Numbers of Age-2 Shrimps in West Greenland—again.

by

Michael C. S. Kingsley

Pinngortitaleriffik, Greenland Institute of Natural Resources
Box 570, DK-3900 Nuuk, Greenland

Summary

Numbers of shrimps at age 2 years are estimated by decomposing the length distributions, fitting Normally distributed components. The poor fit of a third component above the age-1 and age-2 distributions to what is probably a mixture of age classes often impairs the fit to the age-2 length distribution and consequently also the estimate of age-2 numbers. Fitting the third component only as far as 19 mm, which is the limit of our interest anyway, can in many cases improve the fit and the estimate for the age-2s. This method has been implemented in 2014 in analysing length frequencies in West Greenland.

Introduction

The number of ‘age-2’ shrimps in the West Greenland stock is estimated by decomposing the distribution of the carapace lengths of males and juveniles recorded in the survey catch. This length distribution is typically seen, or expected, to comprise 3 components: ‘age-1’, ‘age-2’ and a third component probably comprising more than one age class, but presumably in decreasing numbers as they progressively change sex. The third component vastly outnumbers the other two.

The length distribution is decomposed by fitting Normally distributed components, usually three in number. Under ideal circumstances the age-1 and age-2 components can be clearly seen as modes in the length distribution, while the third component forms a nicely symmetrical Normally distributed mode of its own. An example of this happy situation is shown in the first Figure below.
Figure 1: Unconstrained fit of three Normal components to length frequency data for juveniles and males in Disko Bay and Vaigat in 2014.

This shows an unconstrained fit\(^1\) of three components to the male data from Disko Bay in 2014. The overall fit is good. The data allows a symmetrical third component to be fitted without any interference with the fit of the age-2 distribution to its evident mode. There are enough age-1s to define a first component clearly.

**Problem**

The age-1 shrimps, even if few, usually compose a distinctly separate fraction, but the age-2 shrimps often appear as merely a bulge on the lower slopes of the left-hand limb of the length distribution of the third component. When this is the case, the length distribution of the age-2s, and consequently their number, is mostly just the remainder of the overall length distribution after fitting the first and third components, so the way in which the third component is fitted, and its resulting fit, become decisive in estimating the number of age-2s.

An example of the kind of problem that can arise is shown below.

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\(^1\) size-distribution curves in this document were fitted by maximum likelihood using Excel Solver, and results might therefore differ from those obtained by other fitting programs.
Figure 2: Unconstrained fit of three Normal components to the offshore length distribution from the 2013 West Greenland survey.

In the data from the offshore survey in 2013 the age-2 component barely shows as a distinct mode, forming only a plateau on the left-hand limb of the third component. When three Normal components are fitted, the overall fit of the three components to the data is good. However, the third component is fitted only to its right-hand limb, and fits less well on its left-hand side. The centre, width and height of the age-2 distribution are adjusted to fill this gap, resulting in values—mean of 14.9 mm and s.d. of 2.2, giving contributions up to 20 mm, and a total number of 5.14 bn—that are unrealistic. However, it is precisely because the age-2s are available to fill the gap in this way that the third component can get away with being fitted only on its right-hand side.

Another example, from the offshore survey in 2014, is similar:

Figure 3: Unconstrained fit of three Normal components to length frequency data for juveniles and males in the offshore survey in West Greenland in 2014.

Again, the third component fits on its right-hand side, and the age-2 size distribution is adjusted to fill the gap on its left-hand side, even though in this case there appears to be a distinct mode of age-2 shrimps with a modal CPL at 12½ mm. The left-hand limb of the third component fits very badly, so the age-2 distribution is very unrealistic, with contributions even over 23 mm. The overall fit to the data is also poor.
This problem arises regardless of the method used to fit the distributions, due as it is to the fitting of complete Normal distributions to the entire size range.

Solution

The standard approach to dealing with these difficulties is to fit more than one—two or several—components in the ‘third component’. This is a way to separate the fit on its right-hand side from the fit on its left, so that the age-2 component doesn’t have to be called in to palliate a misfit on the left-hand side of the third component. Various other measures are also available or have been tried: constrain the average length of the age-2s, constrain the CV of length. The problem with constraints is that they tend to be subjective and not reproducible: one analyst might constrain the solution in a way that might not seem reasonable to another.

The present suggestion is that fitting the entire third component in this way is causing unnecessary problems with fitting the age-2s in their proper part of the length distribution. Fitting two or more components above the age-2s is essentially a way to separate the fitting of the two sides of the large-size mode.

So I question why we should fit both sides of the third component. We have no great interest in it, suspecting as we do that it is a biologically unspecified mixture of age-classes. The fitting of its right-hand limb has a large effect on estimating the number of age-2s, as well as other properties of their size distribution. Therefore, I suggest that we should continue to fit three Normal components, but fit to observed sizes only as far as 19 mm CPL. Thereby, we have no fit of the right-hand limb of the third component to govern the parameters of its size distribution and its left-hand limb becomes free to get out of the way of the age-2s.

Results

The three examples above then look like:

Figure 4: Unconstrained fit of three Normal components up to 19 mm to length frequency data for juveniles and males in Disko Bay and Vaigat in 2014.
In the first case, which fitted nicely anyway, the fit is not much changed; the estimated number of age-2s decreases from 3.71 bn to 3.51 bn. In the second case, the problem is not resolved. The mean length estimated for the age-2s decreases only from 14.9 mm to 14.6 and their SD from 2.20 to 2.14, and they are still contributing to the overall length distribution up to 19-20 mm.

In the third case, the problem has been largely resolved. The mean length of age-2s is now 12.5 mm instead of 15.9 and the SD 1.05 instead of 3.18. They are contributing only up to 15 mm instead of 23. However, it appears that the wide distribution of the third component is pushing the age-2s out of the way to the left and spoiling their fit to their apparent mode.

This appears to be a partial solution that works in some cases but not all. A possible further fix would be to constrain the CVs of length to be the same for all components. The results then look like:
For Disko in 2014, the changes are largely driven by the well defined CV for the age-1 shrimps, and the other two distributions change slightly to fit that. Principally, the SD for the partially fitted third component increases slightly and pushes the age-2 distribution to the left; the mean, SD and height of the age-2 distribution all decrease and the estimated number decreases to 3.3 bn.

The second case is still not happy; but it is not obvious, either, what constraints should be applied to get a better fit, seeing that the overall fit to the data is already good.
In the third case, the fit looks more satisfactory. The previously excessive SD of the partially fitted third component has been lowered to be consistent with the other two, so its left-hand tail no longer pushes the age-2s out of the way, allowing them to fit better to their apparent mode. Their mean and SD increase and the estimated number increases from 1.06 bn to 1.38 bn.

Conclusions

It is not at all necessary to fit complete Normal distributions to all the lengths, and fitting a partial distribution above the age-2s solves fitting problems in some cases, but not all. This method has been implemented in Excel Solver and applied in 2014 to calculating, and for some earlier years recalculating, numbers of shrimps at age 2 in the West Greenland stock.