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An assessment of the North Sea shrimp stock using a Bayesian surplus production model

by

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Abstract

A Bayesian surplus production modelling framework was applied in order to provide a quantitative assessment of the North Sea shrimp stock. There were no major problems in model diagnostics. The results indicated that the stock declined since 2006 and is now below Bmsy but still above Blim. The estimated risk of stock biomass being below $B_{trigger}$ in 2013 was 21% and there is a 47% risk of the F being above Fmsy. The posterior for *MSY* was positively skewed with a median at 14 ktons and upper and lower quartiles at 11 ktons and 17 ktons.

Background

The purpose of the study was to investigate the suitability of a biomass dynamic model for the assessment of the North Sea/Skagerrak shrimp stock and to provide a basis for management advice. The model was similar in structure to the one currently used in the assessment of the Barents Sea- and the West Greenland shrimp stocks (Hvingel 2011, Hvingel and Kingsley 2006) and the Greenland halibut stock off East Greenland (Hvingel et al. 2008). It is based on the general assumption that the production curve of the stock is dome shaped, i.e. population growth is logistic.

Predation, although an important source of mortality for shrimp (Hvingel 2006 and references therein), was not included as an explicit variable because the available composite predator abundance indices varied little over time and was found not to hold any information regarding shrimp stock dynamics (Hvingel 2005).

A similar investigation was done in 2005 (Hvingel 2005) and in essence concluded that the available data series of biomass indices and catch held little information regarding model parameters. For the current study the data series has been extended both back- and forward in time and included in the analysis. Informative priors for carrying capacity and survey catchability have also been added

Model

The model was built in a state-space framework (Hvingel and Kingsley 2006, Schnute 1994) with a set of parameters (θ) defining the dynamics of the shrimp stock. The posterior distribution for the parameters of the model, $p(\theta|data)$, given a joint prior distribution, $p(\theta)$, and the likelihood of the data, $p(data|\theta)$, was determined using Bayes' (1763) theorem:

(1) $$p(\theta \mid data) \propto p(data \mid \theta)\, p(\theta)$$

The posterior was derived by Monte-Carlo-Markov-Chain (MCMC) sampling methods using OpenBUGS v.3.2.2 (www.openbugs.info/w.cgi/FrontPage; Spiegelhalter et al. 2003; Lunn et al 2009).

The equation describing the state transition from time t to t+1 was a discrete form of the logistic model of population growth including fishing mortality (e.g. Schaefer (1954), and parameterised in terms of *MSY* (Maximum Sustainable Yield) rather than *r* (intrinsic growth rate) (cf. Fletcher 1978):

(2) $$B_{t+1} = B_t - C_t + 4MSY\frac{B_t}{K}\left(1-\frac{B_t}{K}\right)$$

K is the carrying capacity, or the equilibrium stock size in the absence of fishing. B_t is the stock biomass. C_t is the catch taken by the fishery.

To cancel out the uncertainty of the "catchability" (the parameter that scales biomass indices to real biomass) equation (2) was divided throughout by B_{MSY}, (Hvingel and Kingsley 2006). Finally a term for the process error was applied and the state equation took the form:

(3) $$P_{t+1} = \left(P_t - \frac{2C_t}{K} + \frac{4MSY\ P_t}{K}\left(1-\frac{P_t}{2}\right)\right)\cdot \exp(\nu_t)$$

where P_t is the stock biomass relative to biomass at MSY ($P_t=B_t/B_{MSY}$) in year t. This frames the range of stock biomass (*P*) on a relative scale where P_{MSY}=1 and P_K (carrying capacity)=2. The 'process errors', *v*, are normally, independently and identically distributed with mean 0 and variance σ_v^2 .

The model synthesized information from input priors and four independent series of shrimp biomasses and one series of shrimp catches (Table 1). The four series of shrimp biomass indices were: a standardised series of annual commercial-vessel catch rates for Danish vessels 1987–2013, $CPUEdk_t$, and Norwegian vessels 2000-2013 $CPUEnor_t$; and two trawl-survey biomass index for 1984–2002, $surv1_t$, and 2006-2013, $surv2_t$. These indices were scaled to true biomass by catchability parameters, q_{dk}, q_{nor}, q_1 and q_2. Lognormal observation errors, ω,η, κ and ε were applied, giving:

(4) $$\begin{aligned} CPUEdk_t &= q_{dk}B_{MSY}P_t\exp(\omega_t) \\ CPUEnor_t &= q_{nor}B_{MSY}P_t\exp(\eta_t) \\ surv1_t &= q_1 B_{MSY}P_t\exp(\kappa_t) \\ surv2_t &= q_2 B_{MSY}P_t\exp(\varepsilon_t) \end{aligned}$$

The error terms, ω, η, κ and ε are normally, independently and identically distributed with mean 0 and variance σ_ω^2, σ_η^2 σ_κ^2 and σ_ε^2 . Total reported catch 1970-2012 and the TAC for 2013 was used as yield data (Table 1) and entered into the model as error-free.

Run 1.
A base run with uninformative priors on all parameters was able to reproduce the point estimates of the input data, however with wide confidence limits. The priors of model parameters got somewhat updated, but their posteriors had very low precision. This indicated noisy data and/or that the information contained in the data with respect to some of the model parameters was relatively low. More information could be added to the model through the priors and this was subsequently done in a second run:

Run 2.
Low-information priors (reference priors) were given to *MSY*, the process error, σ_v, and the observation error for the two CPUE biomass index series, σ_ω and σ_η, as there was little or no information on what their probability distributions might look like (Table 2). *MSY* was given a generously wide uniform prior between 0 and 100 kt. A prior for *K* was constructed based on the estimated K for the Barents Sea stock (Hvingel 2012) and the relative size of the two survey areas: 57,000 km^2 in the North Sea and 1500,000 km^2 in the Barents Sea. The posterior estimate of K for the Barents Sea was accordingly scaled down by approximately 1/27 and used as the prior for the North Sea (Table 2). The prior for the stock size in the initial year, P_0, was Norm(1.5, 25) a relatively wide distribution indicating a higher probability of the stock being above than below Bmsy as the fishery at and prior to that time was

comparatively small; in any case, the model showed little sensitivity to the setting of this prior: only the estimated biomass trajectory of the first ca. 10 years (1970-1980) would differ, after that they would converge.

The prior distributions for the error terms associated with the survey biomass indices were assigned inverse gamma distributions with a mode at 0.2, comparable to the CVs typically found in such surveys. Berenboim et al. (1980) estimated a catchability of 0.173 by calibrating trawl catches to the results of a photo survey. This was chosen as basis for an informative prior by giving q a lognormal distribution with a median of 0.173 and a variance of 0.3.

Convergence diagnostics

In order to check whether the sampler had converged to the target distribution a number of parallel chains with different starting points and random number seeds were analysed by the Brooks, Gelman and Rubin convergence diagnostic (Gelman and Rubin 1992; Brooks and Gelman 1998) A stationarity test (Heidelberger and Welch 1983) was applied to individual chains. If evidence of non-stationarity is found iterations were discarded from the beginning of the chain until the remaining chain passed the test. Raftery and Lewis's (1992) tests for convergence to the stationary distribution and estimation of the run-lengths needed to accurately estimate quantiles were used, and finally the Geweke convergence diagnostic was applied (Geweke 1992). A visualisation of the converged chains can be seen in Fig. 1.

Model check

In order to check whether the model was a 'good' fit to the data, different goodness-of-fit statistics were computed. Firstly, I calculated the simple difference between each observed data point and its trial value in each MCMC sampling step. The summary statistics of the distributions of these residuals indicated by their central tendency whether the modelled values were biased with respect to the observations.

Secondly, the overall posterior distribution was investigated for potential effects of model deficiencies by comparing each data point with its posterior predictive distribution (Posterior Predictive Checks; Gelman et al. 1995, 1996). If the model fitted the observed data well, the observed data and the replicate data should look alike. The degree of similarity between the original and the replicate data points was summarised in a vector of p-values, calculated as the proportion of n simulations in which a sampling of the posterior distribution for an observed parameter exceeded its input value:

$$p.value = \frac{1}{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{N}} I((data_{\mathrm{j}}^{rep}, \theta_{\mathrm{j}}) - (data^{obs}, \theta_{\mathrm{j}})) \quad ,$$

where $I(x)$ is 1 if x is true, 0 if x is false. Values close to 0 or 1 in the vector *p-value* would indicate that the observed data point was an unlikely drawing from its posterior distribution.

Results, model performance

The sampler was therefore set to do 10 million iterations. Only each 1000$^{\text{th}}$ value of the sampled chains for the model parameters was stored and used for further analyses in order to remove within chain autocorrelation (Fig. 1). After 50 stored iterations the sampler had converged to the target distribution (Fig. 2) leaving 9950 samples for each parameter for the final analysis.

Model process error standardised to the estimated relative biomass (P_i) was variable with maximum values around 20% (Fig. 4) and a serial correlation of 0.24. This indicated that there are factors other than those included in the model that affects the dynamics of the stock. These effect are, however, relatively small with a low correlation and would have a minor effect on model predictions.

In the Bayesian framework fundamental absence of information in the data will yield posteriors as a copy of the input priors. For the data to carry information on all the parameters of any such model the biomass should vary widely both above and below B_{MSY} (Hvingel and Kingsley 2006). If the available data does not span these conditions, problems in fitting stock-production models by any method can be expected (Hilborn and Walters 1992). The available time series of indexed stock biomass does not span the range from 0 to K (Fig. 5). Even though the conditions for estimation of some parameters are not optimal it may still be possible to get good estimates of parameters relevant for management. Fortunately MSY is the easiest single parameter to estimate. If the range of biomass includes B_{MSY}, good estimates of *MSY* can be obtained independently of other parameters. K is notoriously difficult to estimate from data alone.

The model was able to produce a reasonable simulation of the observed data (Fig. 3). The probabilities of getting more extreme observations than the realised ones given in the data series on stock size were in the range of 0.02 to 0.98 – few observations was found to lie in the extreme tails of their posterior distributions (Table 4) i.e. the 1988 Survey1 data point and the 2007 point for Survey2. The CPUE series was generally better estimated than the survey series. Otherwise no major problems in capturing the variability of the data were detected.

For the parameters K and P_0 the posterior distributions tended to approximate the input priors. The prior for the "initial" shrimp stock biomass (P_0) was slightly informative giving credit to "low-exploited stock conditions" at the start of the series in 1969. Making this prior low-informative by giving P_0 a uniform prior between 0 and 2 have previously been shown to have little or no effects on the posterior of other parameters in the model – except for the first 9-10 years of P (relative biomass). After this period the series converge.

Assessment results

Reference points are as used for the Barents Sea shrimp stock.

Since the late 1980s the stock has varied with a slightly increasing trend until 2006 when it started to decline (Fig 5+7). The median 2013 level is below Bmsy but above Blim (Table 6). The estimated risk of stock biomass being below $B_{trigger}$ in 2013 was 21% and 7% of being below B_{lim} (Table 6).

The estimated median Fishing mortality has remained close to Fmsy in recent years (Fig. 6). In 2013 there is a 47% risk of the F being above Fmsy (Table 6).

The posterior for *MSY* was positiveely skewed with a median at 14 ktons and upper and lower quartiles at 11 ktons and 17 ktons (Table 5).

Concluding comments

The model can reproduce the data, was little serial pattern process errors and can produce reference points, projections and risk analyses to guide management.

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Table 1. Model input data series: Catch by the fishery (Catch 2013=TAC 2013); four indices of fishable biomass – two standardized catch rate index series based on fishery data (CPUE) from Denmark (dk) and Norway (nor) respectively, a research survey index discontinued in 2002 (Survey 1) and the current survey started in 2006 (Survey 2).

Year	Catch (ktons)	CPUEdk (index)	CPUEnor (index)	Survey 1 (ktons)	Survey 2 (ktons)
1970	5.573	-	-	-	-
1971	6.582	-	-	-	-
1972	6.018	-	-	-	-
1973	5.218	-	-	-	-
1974	4.342	-	-	-	-
1975	5.159	-	-	-	-
1976	7.081	-	-	-	-
1977	6.153	-	-	-	-
1978	5.520	-	-	-	-
1979	5.888	-	-	-	-
1980	8.399	-	-	-	-
1981	10.021	-	-	-	-
1982	10.638	-	-	-	-
1983	8.310	-	-	-	-
1984	7.592	-	-	17.60	-
1985	12.619	-	-	25.18	-
1986	12.821	-	-	11.55	-
1987	14.153	1.22	-	18.83	-
1988	12.177	0.99	-	6.83	-
1989	11.249	1.01	-	10.64	-
1990	10.239	1.28	-	12.70	-
1991	11.595	1.42	-	18.40	-
1992	13.081	1.40	-	21.34	-
1993	12.753	1.28	-	17.77	-
1994	11.549	1.40	-	18.50	-
1995	13.361	1.55	-	17.59	-
1996	14.149	1.69	-	24.15	-
1997	15.074	2.06	-	32.02	-
1998	15.504	2.03	-	20.19	-
1999	11.254	1.50	-	17.79	-
2000	11.038	1.42	1.28	17.40	-
2001	11.328	1.45	1.35	24.56	-
2002	12.474	1.76	1.67	24.81	-
2003	13.836	1.79	1.71	-	-
2004	15.952	2.33	1.96	-	
2005	14.207	1.55	1.83	-	
2006	14.268	2.10	1.79	-	19.55
2007	13.552	2.30	2.15	-	37.48
2008	13.554	1.75	2.08	-	19.5
2009	11.542	1.34	1.55	-	14.86
2010	8.333	1.00	1.14	-	10.1
2011	9.049	1.02	1.19	-	8.62
2012	8.834	0.82	1.02	-	6.25
2013	9.500	1.00	1.00	-	7.0

Table 2. Priors used in the model run 2. ~ means "distributed as..", dunif = uniform-, dlnorm = lognormal-, dnorm= normal- and dgamma = gammadistributed. Symbols as in text.

Parameter		Prior	
Name	Symbol	Type	Distribution
Maximal Suatainable Yield	*MSY*	reference	~dunif(1,100)
Carrying capacity	*K*	informative	~dlnorm(4.65,3.16)
Catchability survey 1	q_1	informative	~dlnorm(-1.75,11)
Catchability survey 2	q_2	informative	~dlnorm(-1.75,11)
Catchability CPUEdk	$\ln(q_{dk})$	reference	~dunif(-10,1)
Catchability CPUEnor	$\ln(q_{nor})$	reference	~dunif(-10,1)
Initial biomass ratio	P_0	informative	~dnorm(1.5,25)
Precision survey 1	$1/\sigma_\kappa^2$	low-informative	~dgamma(4,0.1125)
Precision survey 2	$1/\sigma_\varepsilon^2$	low-informative	~dgamma(4,0.1125)
Precision CPUEdk	$1/\sigma_\omega^2$	reference	~dgamma(0.1,0.1)
Precision CPUEnor	$1/\sigma_\eta^2$	reference	~dgamma(0.1,0.1)
Precision model process	$1/\sigma_\nu^2$	reference	~dgamma(0.1,0.1)

Table 3. Model diagnostics: residuals (% of observed value) and probability of getting a more extreme observation (Pr).

	$CPUE_{dk}$		$CPUE_{nor}$		Survey 1		Survey 2	
Year	resid (%)	Pr	resid (%)	Pr	resid (%)	Pr	resid (%)	Pr
1984	-	-	-	-	7.04	0.42	-	-
1985	-	-	-	-	-17.38	0.78	-	-
1986	-	-	-	-	28.03	0.17	-	-
1987	0.57	0.50	-	-	-18.39	0.82	-	-
1988	-10.54	0.73	-	-	62.43	0.02	-	-
1989	-3.58	0.59	-	-	14.66	0.29	-	-
1990	-8.04	0.68	-	-	16.11	0.26	-	-
1991	-0.82	0.53	-	-	-4.11	0.59	-	-
1992	5.59	0.39	-	-	-13.21	0.74	-	-
1993	7.15	0.36	-	-	-3.31	0.56	-	-
1994	2.85	0.44	-	-	-2.50	0.55	-	-
1995	-0.79	0.54	-	-	9.54	0.35	-	-
1996	5.35	0.40	-	-	-7.65	0.64	-	-
1997	1.99	0.47	-	-	-17.79	0.82	-	-
1998	-9.15	0.71	-	-	14.43	0.29	-	-
1999	0.20	0.50	-	-	5.85	0.41	-	-
2000	-0.87	0.53	11.07	0.32	1.37	0.49	-	-
2001	6.70	0.36	15.74	0.26	-21.05	0.86	-	-
2002	0.29	0.50	6.74	0.39	-10.84	0.70	-	-
2003	0.38	0.51	6.09	0.40	-	-	-	-
2004	-11.62	0.76	6.07	0.40	-	-	-	-
2005	12.53	0.27	-3.77	0.58	-	-	-	-
2006	-5.40	0.63	12.05	0.30	-	-	-1.79	0.54
2007	2.14	0.47	10.30	0.34	-	-	-39.45	0.98
2008	7.58	0.35	-8.62	0.66	-	-	-6.73	0.62
2009	7.29	0.36	-6.37	0.63	-	-	-6.55	0.62
2010	8.49	0.33	-3.94	0.58	-	-	3.76	0.45
2011	0.48	0.51	-13.06	0.75	-	-	14.83	0.30
2012	5.84	0.39	-14.11	0.76	-	-	34.11	0.13
2013	-5.99	0.65	-5.10	0.60			29.71	0.17

Table 4. **S**ummary of parameter estimates: mean, standard deviation (sd) and 25, 50, and 75 percentiles of the posterior distribution of selected parameters (symbols are as in the text).

	Mean	sd	25 %	Median	75 %
MSY (ktons)	15	5	11	14	17
K (ktons)	192	93	129	158	221
r	0.35	0.16	0.24	0.33	0.44
q_{dk}	0.02	0.00	0.01	0.02	0.02
q_{nor}	0.02	0.00	0.01	0.02	0.02
q_1	0.20	0.04	0.17	0.20	0.23
q_2	0.16	0.03	0.13	0.15	0.18
P_0	1.48	0.20	1.35	1.48	1.61
P_{2013}	0.72	0.23	0.57	0.75	0.88
σ_{dk}	0.15	0.03	0.13	0.15	0.17
σ_{nor}	0.20	0.05	0.16	0.19	0.22
σ_1	0.20	0.04	0.18	0.20	0.23
σ_2	0.23	0.05	0.19	0.22	0.26
σ_P	0.26	0.06	0.22	0.25	0.29

Table 5. Stock status and short term predictions: *Upper*: stock status for 2012-13. *Lower*: predictions for 2014 given catch options ranging from 6 to 16 ktons.

Status	2012	2013*
Risk of falling below B_{lim} ($0.3B_{MSY}$)	6 %	7 %
Risk of falling below *Btrig* ($0.5B_{MSY}$)	20 %	22 %
Risk of falling below B_{MSY}	91 %	75 %
Risk of exceeding F_{MSY}	43 %	47 %
Stock size (B/Bmsy), median	0.75	0.76
Fishing mortality (F/Fmsy), median	0.93	0.95
Productivity (% of MSY)	94 %	94 %

*Predicted catch = TAC

Catch option 2014 (ktons)	6	8	10	12	14
Risk of falling below B_{lim} ($0.3B_{MSY}$)	6 %	6 %	6 %	7 %	7 %
Risk of falling below *Btrig* ($0.5B_{MSY}$)	18 %	20 %	21 %	22 %	24 %
Risk of falling below B_{MSY}	65 %	67 %	69 %	73 %	75 %
Risk of exceeding F_{MSY}	17 %	31 %	47 %	61 %	72 %
Risk of exceeding $1.7F_{MSY}$	4 %	10 %	18 %	29 %	39 %
Stock size (B/Bmsy), median	0.84	0.82	0.79	0.76	0.73
Fishing mortality (F/Fmsy),	0.54	0.74	0.96	1.19	1.45
Productivity (% of MSY)	97 %	97 %	96 %	94 %	93 %

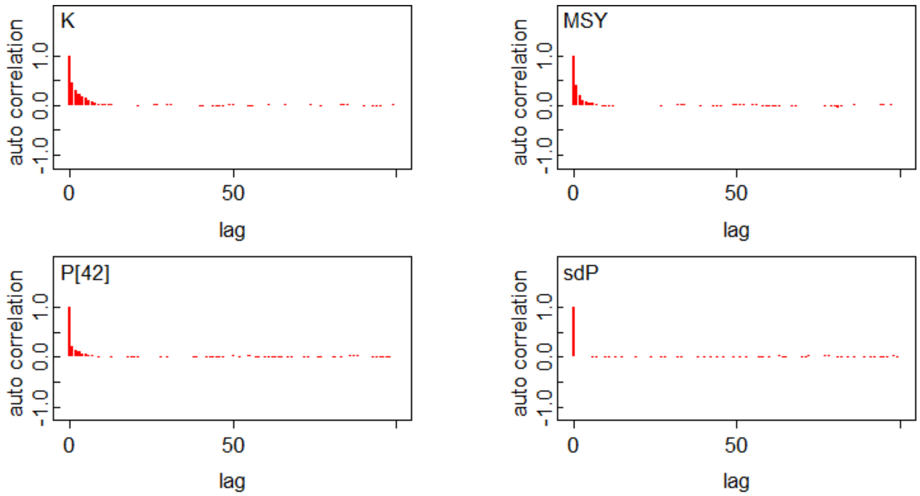

Fig. 1. Autocorrelation function of values sampled for four selected variables out to lag 50. K is the carrying capacity, MSY is maximum sustainable yield, P[42] is the relative biomass in year 2011and sdP is standard error of P i.e. the process error.

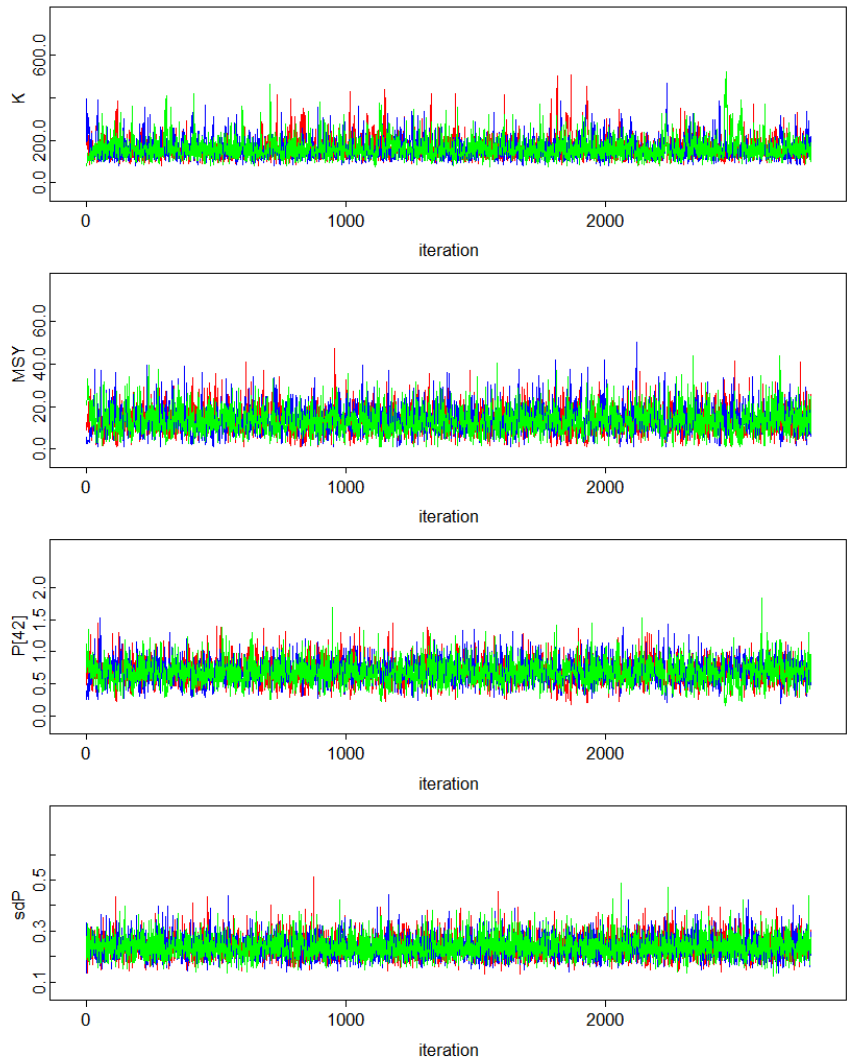

Fig. 2. Three traces (red, green, blue) with different initial values of dour selected variables. K is the carrying capacity, P[42] is the relative biomass in year 2011, MSY is maximum sustainable yield and sdP is the process error.

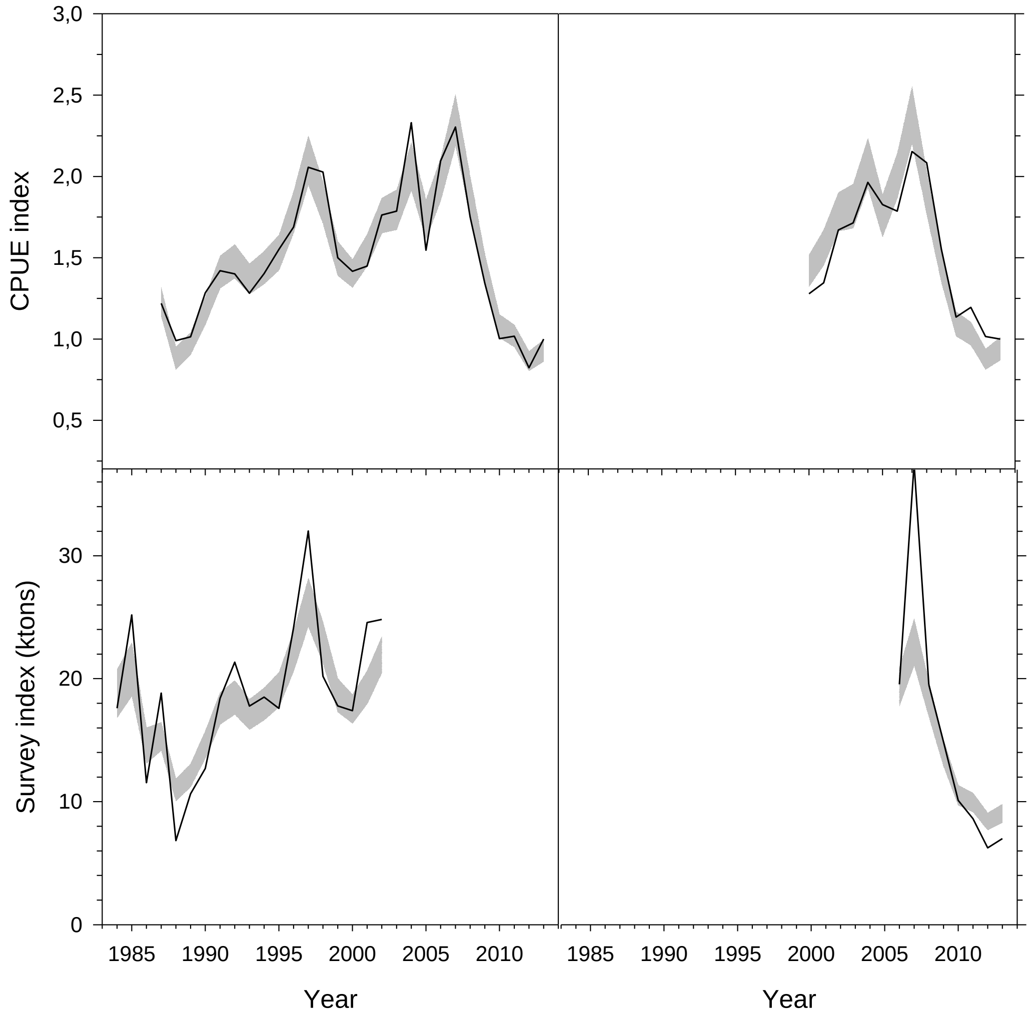

Fig. 3. Observed (solid line) and estimated (shaded) series of the biomass indices. Gray shaded areas are inter-quartile range of the posteriors.

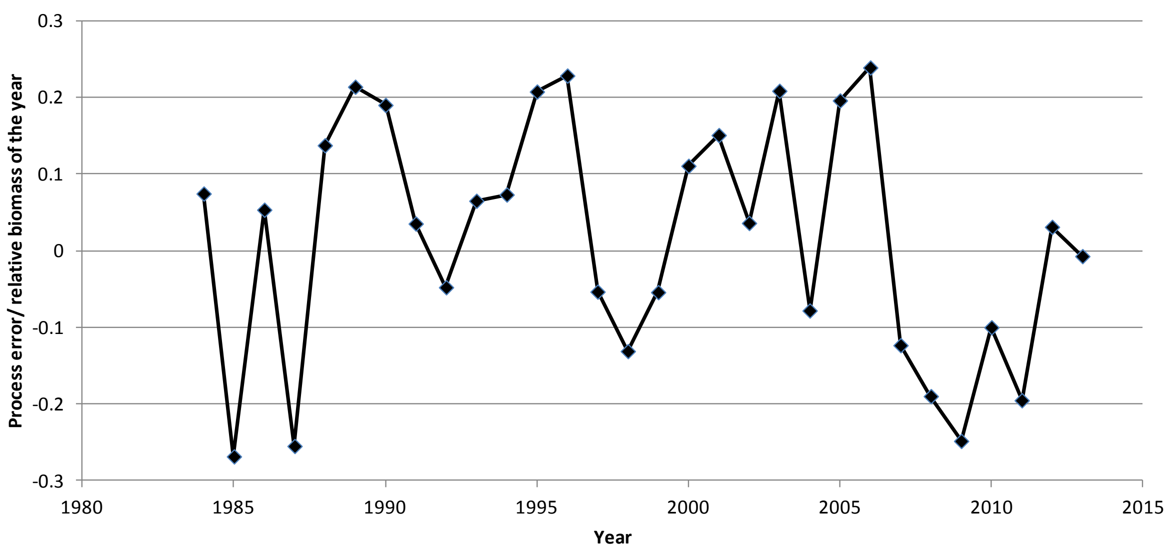

Fig 4- Model process error

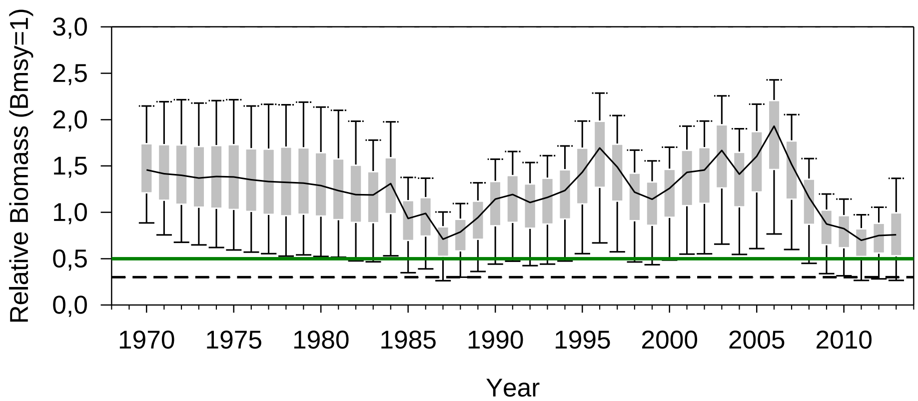

Fig. 5. Estimated time series of relative biomass (B_t/B_{msy}) 1970-2013. The solid black line is the median; boxes represent quartiles; the whiskers cover the central 90 % of the distribution. Dashed black line represents B_{lim}.

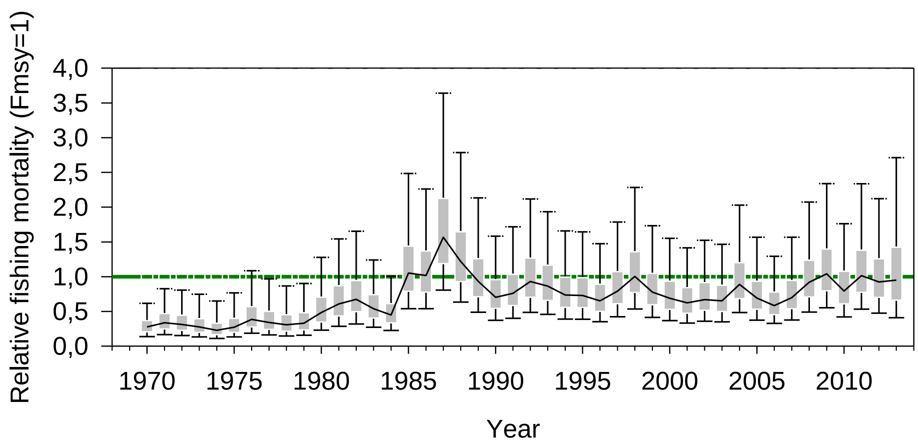

Fig. 6. Estimated time series of relative fishing mortality (Ft/Fmsy) 1970-2013. The solid black line is the median; boxes represent quartiles; the whiskers cover the central 90 % of the distribution. Green line marks F_{msy}

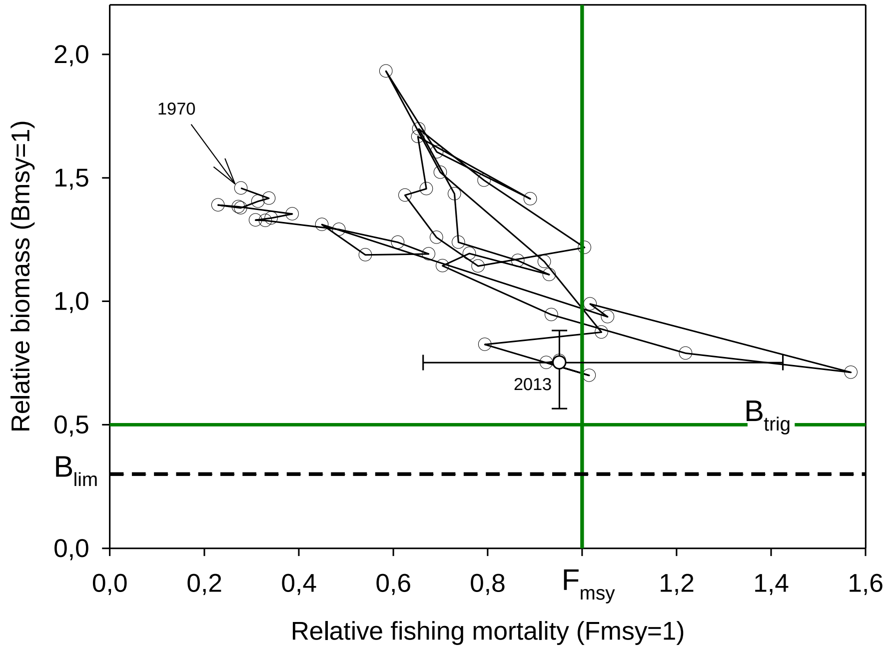

Fig. 7. Estimated annual median biomass-ratio (B/B_{MSY}) and fishing mortality-ratio (F/F_{MSY}) 1970-2013. The reference points for stock biomas, $B_{trigger}$, and fishing mortality, F_{msy}, are indicated by green lines, B_{lim}, by green a dotted line. Error bars on the 2013 value are inter-quartile range