2.4. Survey catches-at-age

The survey catches-at-age are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation (B21)) where:

$p_{y,a} = C_{y,a}^{surv} / \sum_{a'} C_{y,a'}^{surv}$ is the observed proportion of fish of age *a* in year *y*,

$\hat{p}_{y,a}$ is the expected proportion of fish of age *a* in year *y* in the survey, given by:

$$\hat{p}_{y,a} = S_a^{surv} N_{y,a} \Big/ \sum_{a'=0}^{m} S_a^{surv} N_{y,a} \qquad \text{for begin-year surveys.} \qquad \text{(B24)}$$

The residual standard deviation (analogous to σ_{com}, alternatively termed "σ_{comCAA}" in the previous section, is termed σ_{survCAA} in Table 1.

B.2.5. Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell nL^{SRpen} = \sum_{y=y1}^{y2} \left[\varepsilon_y^2 / 2\sigma_R^2 \right] \quad \text{(B25)}$$

where

ε_y from $N\left(0,(\sigma_R)^2\right)$, which is estimated for year $y1$ to $y2$ (see equation (B4)), and

σ_R is the standard deviation of the log-residuals, which is input.

B.2.6. Selectivity residuals

In some instances, variations around the fishing selectivity functions are estimated in two-year periods:

$$S_a \rightarrow S_{y,a} = S_a e^{\Omega_{y,a}} \quad \text{(B26)}$$

The contribution of the selectivity residuals to the negative of the penalised log-likelihood is given by:

$$-\ell nL^{Selpen} = \sum_{y=y1}^{y2} \sum_{a=a1}^{a2} \left[\Omega_{y,a}^2 / 2\sigma_\Omega^2 \right] \quad \text{(B27)}$$

where

$\Omega_{y,a}$ from $N\left(0,(\sigma_\Omega)^2\right)$, which is estimated for year $y1$ to $y2$ and age $a1$ to $a2$, and

σ_Ω is the standard deviation of the residuals, which is input.

B.2.7. Annual catch residuals

In some instances, differences between the reported and the actual annual catches are estimated.

$$C_y \rightarrow C_y^{reported} = C_y^{actual} e^{\omega_y} \quad \text{(B28)}$$

The contribution of the catch residuals to the negative of the penalised log-likelihood is given by:

$$-\ell nL^{Cpen} = \sum_{y=y1}^{y2} \left[\omega_y^2 / 2\sigma_C^2 \right] \quad \text{(B29)}$$

where

ϖ_y from $N\left(0,(\sigma_C)^2\right)$, which is estimated for year $y1$ to $y2$, and

σ_ω is the standard deviation of the residuals, which is input.

B.2.7. Mortality residuals

In some instances, variations about the default age- and year-independent natural mortality are estimated.

$$M_a \rightarrow M_a e^{\varsigma_{y,a}} \qquad \text{(B30)}$$

The contribution of the catch residuals to the negative of the penalised log-likelihood is given by:

$$-\ell n L^{Mpen} = \sum_{y=y1}^{y2} \sum_{a=a1}^{a2} \left[\varsigma_{y,a}^2 / 2\sigma_M^2 \right] \qquad \text{(B31)}$$

where

$\varsigma_{y,a}$ from $N\left(0,(\sigma_M)^2\right)$, which is estimated for year $y1$ to $y2$, and age $a1$ to $a2$, and

σ_M is the standard deviation of the residuals, which is input.

B.3. Estimation of precision

Where quoted, CVs are Hessian-based.

B.4. Model parameters

B.4.1. Fishing selectivity-at-age:

The commercial fishing selectivity, S_a, is estimated separately for ages 5-11. The estimated decreases from ages 6 to 5 and ages 11 to 12 are assumed to continue exponentially to ages 0 and 14+ respectively. Similarly, the selectivities for the surveys are estimate separately for ages 1-11 for the Canadian Fall and EU surveys and for ages 1-8 for the Canadian spring surveys. The estimated decreases from ages 2 to 1 and from ages 10 to 11 (7 to 8 for the Canadian spring survey) are assumed to continue exponentially to ages 0 and 14+ respectively.

B.4.2. Other parameters

Plus group:	
m	14
Commercial CAA:	
a_{minus}	5
a_{plus}	12
Survey CAA:	
a_{minus}	1
a_{plus}	11/8
Stock-recruitment residuals:	
Δ_R	0.25
$y1$	1961
$y2$	2007
Natural mortality:	
M	age independent, fixed
Age-at-maturity:	
$f_{y,a}$	input, see Table A4
Weight-at-age:	
$w_{y,a}$	input, same for start-and mid-year, see Table A3
Initial conditions:	
Δ	1 (unless otherwise specified)
Δ	0 (unless otherwise specified)
Survey serial correlation:	
Δ	0 (unless otherwise specified)

Addendum

In the calculations reported here, the following modifications to the specifications above were incorporated:

1) Taking the modelled population age structure to a plus-group age *m* of 20 rather than 14 so that any decreasing selectivity trend continues to a larger age. For the commercial selectivity, the estimated decrease from ages 11 to 12 are assumed to continue exponentially to age 20+ for the Reference Case Similarly for the survey selectivities, the estimated decrease from ages 10 to 11 (7 to 8 for the Canadian spring survey) is assumed to continue exponentially to age 20+.

2) The inclusion of correlation, in both year and age, in survey proportions-at-age data. To allow for serial correlation between the survey proportion-at-age residuals, equation B21 above is replaced by:

$$-\ell n\, L^{CAA} = \sum_{i=1}^{n_{surv}} \left\{ \begin{array}{l} \displaystyle\sum_{y=2}^{n} \sum_{a=a_{\min us}+1}^{a_{plus}} \left[\ell n \left(\sigma^i_{surv} / \sqrt{p^i_{y,a}} \right) + p^i_{y,a} \left(\eta^i_{y,a} \right)^2 / 2 \left(\sigma^i_{surv} \right)^2 \right] \\ \displaystyle + \sum_{a=a_{\min us}}^{a_{plus}} \left(\sum_{y=1}^{n} p^i_{y,a} - \sum_{y=1}^{n} p^i_{y,a} \right)^2 \end{array} \right\} \qquad \text{(B32)}$$

with

$$\eta^i_{y,a} = \left(\varepsilon^i_{y,a} - \rho^i_{CAAage} \varepsilon^i_{y,a-1} \right) - \rho^i_{CAAyr} \left(\varepsilon^i_{y-1,a} - \rho^i_{CAAage} \varepsilon^i_{y-1,a-1} \right) \qquad \text{(B33)}$$

$$\varepsilon^i_{y,a} = \sqrt{p^i_{y,a}} \left(\ell n\, p^i_{y,a} - \ell n\, \hat{p}^i_{y,a} \right) \qquad \text{(B34)}$$

and

$$\hat{\sigma}^i_{surv} = \sqrt{\sum_y \sum_a \eta^2_{y,a} / \sum_y \sum_a 1} \qquad \text{(B35)}$$

ρ^i_{CAAage} is the age serial correlation coefficient for survey series *i*, which is estimated, and

ρ^i_{CAAyr} is the year serial correlation coefficient for survey series *i*, which is estimated.

The second term in the likelihood has been added so that the average predicted proportion-at-age is close to that observed. This is necessary here because taking account of serial correlation loses the information otherwise in the likelihood to fit to data for the youngest age-class.