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North Sea Pandalus benchmark stock assessment -a Bayesian surplus production model

by

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Background

The purpose of the study was to investigate the suitability of a biomass dynamic model for the assessment of the North Sea/Skagerrak shrimp stock. The model was similar in structure to the one currently used in the assessment of the Barents Sea shrimp stock (Hvingel 2011, Hvingel and Kingsley 2005) and is based on the general assumption that the production curve of the stock is dome shaped, i.e. population growth is logistic.

Predation, although an important source of mortality for shrimp (Hvingel 2005 and references therein), was not included as an explicit variable because the available composite predator abundance indices (ref) varied little over time and was found not to hold any information regarding shrimp stock dynamics.

A similar investigation was done in 2005 (Hvingel 2005) and in essence concluded that the available data series of biomass indices and catch held little information regarding model parameters. For this study the data series had been extended both back- and forward in time and included.

Model

Framework

The model was built in a state-space framework (Hvingel and Kingsley 2006, Schnute 1994) with a set of parameters (θ) defining the dynamics of the shrimp stock. The posterior distribution for the parameters of the model, $p(\theta|data)$, given a joint prior distribution, $p(\theta)$, and the likelihood of the data, $p(data|\theta)$, was determined using Bayes' (1763) theorem:

(1) $$p(\theta \mid data) \propto p(data \mid \theta)\, p(\theta)$$

The posterior was derived by Monte-Carlo-Markov-Chain (MCMC) sampling methods using WinBUGS v.1.4 (www.mrc-bsu.cam.ac.uk/bugs; Spiegelhalter et al. 2003).

State equations

The equation describing the state transition from time t to t+1 was a discrete form of the logistic model of population growth including fishing mortality (e.g. Schaefer (1954), and parameterised in terms of *MSY* (Maximum Sustainable Yield) rather than *r* (intrinsic growth rate) (cf. Fletcher 1978):

(2) $$B_{t+1} = B_t - C_t + 4MSY \frac{B_t}{K}\left(1 - \frac{B_t}{K}\right)$$

K is the carrying capacity, or the equilibrium stock size in the absence of fishing. B_t is the stock biomass. C_t is the catch taken by the fishery.

To cancel out the uncertainty of the "catchability" (the parameter that scales biomass indices to real biomass) equation (2) was divided throughout by B_{MSY}, (Hvingel and Kingsley 2006). Finally a term for the process error was applied and the state equation took the form:

(3)
$$P_{t+1} = \left(P_t - \frac{C_t}{B_{MSY}} + \frac{2\, MSY\, P_t}{B_{MSY}} \left(1 - \frac{P_t}{2} \right) \right) \cdot \exp(\nu_t)$$

where P_t is the stock biomass relative to biomass at MSY ($P_t = B_t / B_{MSY}$) in year t. This frames the range of stock biomass (P) on a relative scale where P_{MSY}=1 and K=2. The 'process errors', v, are normally, independently and identically distributed with mean 0 and variance σ_ν^2 .

Observation equations

The model synthesized information from input priors and four independent series of shrimp biomasses and one series of shrimp catches (Table 1). The four series of shrimp biomass indices were: a standardised series of annual commercial-vessel catch rates for Danish vessels 1987–2010, $CPUEdk_t$, and Norwegian vessels 2000-2010 $CPUEnor_t$, (Guldborg et al WGdoc); and two trawl-survey biomass index for 1984–2002, $surv1_t$, and 2006-2012, $surv2_t$ (Guldborg al WGdoc). These indices were scaled to true biomass by catchability parameters, q_{dk}, q_{nor}, q_1 and q_2. Lognormal observation errors, ω, η, κ and ε were applied, giving:

(4)
$$\begin{aligned} CPUEdk_t &= q_{dk} B_{MSY} P_t \exp(\omega_t) \\ CPUEnor_t &= q_{nor} B_{MSY} P_t \exp(\eta_t) \\ surv1_t &= q_1 B_{MSY} P_t \exp(\kappa_t) \\ surv2_t &= q_2 B_{MSY} P_t \exp(\varepsilon_t) \end{aligned}$$

The error terms, ω, η, κ and ε are normally, independently and identically distributed with mean 0 and variance σ_ω^2, σ_η^2 σ_κ^2 and σ_ε^2 .

Total reported catch 1970-2011 was used as yield data (Table 1) and entered into the model as error-free.

Similar models have been applied to the shrimp fisheries off West Greenland (Hvingel and Kingsley 2005, Hvingel 2004, Anon. 2004b) and in the Barents Sea (Hvingel 2011).

Run 1.
A base run with uninformative priors on all parameters was able to reproduce the point estimates of the input data, however with very wide confidence limits. The priors of model parameters got somewhat updated, but again, their posteriors had very low precision. This indicated noisy data and/or that the information contained in the data with respect to some of the model parameters was low.

More information can be added to the model through the priors and this was subsequently done in a second run:

Run 2.
Low-information priors (reference priors) were given to MSY, K, σ_v and the σ for the two CPUE biomass index series as there was little or no information on what their probability distributions might look like (Table 2). MSY was given a generously wide uniform prior between 0 and 100 000 tons. A similar distribution was used for K between 1 and 665 000 tons (The upper limit corresponds to about 11g or about 5-10 shrimp per m^2 over the survey area of 57 300 km^2 which by shrimp experts is considered to be high). The prior for the stock size in the initial year, P_0, was Norm(1.5, 5) a relatively wide distribution indicating a higher probability of the stock being above than below Bmsy as the fishery at and prior to that time was relatively small; in any case, the model showed little sensitivity to the setting of this prior: only the trajectory of the first ca. 10 years (1970-1980) would differ, after that they would converge.

The prior distributions for the error terms associated with the survey biomass indices were assigned inverse gamma distributions with a mode at 0.2, comparable to the CVs typically found in such surveys. Berenboim et al. (1980) estimated a catchability of 0.173 by calibrating trawl catches to the results of a photo survey. This was chosen as basis for an informative prior by giving *q* a lognormal distribution with a median of 0.173 and a variance of 0.3.

Convergence diagnostics

In order to check whether the sampler had converged to the target distribution a number of parallel chains with different starting points and random number seeds were analysed by the Brooks, Gelman and Rubin convergence diagnostic (Gelman and Rubin 1992; Brooks and Gelman 1998) A stationarity test (Heidelberger and Welch 1983) was applied to individual chains. If evidence of non-stationarity is found iterations were discarded from the beginning of the chain until the remaining chain passed the test. Raftery and Lewis's (1992) tests for convergence to the stationary distribution and estimation of the run-lengths needed to accurately estimate quantiles were used, and finally the Geweke convergence diagnostic was applied (Geweke 1992). A visualisation of the converged chains can be seen in Fig. 1.

Model check

In order to check whether the model was a 'good' fit to the data, different goodness-of-fit statistics were computed. Firstly, we calculated the simple difference between each observed data point and its trial value in each MCMC sampling step. The summary statistics of the distributions of these residuals indicated by their central tendency whether the modelled values were biased with respect to the observations.

Secondly, the overall posterior distribution was investigated for potential effects of model deficiencies by comparing each data point with its posterior predictive distribution (Posterior Predictive Checks; Gelman et al. 1995, 1996). If the model fitted the observed data well, the observed data and the replicate data should look alike. The degree of similarity between the original and the replicate data points was summarised in a vector of *p*-values, calculated as the proportion of n simulations in which a sampling of the posterior distribution for an observed parameter exceeded its input value:

$$p.value = \frac{1}{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{N}} I((data_{\mathrm{j}}^{rep}, \theta_{\mathrm{j}}) - (data^{obs}, \theta_{\mathrm{j}})) \quad ,$$

where $I(x)$ is 1 if x is true, 0 if x is false. Values close to 0 or 1 in the vector *p-value* would indicate that the observed data point was an unlikely drawing from its posterior distribution.

Results, model performance

The sampler was therefore set to do 5 million iterations. Only each 500^{th} value of the sampled chains for the model parameters was stored and used for further analyses in order to remove within chain autocorrelation (Fig. 1). After 50 stored iterations the sampler had converged to the target distribution (Fig. 2) leaving 9950 samples for each parameter for the final analysis.

The model was able to produce a reasonable simulation of the observed data (Fig. 4). The probabilities of getting more extreme observations than the realised ones given in the data series on stock size were in the range of 0.03 to 0.88 i.e. a few observations was found to lie in the extreme tails of their posterior distributions (Table 4). The CPUE series was generally better estimated than the survey series. Otherwise no major problems in capturing the variability of the data were detected.

For the parameters K and P_1 the posterior distributions tended to approximate the input priors. The prior for the "initial" shrimp stock biomass (P_0) was slightly informative giving credit to "low-exploited stock conditions" at the start of the series in 1969. Making this prior low-informative by giving P_0 a uniform prior between 0 and 2 have previously been shown to have little or no effects on the posterior of other parameters in the model – except for the first 9-10 years of P (relative biomass). After this period the series converge.

The model was having problems estimating absolute stock size without the help of an informative prior on either K or survey qs. In this case there was some information available to set q (it might also be possible to construct a prior for K with help from stock experts) in either case the model will show some sensitivity to the choice of prior this "scaling" prior.

Assessment results

The estimated biomass-ratio has been above its MSY-level until the late 1980s (Fig. 6+7) where a steep decline was noted and the median biomass went below Bmsy (Fig. 7). Since then the stock has varied with a slightly increasing trend until 2006 when it started to decline. The median 2011 level is likely below Bmsy but above Blim (Table 6). The estimated risk of stock biomass being below $B_{trigger}$ in 2011 was 16% (Table 6).

There is a high uncertainty associated with the estimates of F (Fig 7). In 2011 there is a 45% risk of the F being above Fmsy (Table 6).

The posterior for *MSY* was positively skewed with a median at 13 ktons and upper and lower quartiles at 9 ktons and 17 ktons (Table 5).

Concluding comments

The model works and can produce results to guide management

There are relatively large uncertainties associated with model estimates which will be reflected in risk analyses of management performance.

Precision can be improved if an informative prior on K can be constructed. Further, the model points to some data points as being large outliers, they should be further investigated.

References

Bayes, T. 1763. An essay towards solving a problem in the doctrine of chances. Philosophical Transactions of the Royal Society, 330–418. Reprinted in Biometrika 1958, 45: 293–315.

Brooks, S. and Gelman, A. 1998. General methods for monitoring convergence of iterative simulations. Journal of Computational and Graphical Statistics 7, 4: 434–455.

Fletcher, R.I. 1978. Time-dependent solutions and efficient parameters for stock-production models. Fisheries Bulletin, 76: 377–388.

Gelman, A. and Rubin, D. B. 1992. Inference from iterative simulation using multiple sequences. Statistical Science, 7: 457–511.

Gelfand, A.E. and Dey, D.K. 1994. Bayesian model choice: asymptotics and exact calculations. Journal of the Royal Statistical Society, B 56: 501–514.

Gelman, A., Meng, X.L., and Stern, H.S. (1996) Posterior predictive assessment of model fitness via realized discrepancies, Statistica Sinica, 6: 733–807

Gelman, A., Carlin, J.C., Stern, H. and Rubin D.B. 1995. Bayesian Data Analysis. Chapman and Hall, New York. 525 pp.

Gelman, A., Meng, X.L. and Stern H.S. 1996. Posterior predictive assessment of model fitness via realized discrepancies. Statistica Sinica, 6: 733-807.Gilks W.R., Richardson, S and Spiegelhalter, D.J. (Editors). 1996. Markov chain Monte Carlo in Practice. Chapman and Hall, London, UK. 512 pp.

Geweke, J. 1992. Evaluating the accuracy of sampling-based approaches to calculating posterior moments. In Bayesian Statistics 4 (ed JM Bernado, JO Berger, AP Dawid and AFM Smith). Clarendon Press, Oxford, UK: 169-194

Heidelberger, P. and Welch, P. 1983. Simulation run length control in the presence of an initial transient. Operations Research, 31: 1109–1144.

Hvingel, C. 2005. Deriving Quantitative Biological Advice for the Shrimp Fishery in Skagerrak and Norwegian Deep (ICES Divisions IVa east and IIIa). *NAFO SCR Doc. 05/84.* Serial No. N5189

Hvingel , C. 2006a. Construction of biological advice for the management of a northern shrimp fishery – the West Greenland example. UITØ, Norges Fiskerihøgskole, Februar 2006.

Hvingel, C. and M.C.S. Kingsley (2006). A framework to model shrimp (Pandalus borealis) stock dynamics and quantify risk associated with alternative management options, using Bayesian methods. *ICES J. Mar. Sci.* ***63****:68–82*.

Hvingel, C. 2010. Shrimp (*Pandalus borealis*) in the Barents Sea – stock assessment and precautionary approach and MSY based management considerations. *NAFO SCR Doc. 10/61*. Serial No. N5852

Raftery, A.L. and Lewis, S. 1992. How many iterations in the Gibbs sampler? *In* Bayesian Statistics 4. *Edited by* J.M. Bernardo, J.O. Berger, A.P. Dawid, and A.F.M. Smith. Oxford University Press, Oxford pp. 763–774.

Schaefer, M.B. 1954. Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. Bulletin of the Inter-American Tropical Tuna Commission, 1: 27–56.

Schnute, J. 1994. A general framework for developing sequential fisheries models. Canadian Journal of Fisheries and Aquatic Sciences, 51: 1676–1688.

Spiegelhalter, D.J., Thomas, A., Best, N., Lunn, D. 2004. WinBUGS User Manual version 1.4 January 2003 MRC Biostatistics Unit, Inst. of Public Health, Cambridge, England.

Table 1. Model input data series: Catch by the fishery; four indices of fishable biomass – two standardized catch rate index series based on fishery data (CPUE) from Denmark (dk) and Norway (nor) respectively, a research survey index discontinued in 2002 (Survey 1) and the current survey started in 2006 (Survey 2).

Year	Catch (ktons)	CPUEdk (index)	CPUEnor (index)	Survey 1 (ktons)	Survey 2 (ktons)
1970	5.6	-	-	-	-
1971	6.6	-	-	-	-
1972	6.0	-	-	-	-
1973	5.2	-	-	-	-
1974	4.3	-	-	-	-
1975	5.2	-	-	-	-
1976	7.1	-	-	-	-
1977	6.1	-	-	-	-
1978	5.5	-	-	-	-
1979	5.9	-	-	-	-
1980	8.4	-	-	-	-
1981	10.0	-	-	-	-
1982	10.3	-	-	-	-
1983	8.3	-	-	-	-
1984	7.6	-	-	17.00	-
1985	12.7	-	-	28.18	-
1986	12.8	-	-	11.01	-
1987	14.2	1.21	-	18.83	-
1988	12.2	0.99	-	6.83	-
1989	11.1	1.05	-	10.64	-
1990	10.2	1.27	-	12.70	-
1991	11.6	1.47	-	18.40	-
1992	13.0	1.39	-	21.34	-
1993	12.6	1.30	-	17.91	-
1994	11.5	1.42	-	18.50	-
1995	13.4	1.59	-	17.59	-
1996	14.1	1.69	-	24.15	-
1997	15.1	2.09	-	32.02	-
1998	15.4	2.06	-	20.19	-
1999	11.3	1.50	-	17.79	-
2000	11.0	1.38	1.20	17.40	-
2001	11.3	1.45	1.26	24.56	-
2002	12.5	1.75	1.56	16.77	-
2003	13.8	1.81	1.60	-	-
2004	15.9	2.34	1.82	-	
2005	14.2	1.55	1.70	-	
2006	14.2	2.10	1.68	-	19.55
2007	13.5	2.29	2.02	-	36.77
2008	13.0	1.73	1.94	-	19.5
2009	11.0	1.32	1.46	-	14.92
2010	7.7	1.00	1.00	-	10.1
2011	8.0	-	-	-	8.62
2012	8.0	-	-	-	6.25

Table 2. Priors used in the model run 2. ~ means "distributed as..", dunif = uniform-, dlnorm = lognormal-, dnorm= normal- and dgamma = gammadistributed. Symbols as in text.

Parameter		Prior	
Name	Symbol	Type	Distribution
Maximal Suatainable Yield	MSY	reference	~dunif(1,100)
Carrying capacity	K	reference	~dunif(1,665)
Catchability survey 1	q_1	informative	~dlnorm(-1.75,11)
Catchability survey 2	q_2	informative	~dlnorm(-1.75,11)
Catchability CPUEdk	$\ln(q_{dk})$	reference	~dunif(-10,1)
Catchability CPUEnor	$\ln(q_{nor})$	reference	~dunif(-10,1)
Initial biomass ratio	P_0	informative	~dlnorm(1.5,25)
Precision survey 1	$1/\sigma_1^2$	low-informative	~dgamma(4,0.1125)
Precision survey 2	$1/\sigma_2^2$	low-informative	~dgamma(4,0.1125)
Precision CPUEdk	$1/\sigma_{dk}^2$	reference	~dgamma(0.1,0.1)
Precision CPUEnor	$1/\sigma_{nor}^2$	reference	~dgamma(0.1,0.1)
Precision model	$1/\sigma_P^2$	reference	~dgamma(0.1,0.1)

Table 3. Model diagnostics: residuals (% of observed value) and probability of getting a more extreme observation (Pr).

	$CPUE_{dk}$		$CPUE_{nor}$		Survey 1		Survey 2	
Year	resid (%)	Pr	resid (%)	Pr	resid (%)	Pr	resid (%)	Pr
1984	-	-	-	-	15.00	0.27	-	-
1985	-	-	-	-	-25.39	0.82	-	-
1986	-	-	-	-	27.60	0.15	-	-
1987	1.92	0.47	-	-	-24.21	0.84	-	-
1988	-8.55	0.67	-	-	46.62	0.03	-	-
1989	-3.98	0.58	-	-	12.76	0.31	-	-
1990	-6.16	0.62	-	-	12.41	0.31	-	-
1991	-2.01	0.54	-	-	-6.06	0.60	-	-
1992	6.75	0.36	-	-	-18.04	0.77	-	-
1993	7.68	0.34	-	-	-6.34	0.60	-	-
1994	3.63	0.42	-	-	-4.85	0.58	-	-
1995	-0.81	0.51	-	-	7.29	0.39	-	-
1996	6.12	0.37	-	-	-11.66	0.69	-	-
1997	1.14	0.48	-	-	-23.24	0.83	-	-
1998	-9.65	0.69	-	-	10.00	0.34	-	-
1999	1.53	0.47	-	-	2.66	0.46	-	-
2000	2.34	0.45	8.67	0.35	-2.63	0.55	-	-
2001	6.06	0.37	12.98	0.29	-28.03	0.88	-	-
2002	-6.03	0.63	-2.40	0.54	16.28	0.25	-	-
2003	-1.16	0.52	4.02	0.43	-	-	-	-
2004	-13.46	0.76	3.92	0.44	-	-	-	-
2005	11.28	0.28	-5.56	0.59	-	-	-	-
2006	-7.38	0.66	7.58	0.37	-	-	-0.35	0.50
2007	-7.61	0.66	-2.56	0.55	-	-	-0.20	0.51
2008	7.08	0.35	-11.91	0.71	-	-	-5.12	0.61
2009	7.62	0.34	-10.09	0.68	-	-	-4.95	0.60
2010	3.88	0.42	-3.58	0.56	-	-	2.61	0.44
2011	-	-	-	-	-	-	0.62	0.49
2012	-	-	-	-	-	-	13.06	0.28

Table 4. **S**ummary of parameter estimates: mean, standard deviation (sd) and 25, 50, and 75 percentiles of the posterior distribution of selected parameters (symbols are as in the text).

	Mean	sd	25 %	Median	75 %
MSY (ktons)	14	6	9	13	17
K (ktons)	161	49	128	151	182
r	0.38	0.20	0.22	0.35	0.50
q_1	0.17	0.04	0.15	0.17	0.20
q_2	0.14	0.03	0.12	0.14	0.16
q_{dk}	1.4E-02	3.1E-03	1.2E-02	1.4E-02	1.6E-02
q_{dk}	1.3E-02	3.0E-03	1.1E-02	1.3E-02	1.5E-02
P_0	1.50	0.20	1.36	1.50	1.63
P_{2011}	0.68	0.18	0.55	0.67	0.79
σ_1	0.22	0.04	0.19	0.21	0.24
σ_2	0.16	0.04	0.14	0.16	0.18
σ_{dk}	0.16	0.03	0.14	0.15	0.17
σ_{nor}	0.20	0.05	0.16	0.19	0.23
σ_P	0.24	0.04	0.21	0.23	0.26

Table 5. Stock status and short term predictions: stock status at the end of 2011 and predicted to the end of 2012.

Status	2011	2012*
Risk of falling below B_{lim} ($0.3B_{MSY}$)	1.0 %	3.8 %
Risk of falling below $Btrig$ ($0.5B_{MSY}$)	16.4 %	21.4 %
Risk of falling below B_{MSY}	95.3 %	80.1 %
Risk of exceeding F_{MSY}	44.9 %	42.5 %
Stock size (B/Bmsy), median	0.67	0.72
Fishing mortality (F/Fmsy), median	0.92	0.87
Productivity (% of MSY)	89 %	92 %

*Predicted catch = 8 ktons

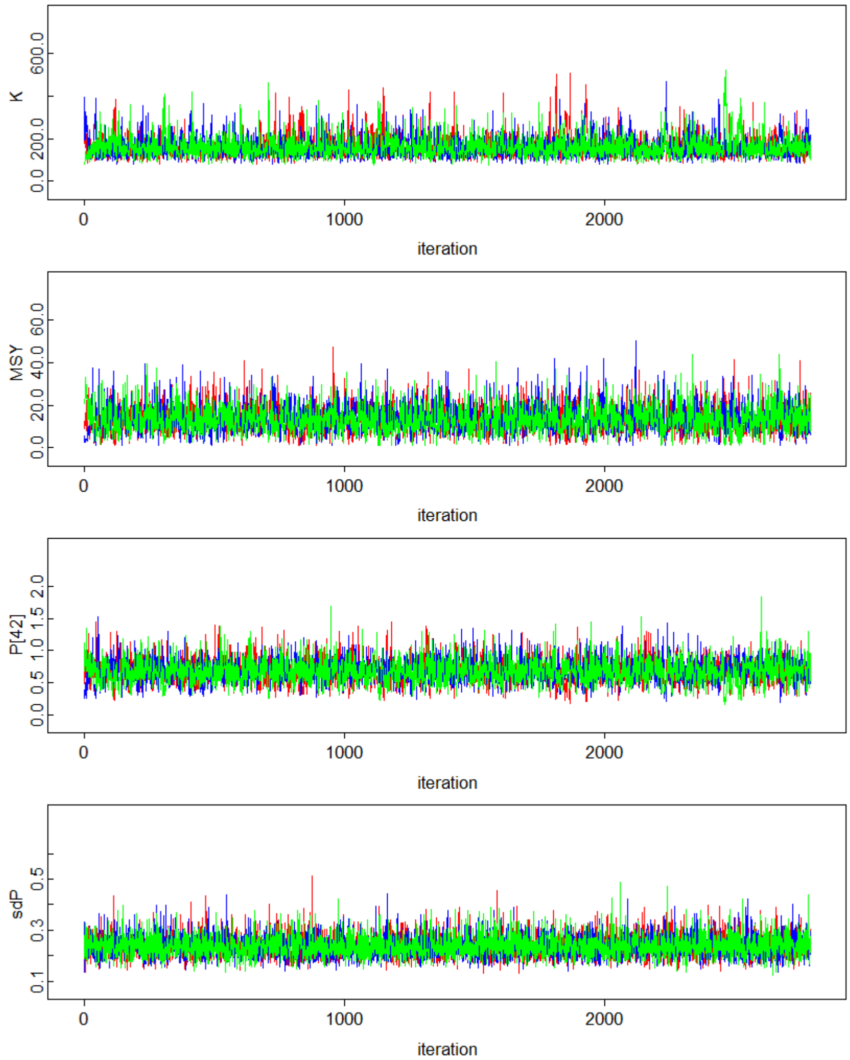

Fig. 1. Autocorrelation function of values sampled for four selected variables out to lag 50. K is the carrying capacity, MSY is maximum sustainable yield, P[42] is the relative biomass in year 2011and sdP is standard error of P i.e. the process error.

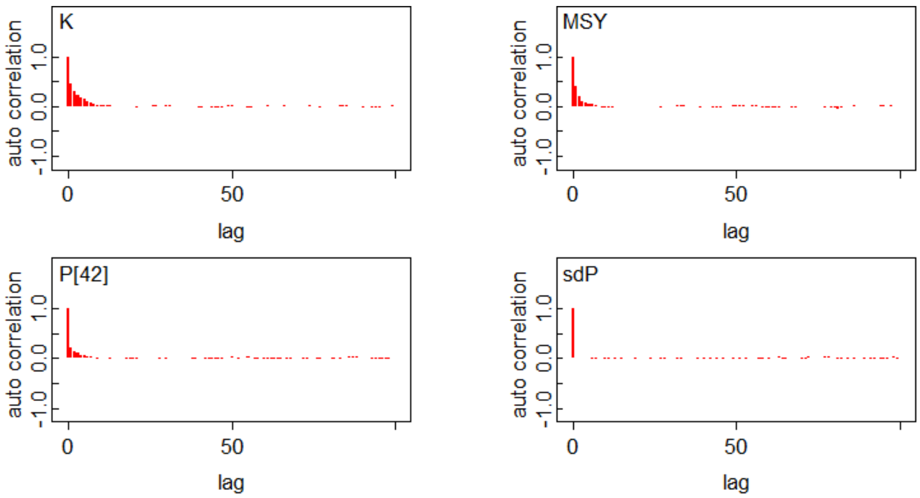

Fig. 3. Three traces (red, green, blue) with different initial values of dour selected variables. K is the carrying capacity, P[42] is the relative biomass in year 2011, MSY is maximum sustainable yield and sdP is the process error.

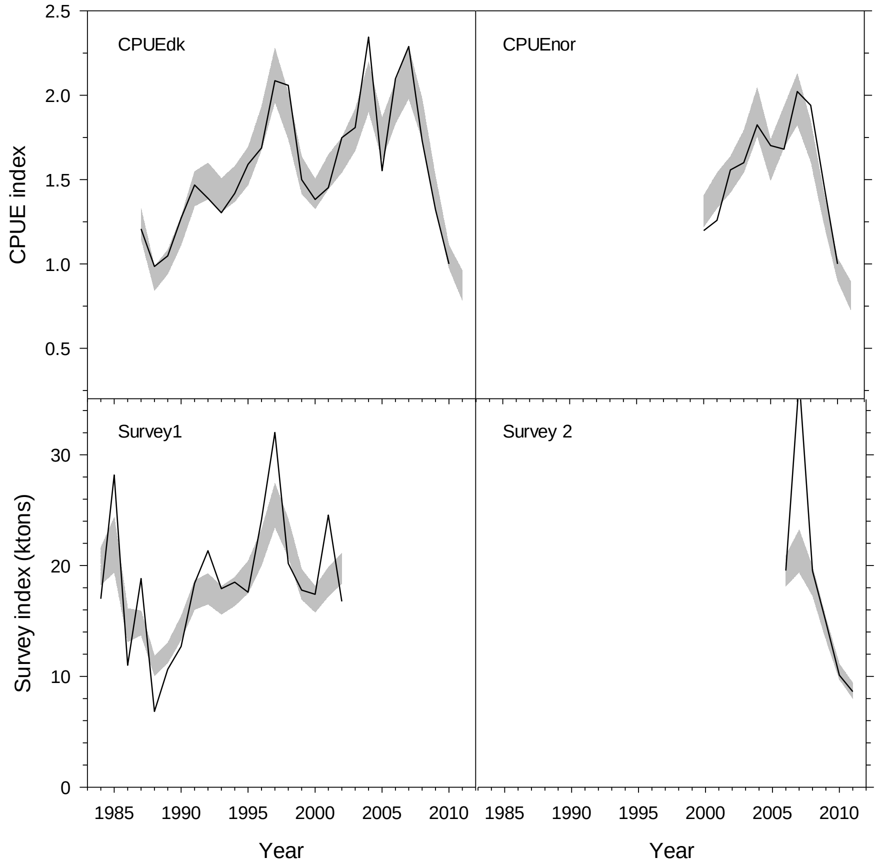

Fig. 4. Observed (solid line) and estimated (shaded) series of the biomass indices. Gray shaded areas are inter-quartile range of the posteriors.

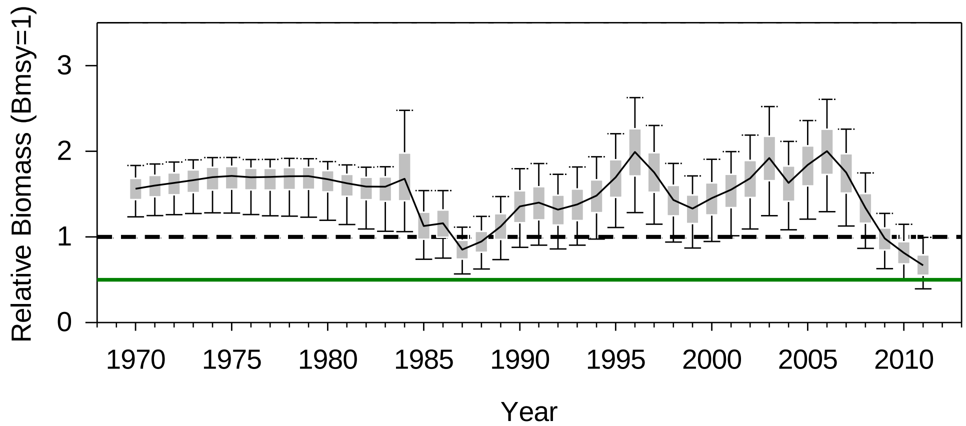

Fig. 5. Estimated time series of relative biomass (B_t/B_{msy}) 1970-2011. Boxes represent inter-quartile ranges and the solid black line running through the (approximate) centre of each box is the median; the arms of each box extend to cover the central 90 % of the distribution.

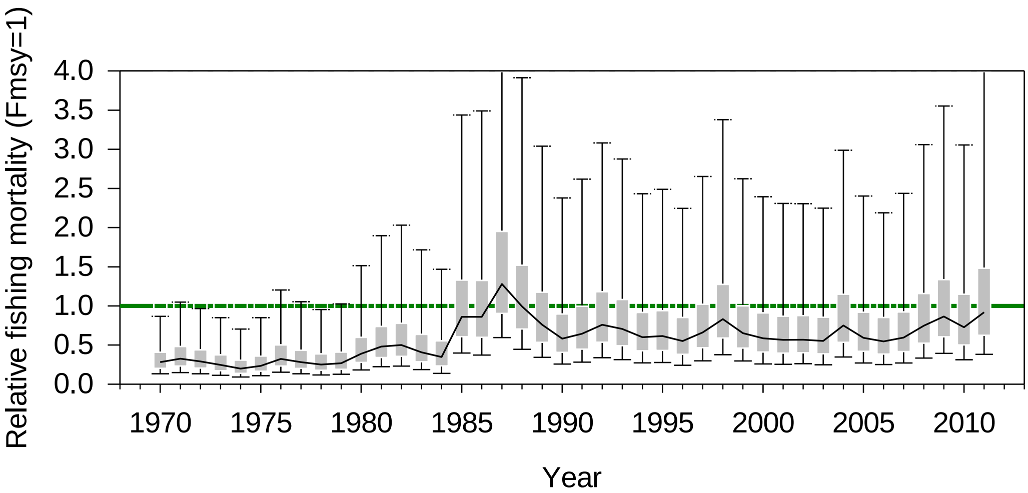

Fig. 6. Estimated time series of relative fishing mortality (F_t/F_{msy}) 1970-2011. Boxes represent inter-quartile ranges and the solid black line running through the (approximate) centre of each box is the median; the arms of each box extend to cover the central 90 % of the distribution.

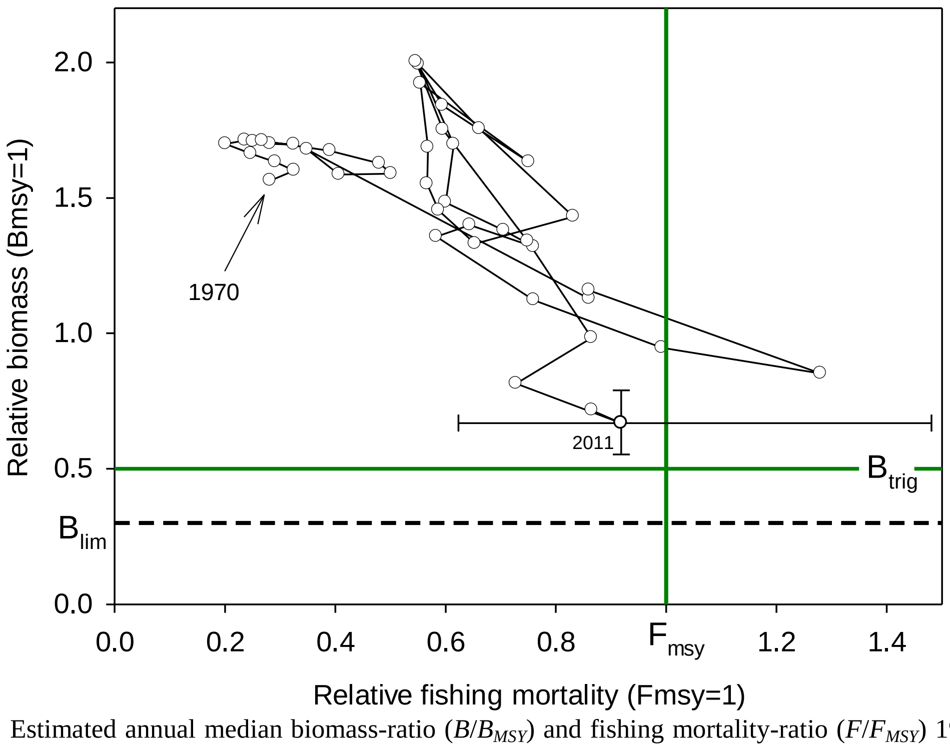

Fig. 7. Estimated annual median biomass-ratio (B/B_{MSY}) and fishing mortality-ratio (F/F_{MSY}) 1970-2011. The reference points for stock biomas, $B_{trigger}$, and fishing mortality, F_{msy}, are indicated by green lines, B_{lim}, by green a dotted line. Error bars on the 2011 value are inter-quartile range