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A stochastic length-based assessment model for the *Pandalus* stock in Skagerrak and the Norwegian Deep

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Abstract

This working document describes a length based stochastic assessment model of *Pandalus* in ICES areas IIIA and IVA. The model describing stock development is age based, but the model also estimates the relation between age and length assuming a von Bertalanffy growth curve.

Input data

Data for this assessment covers the period from 1988 to 2012. Total commercial catch-at-length in numbers $C_{l,y}$ are available only in the years from 1988 to 2010. The total catches are accumulated in 1mm length classes starting with 8mm.

In addition to the total catches a (scientific) survey-index $I_{l,y}$ is available in all years except 2003. The survey length classes start at 7mm.

The natural mortality is assumed to be $M_{a,y} = 0.75$ for all ages in all years. This figure has been used in previous assessments of this stock.

The model

The model is a full parametric statistical model. In this model the stock development from year to year is an age based model, but the data is given in lengths.

The stock dynamics is similar to most fully parametric statistical age based stock assessment models. The stock sizes in the first age class $N_{a=0,y=1988...2012}$ are assumed to be model parameters to be estimated. For the stock sizes in the first year $N_{a=0...4,y=1988}$ $N_{a=1,y=1988}$ and $N_{a=2,y=1988}$ are model parameters and the last two are set by assuming that the total mortality and stock sizes were similar the year before, such that $N_{a=3,y=1988} = N_{a=2,y=1988}e^{Z_{a=2,y=1988}}$ and $N_{a=4,y=1988} = N_{a=3,y=1988}e^{Z_{a=3,y=1988}}$.

A multiplicative model is assumed for the age specific fishing mortalities $F_{a,y} = F_aF_y$. Further, to ensure that the model is identifiable, it is assumed that $F_{a=2} = F_{a=3} = F_{a=4}$. The total mortality is defined as $Z_{a,y} = M_{a,y} + F_{a,y}$.

Having defined the total mortality Z and the marginal stock sizes, all other stock sizes $N_{a,y}$ can be calculated successively by the stock equation $N_{a+1,y+1} = N_{a,y}e^{-Z_{a,y}}$.

The relationship between age and length is assumed to follow the Von Bertalanffy growth function

$$L(a) = L_\infty - (L_\infty - L_0)e^{-ka}$$

Where L_∞, and k are model parameters to be estimated describing the average length of an old individual and the growth rate respectively. L_0 is a model parameter describing individual length at age zero and is fixed to zero in this model. The length distribution of a year old fish are assume to be normally distributed with a mean given by the Von Bertalanffy growth function and a standard deviation estimated by the model. This normal distribution would more logically be assumed on the log-scale, this was attempted, but found too unstable in this model.

The data are observed commercial catch-at-length $C_{l,y}$ and survey catch-at-length $I_{l,y}$, for a number of length groups $g_1, \ldots, g_n$.

The expected number of fish caught by the survey in a length group g_i can be computed by:

$$\widehat{I_{g_i,y}} = \sum_{a=0}^{4} \left(\Phi\left(\frac{\text{upper}(g_i) - L(a+\tau)}{\sigma_L}\right) - \Phi\left(\frac{\text{lower}(g_i) - L(a+\tau)}{\sigma_L}\right)\right) Q(a) N_{a,y} e^{-Z_{a,y}\tau}$$

In this equation the term $Q(a)N_{a,y}e^{-Z_{a,y}\tau}$ is recognized as the expected number caught from age group a, if τ is the time of year where the survey is conducted, and $Q(a)$ is the survey catchability for age group a. The remaining large parenthesis computes what fraction of lengths at age $a+\tau$ that are within length group g_i. Here Φ is the distribution function for a standard normal distribution, lower(g_i) and upper(g_i) are the lower and upper limits defining the length group g_i. $L(a+\tau)$ is, according to the Von Bertalanffy growth function the mean length at age $a+\tau$ and σ_L is the corresponding standard deviation. Finally, the lengths in g_i from all age groups are summed.

It is slightly more difficult to compute the expected commercial catch in each specified length group, since this is not taken at a specific time, but throughout the entire year, while the individuals are growing. Here it is assumed that fishing occurs uniformly in time within each year.

Computing the expected total catch in each length group is done be stepping through the year in sufficiently small steps (here 10 per year). In each step the fraction with lengths in the length group g_i of the total catch from that step is added up. With 10 steps define $\Delta t = 1/10$, then:

$$\widehat{C_{g_i,y}} = \sum_{s=1}^{10}\sum_{a=0}^{4} \left(\Phi\left(\frac{\text{upper}(g_i) - L(a+(s-0.5)\Delta t)}{\sigma_L^{(c)}}\right) - \Phi\left(\frac{\text{lower}(g_i) - L(a+(s-0.5)\Delta t)}{\sigma_L^{(c)}}\right)\right) \cdot \frac{F_{a,y}}{Z_{a,y}}(1 - e^{-Z_{a,y}\Delta t}) N_{a+(s-1)\Delta t,y}$$

Here the last term is recognized as the catch equation applied in each step, and the first term is computing what fraction has lengths within length group g_i.

The remaining part of the model is the observation error which is assumed to be independent normally distributed with separate variance parameters for total catches and for survey catches.

Example calculating reference points

The model is able to produce full stock numbers at age, fishing mortality at age, and as such normal calculations of reference points can also be conducted. As an example the calculation of $F_{0.1}$ is illustrated here.

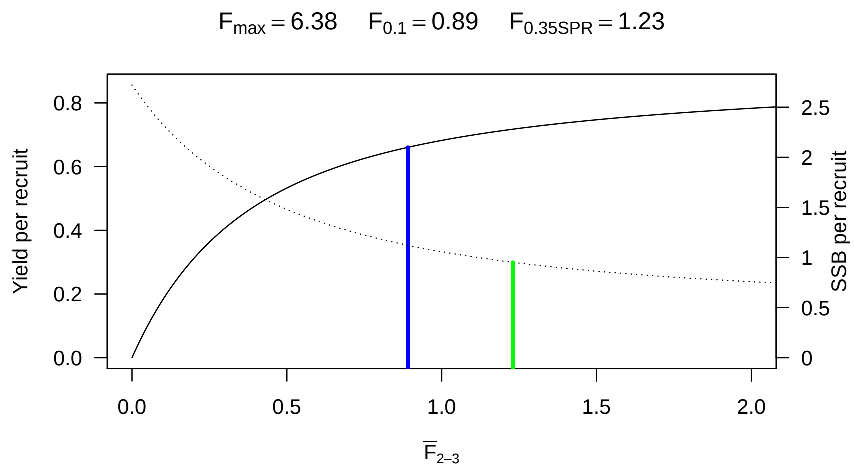

Figure 1: Yield per recruit and reference point calculation example

The yield per recruit curve must be calculated, which should not be based on the plus group selected for the assessment model, as the cohort need to be followed to the end. The maximum age was set to 20, which should be more than enough. The fishing selectivity at age F_a was set as estimated by the model and extended to maximum age by setting the selectively equal to selectivity at age 4 for all ages above 4. The average proportion mature over the last 15 years was used, and set to 1 for ages above 4. The natural mortality was extended by setting it equal age 4 for all ages above 4. The weights-at-ages are calculated via the estimated length at age relationship (details to be updated).

Having extended all the variables that determinate the cohort development, a yield is computed for a number of different levels of $\overline{F}_{2-3}$, here the levels 0,0.01,0.02,...,2.0 were chosen, and the yield was plotted against $\overline{F}_{2-3}$. Finally, the point on the yield per recruit curve, where the slope is 10% of its initial slope (at $\overline{F}_{2-3} = 0$), is identified.

Example calculating forecasts

Following the estimated stock numbers at age and fishing mortality at age it is possible to carry out normal short term forecasts. An example the calculation is shown here. The stock status at the end of the final assessment year (here 2011) is used as the starting point.

To forecast different stock options assumptions about the level of the future recruitment is needed. For this example the median of the last 7 years was used, since there is no obvious stock-recruitment relationship. For a future stock weights, catch weights, proportion mature and natural mortality, an average of the last three years (2009-2011) was used.

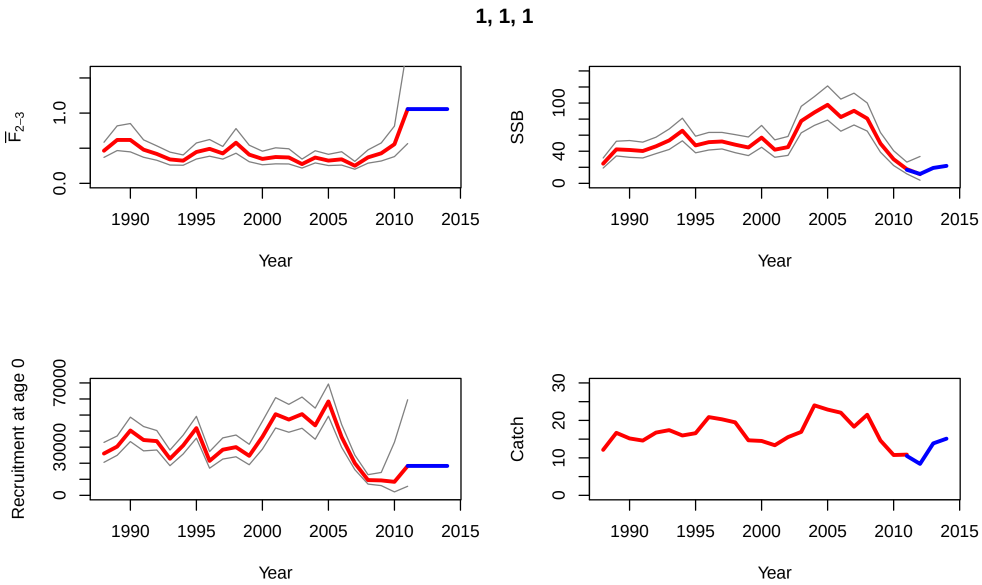

Figure 2: Short term forecast example calculation

Consider now the scenario, where fishing is set to be the same in the three following years. For the first forecast year (2012) the recruitment is set to the median described above, and following the fishing and natural mortalities each N-at-age is updated one step ahead from the previous year. The catch and SSB is calculated. This is repeated three times.

1 Stockassessment.org

All details about the model, its code, and the results can be found at `http://www.stockassessment.org`

Results

Year	$\overline{F}_{2-3}$	**SSB**
1988	0.46	24.5
1989	0.62	42.3
1990	0.62	41.6
1991	0.48	40.3
1992	0.42	46.1
1993	0.34	53.4
1994	0.32	65.5
1995	0.45	47.2
1996	0.49	51.3
1997	0.43	52.1
1998	0.58	48.1
1999	0.41	44.7
2000	0.35	56.9
2001	0.37	41.9
2002	0.37	45.1
2003	0.27	77.6
2004	0.37	88.3
2005	0.32	97.8
2006	0.34	82.5
2007	0.25	90.3
2008	0.37	80.7
2009	0.43	49.6
2010	0.56	30
2011	1.06	17.7
2012	NA	11.4

Table 1: Yearly estimates of $\overline{F}_{2-3}$ and Spawning stock biomass.

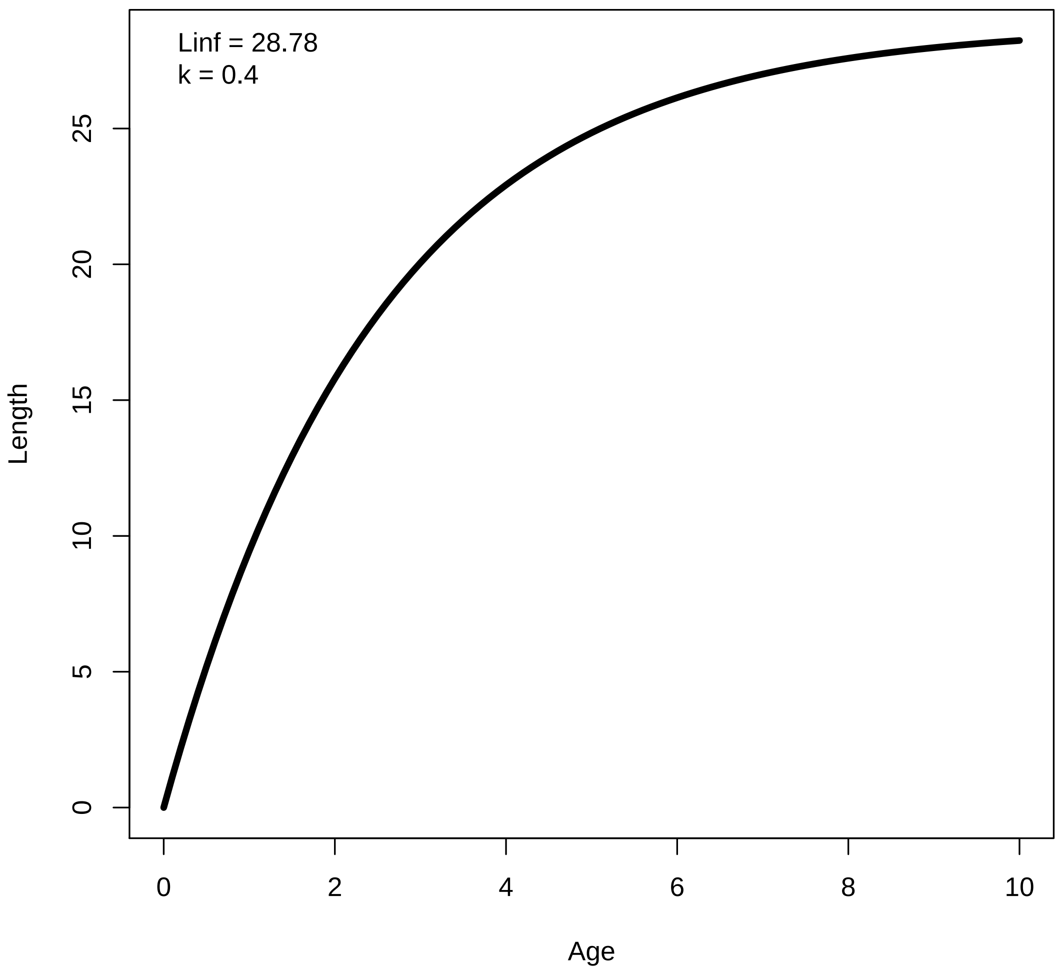

Figure 3: The estimated relation between age and length.

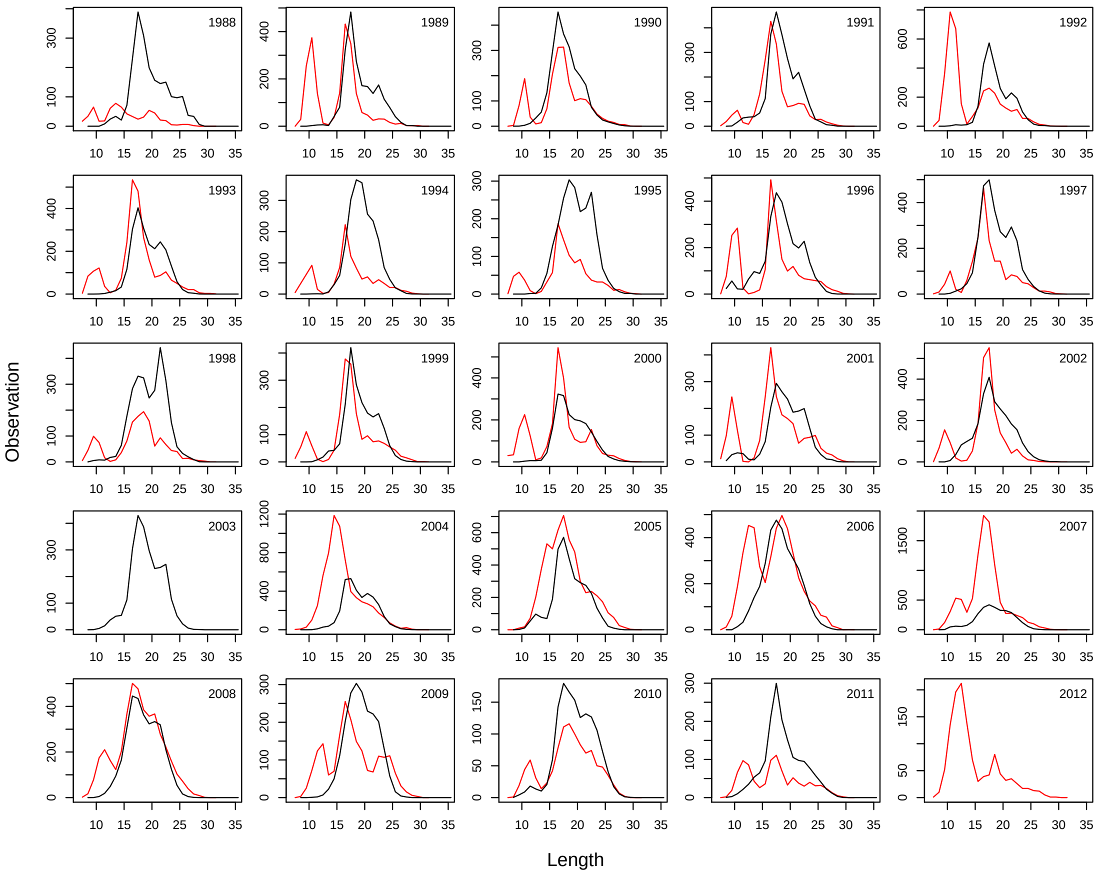

Figure 4: The length observations from surveys (red) and form total catches (black).

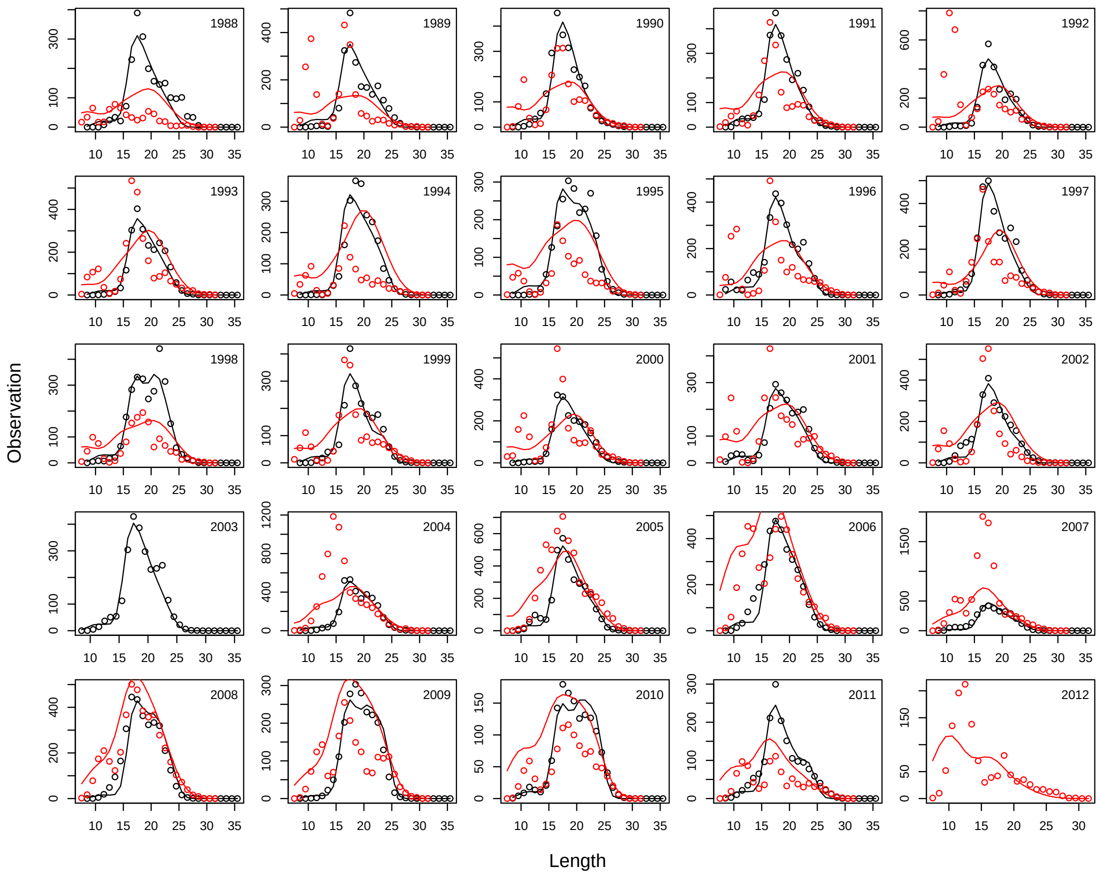

Figure 5: The length observations from survey (red circles) and total catches (black circles) on top of the predicted length distributions from survey (red lines) and total catches (black lines).

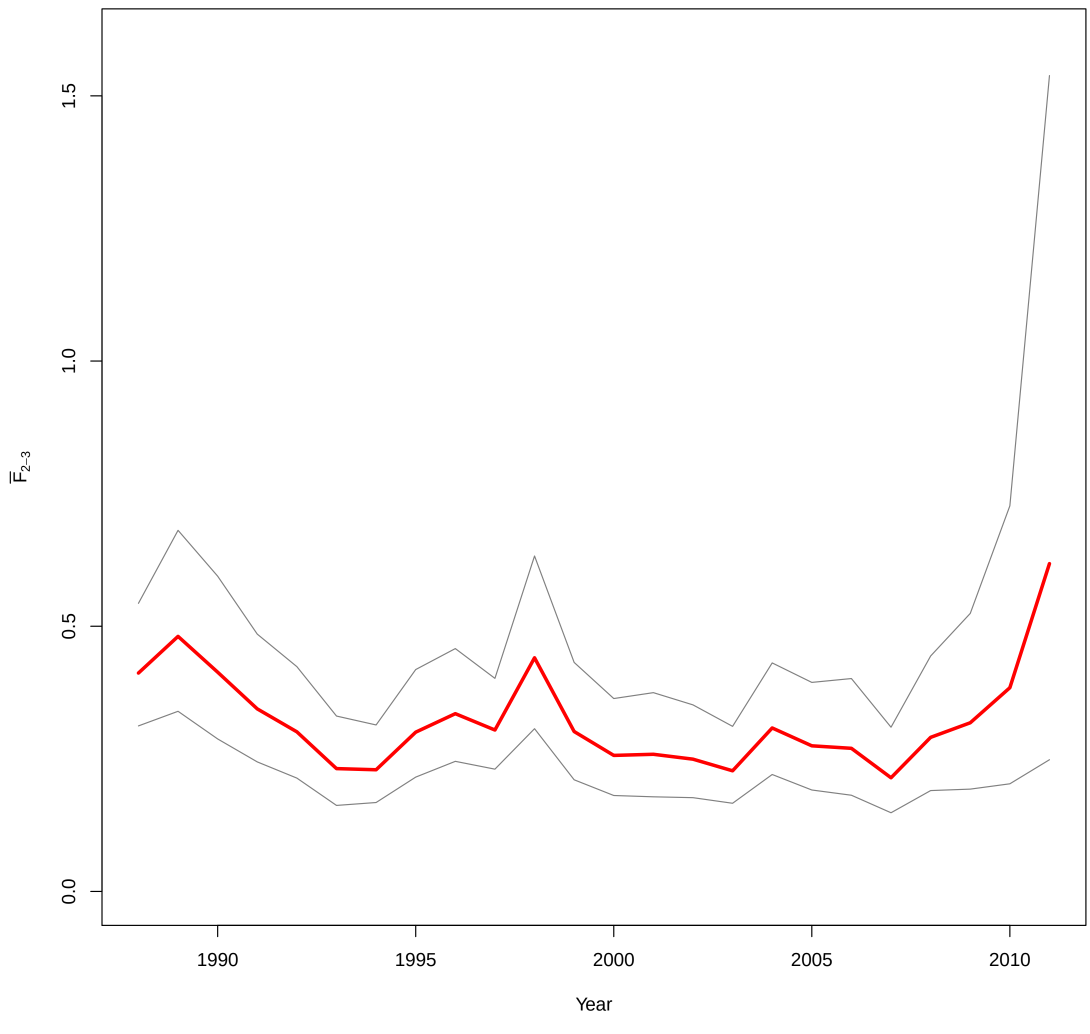

Figure 6: The estimated fishing mortalities

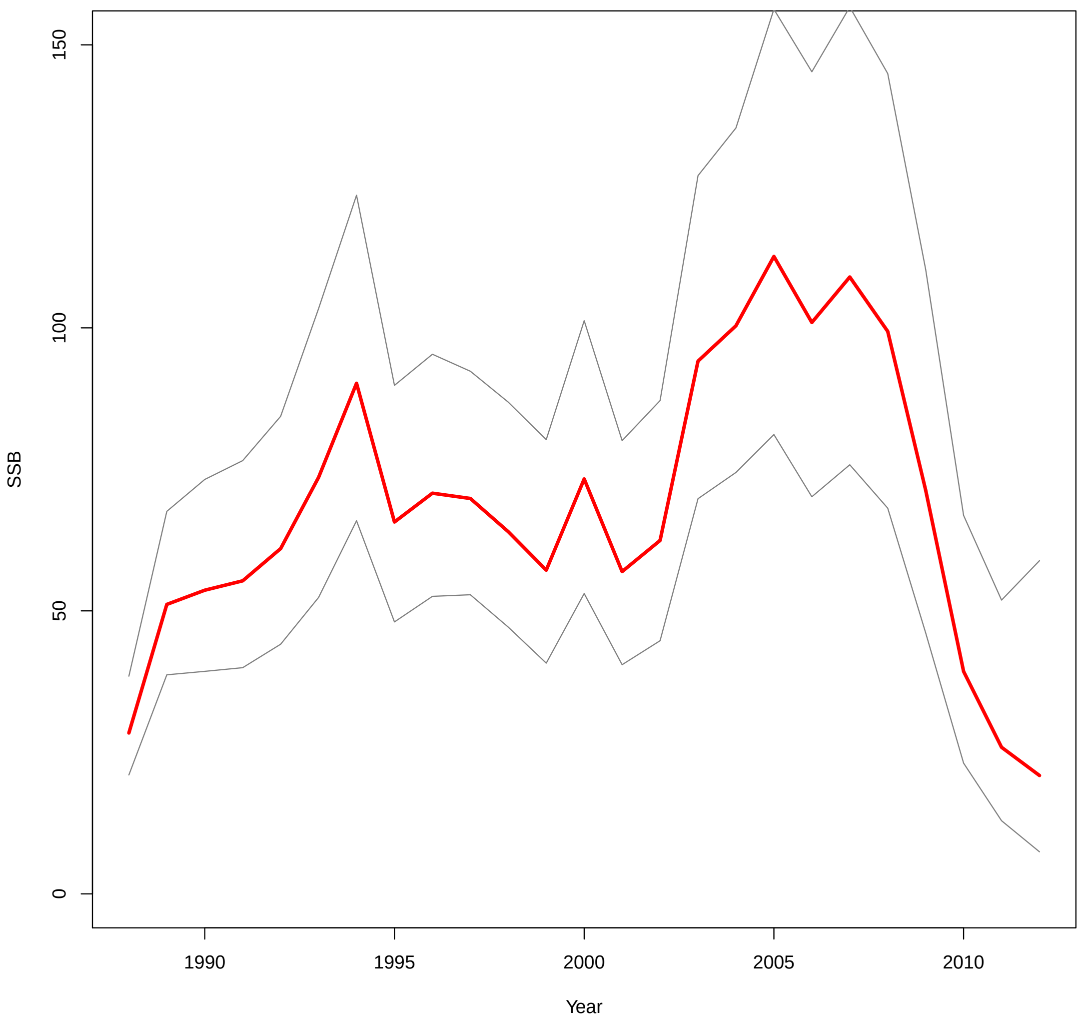

Figure 7: The estimated spawning stock biomass

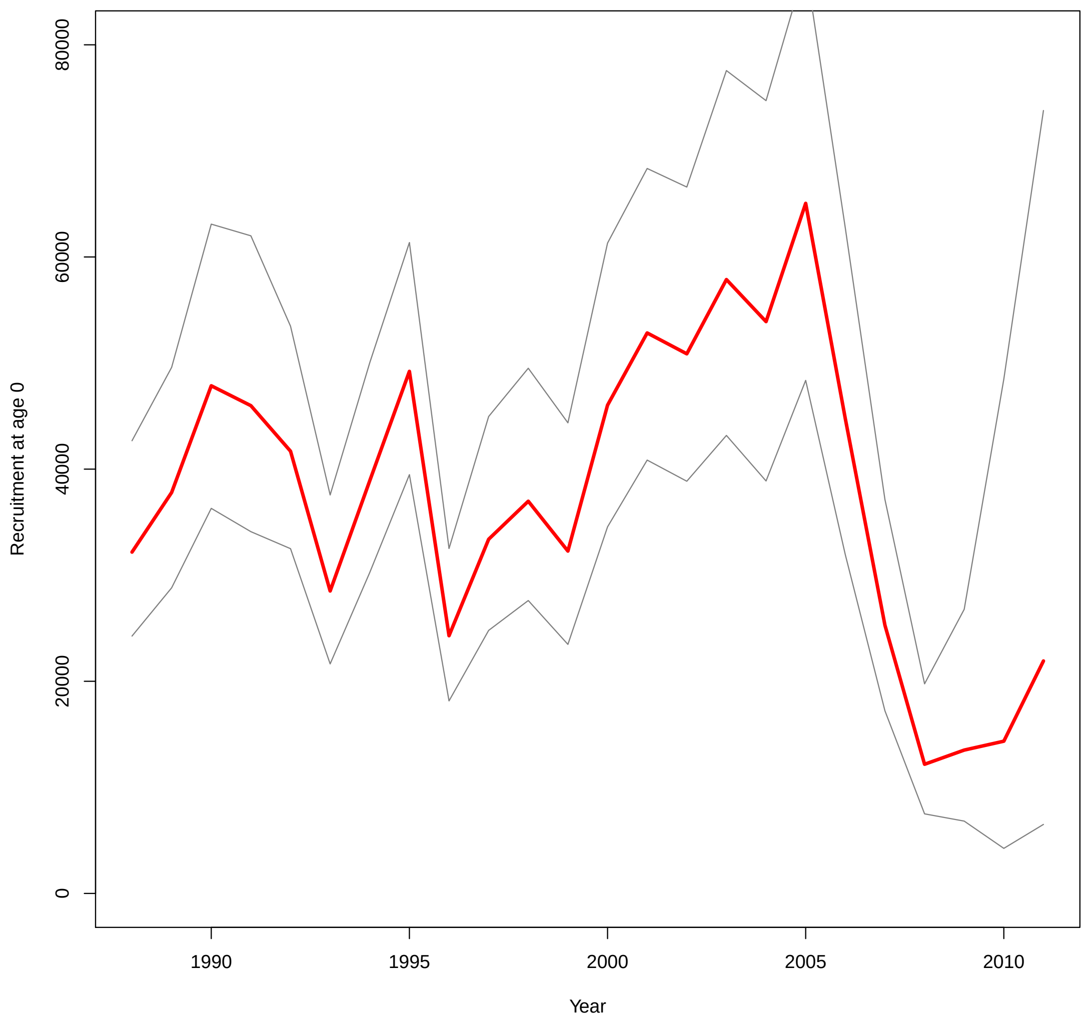

Figure 8: The estimated recruitment (age class 0)

Comments

It is a start, but I still recommend that more than one model is used.

Age-length relationship estimated within.

Cause for concern that the survey fit is so poor.

The catch is fitted well.

Likelihood need further advancement to possible include correlation, and unequal variances.

Easy to try out.