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Surplus production models in a Bayesian framework applied to witch flounder in NAFO Div. 3NO

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Abstract

A series of formulations of a surplus production model in a Bayesian framework were applied to data for witch flounder in NAFO Div. 3NO. The data series included catch from 1960-2012 and several Canadian survey series. The sensitivity of model results to the choice of priors was explored. Model formulations were compared to determine the formulation that best described the data. We suggest a model formulation for the use in the assessment of the status of this stock.

Key words: Bayesian surplus production model, Div. 3NO witch flounder, assessment

Introduction

In the past several years Scientific Council has had a number of requests from the Fisheries Commission related to limit reference points and the potential reopening of a directed witch flounder fishery in Div. 3NO. In 2014 proxy reference points were established based on the Canadian spring survey series.

The stock has had a long history of exploitation, but has been under moratorium since 1994. The maximum catch was 15 000 t in 1971. However the average catch in the pre moratorium period was 5 500 t, and catch was rarely above this level.

The current status of the stock is not well known. The Canadian spring research vessel survey series is the longest continuous series. While this index has increased substantially from the lows of the late 1990s (by about 2 times), it is less than half of the values of the late 1980s (adjusted for survey coverage, see below).

A number of other potential indices exist for this stock. The use of these indices in a surplus production model in a Bayesian framework was explored first explored in 2014. The present work is based on this initial exploration. Several model formulations are explored and the sensitivity of model results to the choice of priors was explored. The purpose was to propose a model for the assessment of the stock.

Methods

The Schaefer (1954) form of a surplus production model used here is:

$$P_t=[P_{t-1}+ r\bullet P_{t-1} (1 - P_{t-1})- C_{t-1}/K]\bullet\eta_t$$

where Pt-1 and Ct-1 denote exploitable biomass (as a proportion of carrying capacity) and catch, respectively, for year t-1 (Meyer and Millar, 1999a, 1999b). Carrying capacity, K, is the level of stock biomass at equilibrium prior to commencement of a fishery, r is the intrinsic rate of population growth, and ηt is a random variable describing stochasticity in the population dynamics (process error). The model utilizes biomass proportional to an estimate of K in order to aid mixing of the Markov Chain Monte Carlo (MCMC) samples and to help minimize autocorrelation between each state and K (Meyer and Millar, 1999a, 1999b).

An observation equation is used to relate the unobserved biomass, Pt, to the research vessel survey indices:

$$I_t = q \bullet P_t \bullet \varepsilon_t$$

where q is the catchability parameter, Pt is an estimate of the biomass proportional to K at time t, and εt is observation error.

A number of model formulations tested in 2014 indicated that some priors needed to be at least partially informative. The results also indicated that estimation of q (and/or error on q) was particularly difficult. Finally the results indicated that the best indices to explore were the Canadian spring and fall series. These results formed the starting point for the formulations explored here.

Data explored were as follows: (TABLE 1)

(1) Landings – 1960-2012

(2) Canadian RV spring Survey Indices: early series with shallow coverage converted to Campelen– 1984-1990

(3) Canadian RV spring Survey Indices: early series adjusted for lack of deep coverage converted to Campelen– 1984-1990 and combined with the Canadian RV spring from 1991-2012 to make one spring series.

(4) Canadian RV spring Survey Indices: latter part of series in Campelen or equivalents– 1991-2012

(5) Canadian RV fall survey indices: Campelen Trawl or equivalent – 1990-2012

Starting point

The starting point for further exploration had priors on q and the observation error taken from the gamma distribution. The data were the Canadian spring series divided into early and late but not adjusted for lack of depth coverage (series 2 and 4) and the Canadian fall survey (series 5).

RUN 1

r ~ dlnorm(-5.3,0.167)I(0,1)

K~dlnorm(4.49,4.48)I(0,3000)

pq~dgamma(1,1)
q<-1/pq
(prior on q the same for all survey series)

sigma ~ dunif(0,10) (process error)

tau~dgamma(1,1)
itau2 <- 1/tau
(prior on observation error the same for all survey series)

Since previous testing had indicated that estimation of q and/or observation error was one of the difficulties in modelling these data, exploration of priors on q and observation error formed a large portion of the modelling work.

The next model formulation that was explored had the same priors as RUN 1 except that the error on q and observation error was taken from a uniform distribution.

RUN 2

pq~dunif(-10,0)

tau~dunif(0,10)

RUN 3 was the same as RUN 2 except that instead of estimating observation error the coefficient of variation of the surveys was used and the observation equations reformulated as:

Indexmean[t] <- log(max(1.E-6,q *K * P[t]))

Index[t] ~ dlnorm(Imean[t], precision index[t])

Precision [t]<-1/(CVindex[t]*CVindex[t])

RUN 4

This was the same as run 3 but with new r, informed by prior derived by Swain 2012 for witch flounder in the southern Gulf of St. Lawrence. We allowed for a higher r than derived by Swain (2012) as some of the morphometric methods indicated a higher r. Therefore we used the mean (0.17) derived by Swain (2012) as the central tendency (i.e. the median) but with a larger standard deviation.

A mean of 0.2 and standard deviation of 0.12 gives a median of 0.17 on the log normal scale. The prior used therefore was:

R~(-1.763,3.252)

RUN 5

This run explored the effect of the prior on K. It was the same as RUN 4 but used a K based on Ecosystem Production Potential modelling (NAFO 2014). This modelling indicated that a reasonable distribution for K would have a mean of 100 and a standard deviation of 30.

K~dlnorm(4.562,11.6)

RUN 6

This formulation explored the effect of splitting the spring series. It was the same as run 5 but using one spring series instead of a split at 1991 with the pre 1991 adjusted for lack of deep water coverage. That is it used survey series 3 and 5.

RUN 7

We continued our exploration of the effect of priors on q by estimating observation error. This was the same as run 6 but with observation error estimated with the prior being uniform.

tau~dunif(0,10)

RUN 8

Several runs explored gamma priors on q and observation error. RUN 8 was the same as RUN 7 but with gamma prior on tau and q

pq ~dgamma(1,1)
q <-1/pq

tau ~dgamma(1,1)
itau2 <- 1/tau

RUN 9

This formulation combined uniform priors on q with gamma on the observation error, all else being the same as RUN 8

pq ~dunif(-10,0)
q <-exp(pq)

tau ~dgamma(1,1)
itau2 <- 1/tau

RUN 10

Gamma priors on both observation error and q as in RUN 8 but with the spring series split i.e. survey series 2, 4 and 5.

RUN 11

This formulation explored less informative gamma priors on q and observation error. Otherwise it was the same as RUN 10.

```
pq~dgamma(0.1,0.1)
q<-1/pq

tau~dgamma(1,1)
itau2 <- 1/tau
```

Results and Discussion

The main parameter estimates from all of the runs, along with the Deviance Information Criteria, are given in Table 2. Most of the formulations (except those where the CV of the surveys was used instead of estimating observation error) estimated a similar productivity for the stock. The formulations using CV of the surveys as the observation error followed survey trends almost exactly and had extremely high levels of relative biomass, particularly in the 1980s. They also had much higher estimates of population productivity than other formulations. These formulations were not explored further.

Other formulations generally gave similar levels of r and MSY but differed in their estimate of the current level of biomass. We compared the different formulations using trends and magnitude of process error in an attempt to identify formulations with better model fit. Formulations with uniform priors on q and/or on observation error, had larger process error with more trend in the process error (Figure 1). The adjustment used for depth coverage in the combined spring index is ad hoc and so we consider it better to estimate q separately for the two parts of the time series if possible. We therefore eliminated RUN 9 from further consideration. RUN 1 has a bimodality in the posterior density of r near the lower end of the distribution, probably as the result of an inappropriate prior on r.

We therefore explored the results and convergence diagnostics of RUN 10 and RUN 11.

Diagnostics RUN 10

Priors for r, K, pq (inverse of q) and tau were all updated from the prior, although the posterior for tau is concentrated to the left of the prior (Figures 2-4). Convergence diagnostics did not indicate any issue with convergence (Appendix I)

Diagnostics RUN 11

Priors for r, K, pq (inverse of q) and tau were all updated from the prior (Figures 5-7). However, the posterior for r is closer to the prior than it is in RUN 10. The posteriors for pq are very flat for RUN 11, indicating that they are not well estimated. The posterior for tau is concentrated to the left of the prior, as it was for RUN 10 as well. Convergence diagnostics for RUN 11 indicated a failure to converge in that the Geweke Convergence Diagnostic which measures the convergence of the mean (Ntzoufras, 2009) was significant or near significance for one or more chains for K, r, q for fall, q for spring late and q for spring early (Appendix II).

A comparison of process error for RUN 10 and RUN 11 shows that process error and pattern in this error is larger for RUN 11 than for RUN 10.

As a result of these comparisons RUN 10 was considered to be the ‘preferred’ formulation.

Acknowledgments

The Bayesian surplus production models are based on programs originally developed by Jason Bailey.

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Table 1. Data used in the Bayesian Surplus Production models. Values are in thousands of tons except for CV which is the coefficient of variation of the corresponding survey series.

Year	Landings	spring late	fall	spring early	CVearly	CVlate	CVfall	spring adjusted
1960	5.799							
1961	4.627							
1962	1.228							
1963	2.183							
1964	1.066							
1965	2.177							
1966	7.522							
1967	11.503							
1968	10.599							
1969	4.7							
1970	6.763							
1971	14.965							
1972	9.177							
1973	6.691							
1974	8.045							
1975	6.168							
1976	6.035							
1977	5.759							
1978	3.473							
1979	3.077							
1980	2.42							
1981	2.425							
1982	3.732							
1983	3.616							
1984	2.802			14.313	0.101			17.823
1985	8.771			24.581	0.196			30.608
1986	9.131			9.214	0.204			11.473
1987	7.596			11.199	0.161			13.946
1988	7.325			24.655	0.22			30.701
1989	3.688			8.988	0.2			11.192
1990	4.179		15.368	10.759	0.401		0.112	13.397
1991	4.847	7.07	5.477			0.231	0.151	7.07
1992	4.96	8.217	9.118			0.196	0.175	8.217
1993	4.414	4.226	9.474			0.167	0.24	4.226
1994	1.119	16.279	7.821			0.37	0.229	16.279
1995	0.3	4.057	11.743			0.18	0.288	4.057
1996	0.358	4.085	12.1			0.151	0.493	4.085
1997	0.512	7.133	5.638			0.566	0.184	7.133

1998	0.612	2.688	6.894	0.155	0.196	2.688
1999	0.763	8.936	13.33	0.156	0.151	8.936
2000	0.545	5.49	7.64	0.164	0.11	5.49
2001	0.694	9.418	7.021	0.223	0.166	9.418
2002	0.45	7.562	11.13	0.216	0.158	7.562
2003	1.544	15.855	10.315	0.658	0.156	15.855
2004	0.627	11.825	18.632	0.542	0.217	11.825
2005	0.257	6.865	18.132	0.171	0.197	6.865
2006	0.481		14.605		0.225	
2007	0.222	7.189	7.715	0.231	0.135	7.189
2008	0.264	8.825	22.739	0.209	0.159	8.825
2009	0.376	9.179	37.708	0.209	0.16	9.179
2010	0.421	6.639	27.039	0.179	0.18	6.639
2011	0.351	9.746	17.939	0.195	0.125	9.746
2012	0.314	12.844	27.033	0.148	0.211	12.844

Table 2. Parameter estimates from different formulations of surplus production models for Div. 3NO witch flounder. See text for details.

Run	R	Bratio in last year	MSY	K	DIC
G4 Gamma 1,1 RUN 1	0.003 0.11 0.28	0.30 0.69 1.66	0.12 3.54 5.95	70.19 130.8 211.6	303.8
Uniform on q RUN 2	2.05E-4 0.09 0.46	0.56 1.73 6.02	0.008 3.54 13.36	77.33 153.0 308.1	293.75
RUN 3 CV instead of estimates of tau	2.86E-4 0.24 0.73	0.99 2.88 6.597	0.009 7.03 26.38	60.14 126.4 278.2	250.46
RUN 4 new r prior CV	0.09 0.27 0.58	1.16 2.85 5.06	2.45 8.39 23.58	63.39 125.0 286.0	267.73 5
RUN 5 new r and K prior CV	0.09 0.27 0.61	0.039 1.10 2.77	2.47 7.53 17.97	66.89 110.6 190.4	280.57 5
RUN 6 springadj CV	0.08 0.22 0.46	0.78 2.15 3.97	2.13 6.15 14.6	68.13 113.7 191.7	281.97
RUN 7 Uniform q obs error estimated	0.07 0.14 0.35	0.41 1.15 3.19	1.91 3.98 10.59	75.58 118.9 185.6	299.12
RUN 8 Gamma q obs error estimated	0.07 0.12 0.23	0.32 0.59 1.35	2.13 3.64 6.01	75.88 119.0 167.9	308.04
RUN 9 uniform q gamma obs error	0.07 0.13 0.33	0.41 1.07 3.12	1.93 3.90 10.38	75.01 119.0 180.7	299.39
RUN 10 as 8 but spring series split	0.08 0.13 0.26	0.35 0.76 1.69	2.38 3.82 6.32	72.73 116.9 167.9	304.82
RUN 11 as 10 but q obs gamma 0.1	0.08 0.16 0.41	0.678 1.65 3.047	2.46 4.49 10.08	72.69 115.8 171.4	303.19

Table 2. continued

Run	Q spring early	Q spring late	Q spring	Q fall	Fmsy
G4 Gamma 1,1 RUN 1	0.23 0.39 0.69	0.17 0.31 0.56	NA	0.27 0.49 0.88	0.001 0.05 0.13
Uniform on q RUN 2	0.04 0.17 0.42	0.02 0.09 0.28	NA	0.04 0.15 0.45	0.001 0.04 0.23
RUN 3 CV instead of estimates of tau	0.03 0.11 0.30	0.02 0.08 0.23	NA	0.04 0.14 0.40	0 0.12 0.36
RUN 4 new r prior	0.03 0.10 0.28	0.03 0.08 0.21	NA	0.05 0.13 0.37	0.04 0.13 0.29
RUN 5 new r and K prior	0.05 0.12 0.32	0.04 0.09 0.23	NA	0.07 0.15 0.40	0.05 0.14 0.30
RUN 6 springadj	NA	NA	0.05 0.11 0.32	0.08 0.19 0.54	0.04 0.11 0.23
RUN 7 Uniform q obs error estimated	NA	NA	0.07 0.22 0.53	0.10 0.32 0.80	0.03 0.07 0.17
RUN 8 Gamma q obs error estimated	NA	NA	0.23 0.42 0.65	0.34 0.62 1.01	0.04 0.06 0.12
RUN 9 uniform q gamma obs error	NA	NA	0.07 0.24 0.54	0.10 0.35 0.83	0.03 0.07 0.17
RUN 10 as 8 but spring series split	0.26 0.42 0.72	0.17 0.33 0.60	NA	0.28 0.51 0.94	0.04 0.07 0.13
RUN 11 as 10 but q obs gamma 0.1	0.12 0.26 0.49	0.07 0.13 0.34	NA	0.11 0.21 0.52	0.04 0.08 0.21

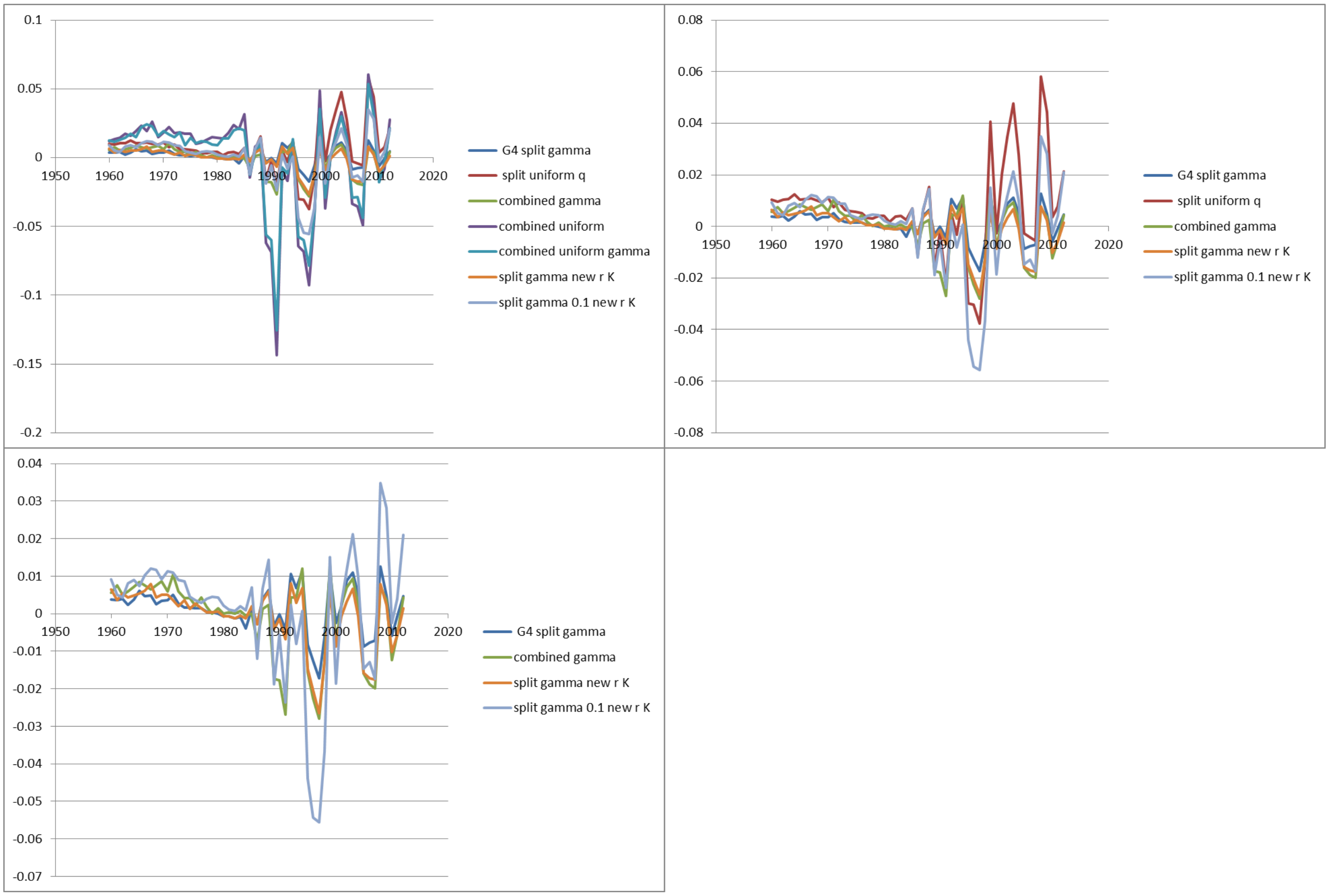

Figure 1. Process error for different formulations of the surplus production model for Div. 3NO witch flounder.

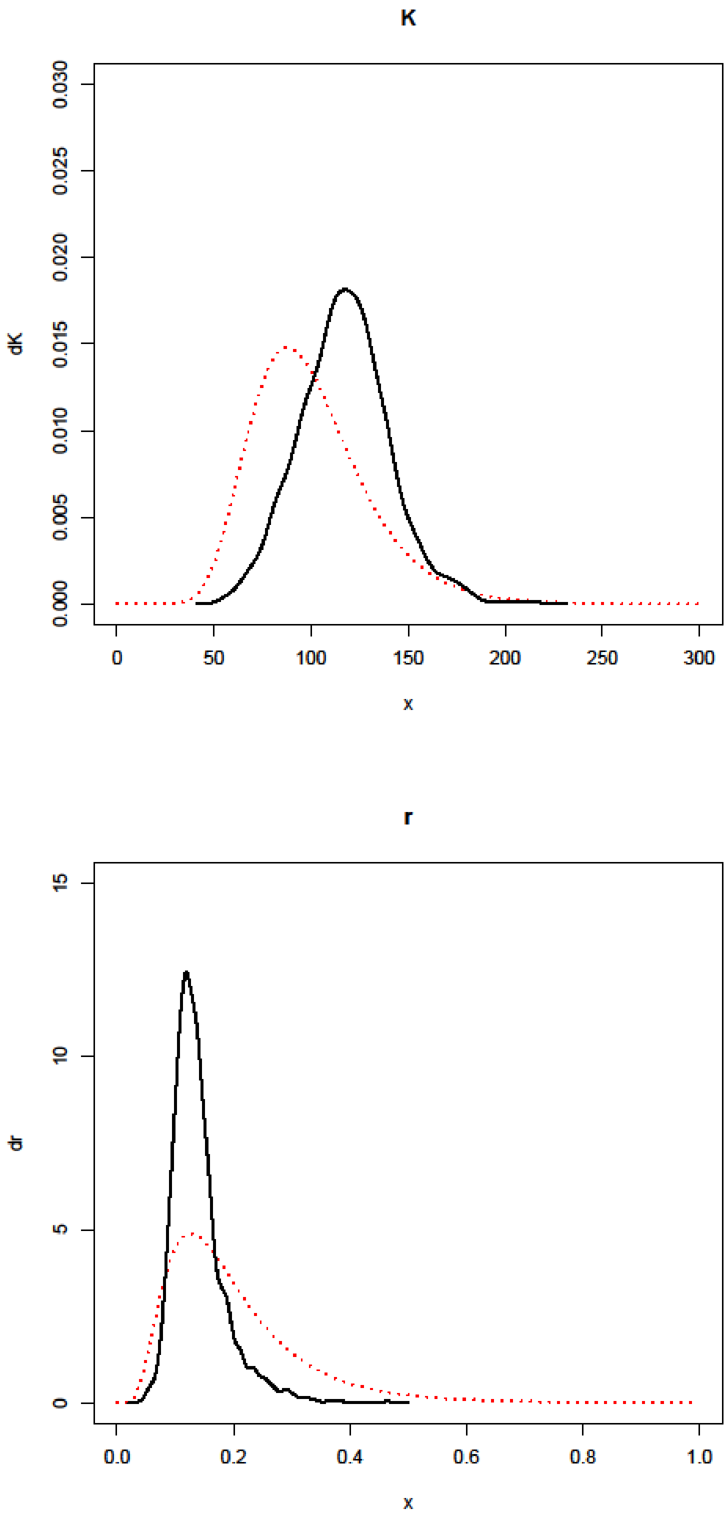

Figure 2. Prior (red dotted line) and posterior (black) for K and r for RUN 10.

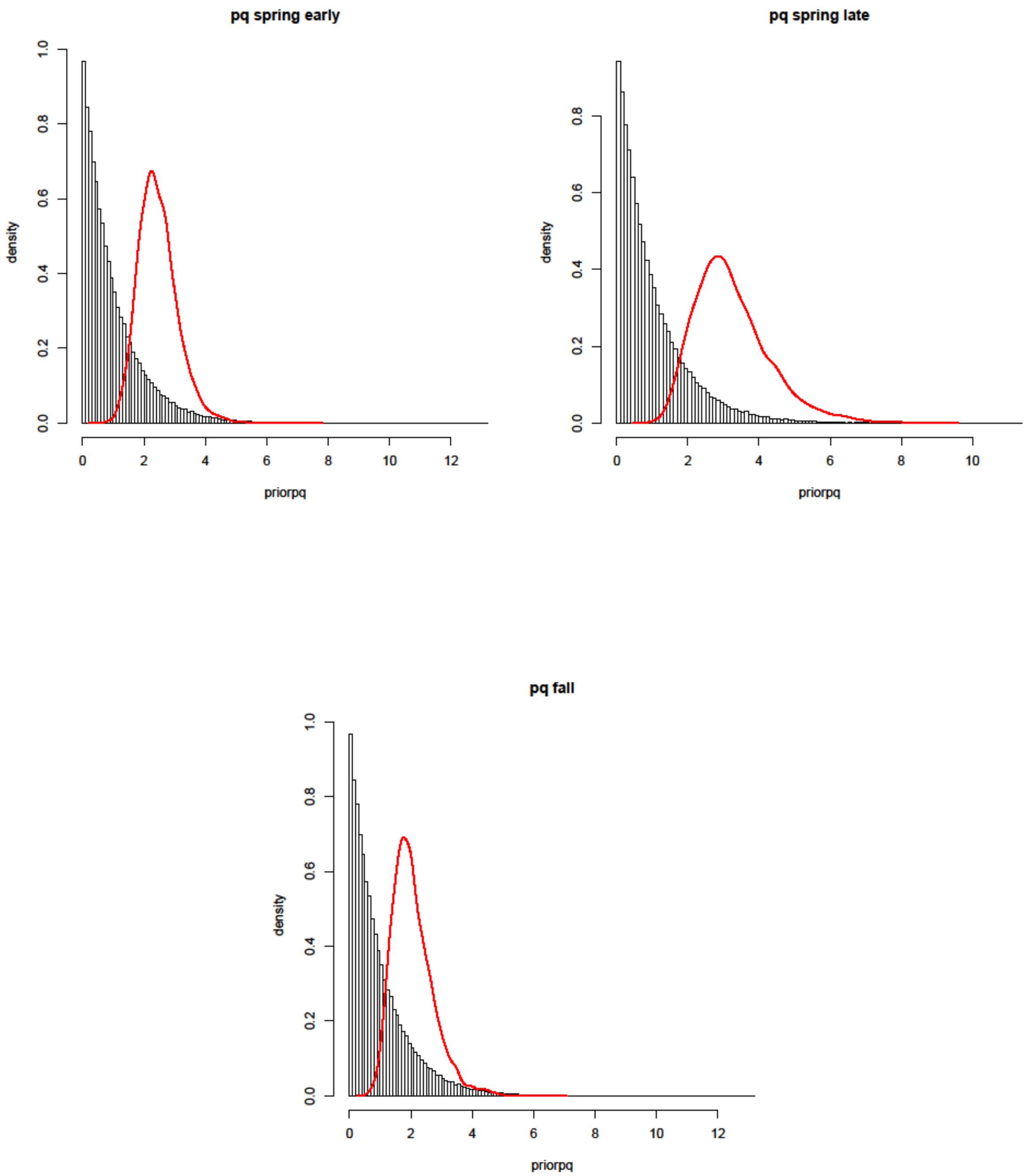

Figure 3 Priors (histogram) and posterior (red line) for pq for the three survey series from RUN 10. Q is 1/pq.

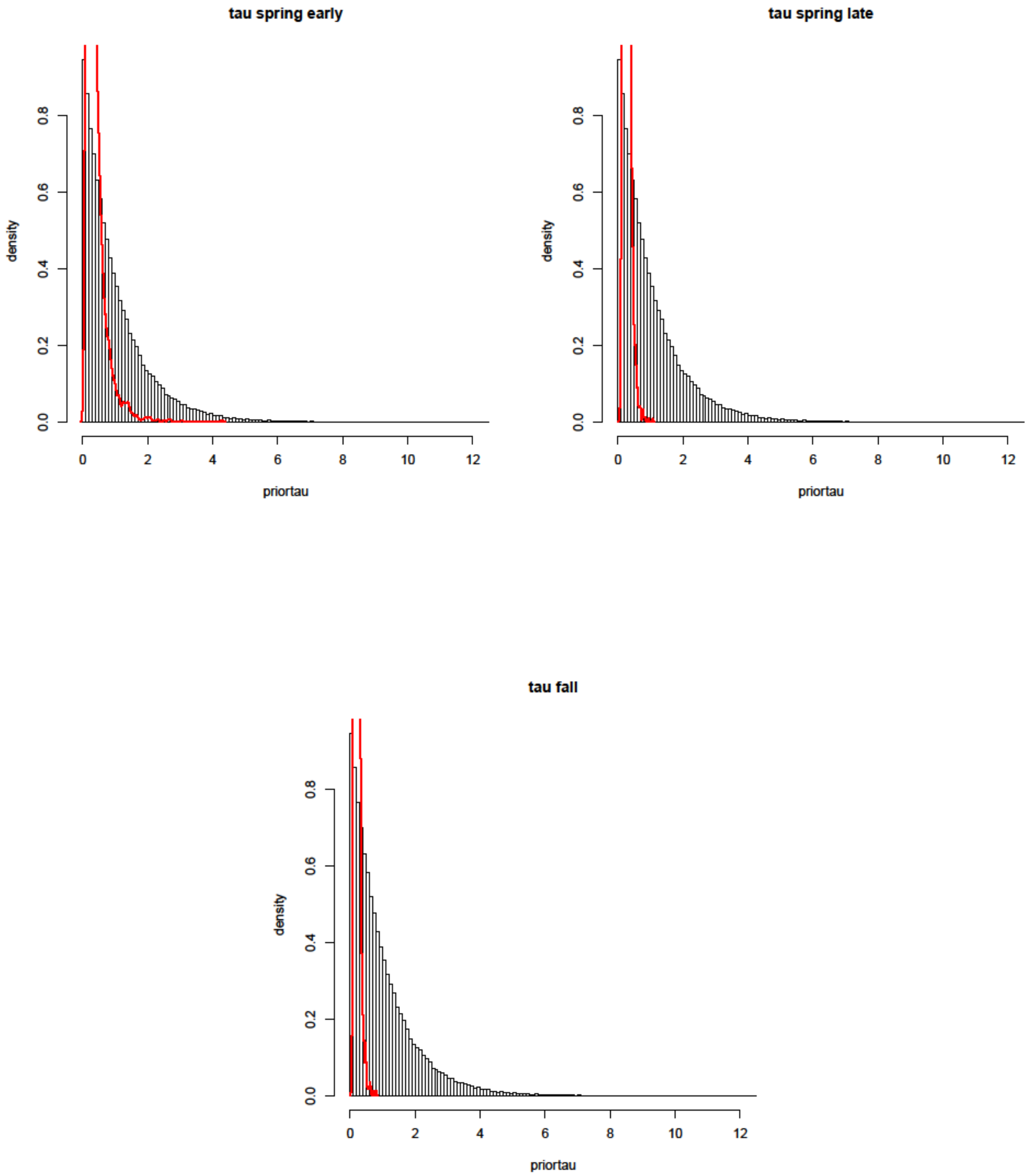

Figure 4. Priors (histogram) and posterior (red line) for tau for the three survey series from RUN 10.

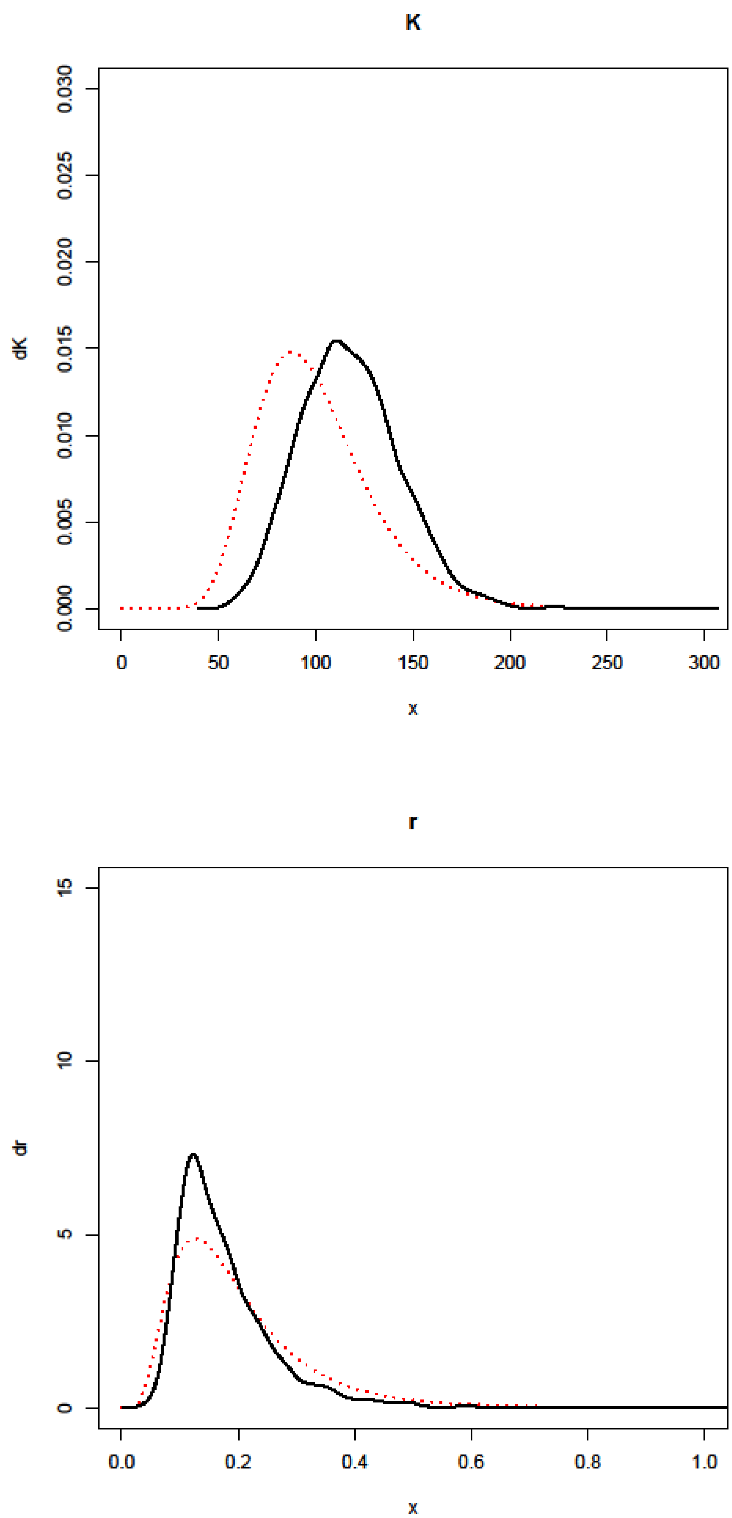

Figure 5. Prior (red dotted line) and posterior (black) for K and r for RUN 11.

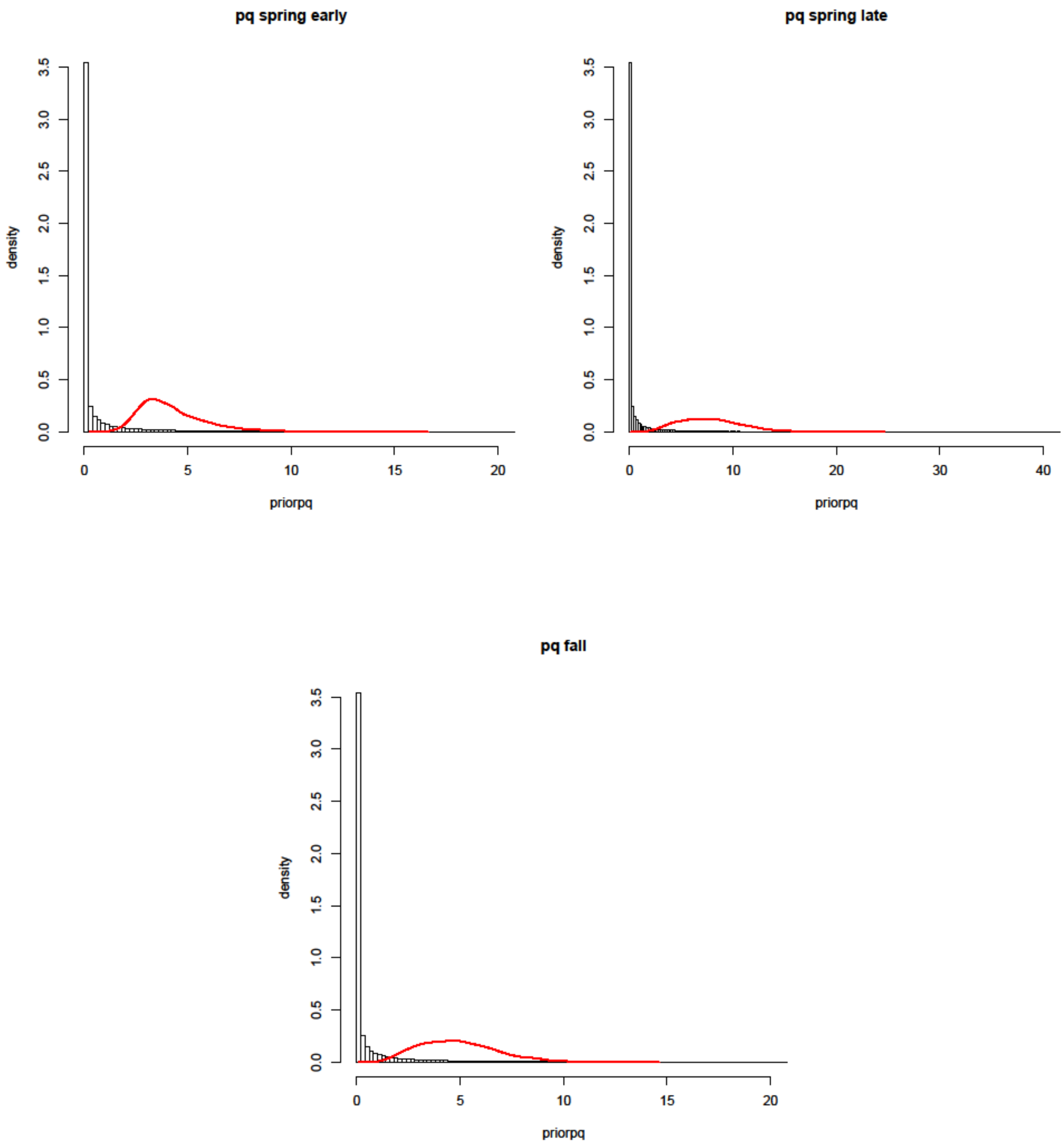

Figure 6 Priors (histogram) and posterior (red line) for pq for the three survey series from RUN 11. Q is 1/pq.

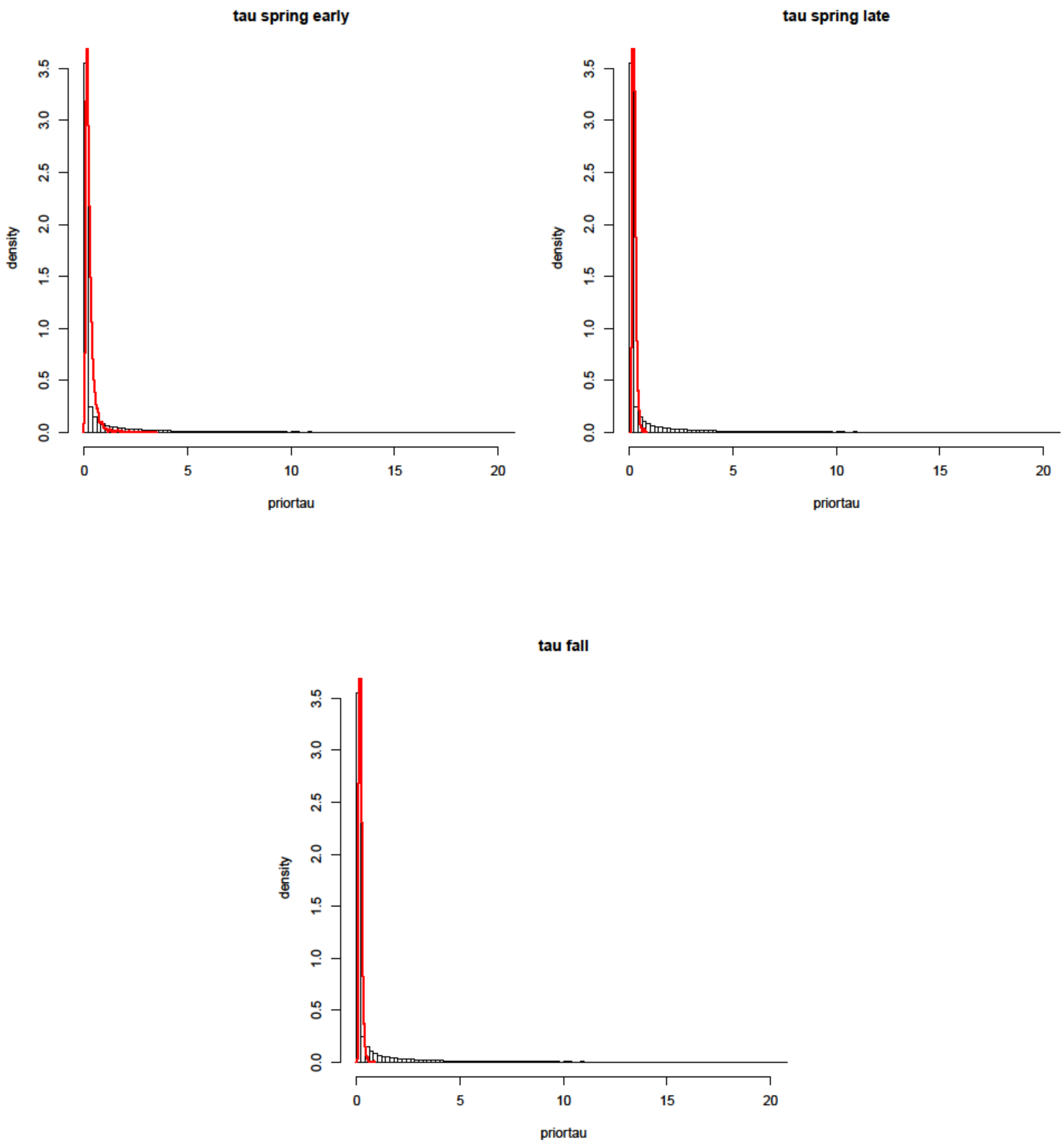

Figure 7. Priors (histogram) and posterior (red line) for tau for the three survey series from RUN 11.

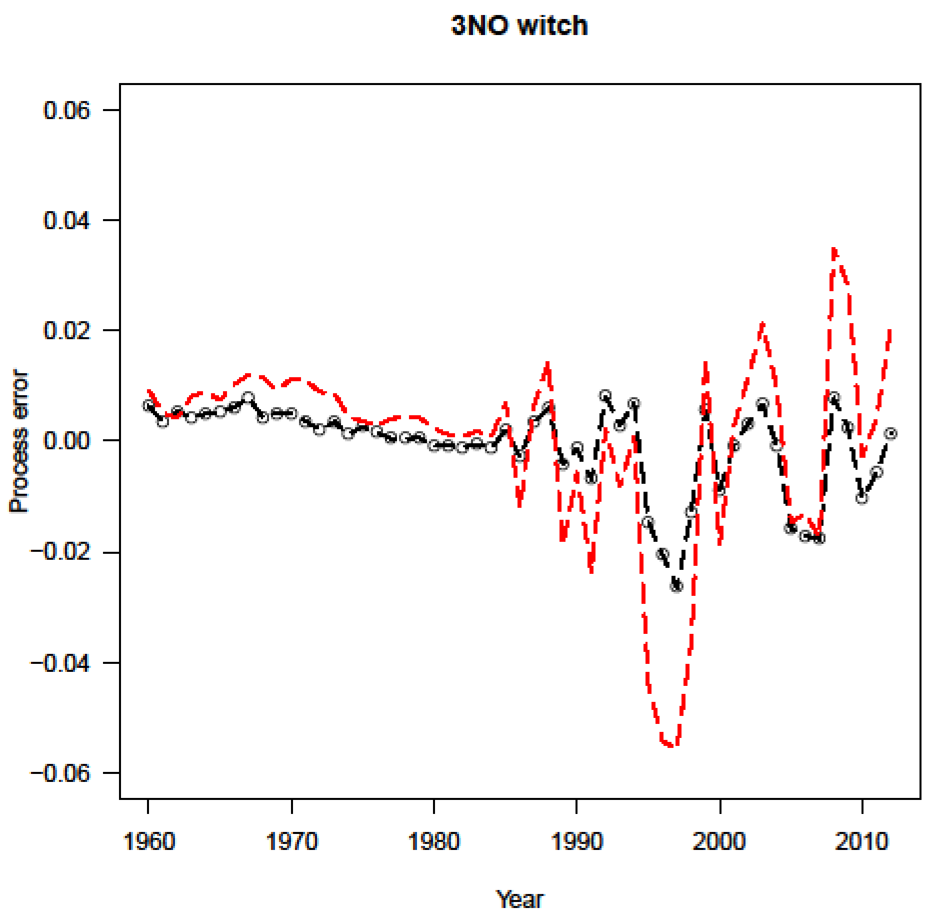

Figure 8. Process error for RUN 10 (black) and RUN 11 (dotted red line).

Appendix I

Convergence Diagnostics for RUN 10.

Diagnostics r

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.137	0.04195851	0.001326845	0.002139437	0.001865911	-0.0535869	0.0787985	0.12895	0.24002

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.138	0.04425724	0.001399537	0.002074535	0.001967648	-0.09745382	0.07545225	0.13085	0.254945

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.144	0.05091964	0.00161022	0.00313878	0.002573522	-0.02958648	0.08119075	0.1325	0.2853725

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 501:1000

Potential Scale Reduction Factors

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x

1.001325

Multivariate Potential Scale Reduction Factor = 1.002486

Corrected Scale Reduction Factors

Estimate 0.975

x 1.003645 1.009959

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -1.1853220

p-value 0.2358902

Chain: witchchain2

----

x

Z-Score -0.9190841

p-value 0.3580516

Chain: witchchain3

----

x

Z-Score 0.0538858

p-value 0.9570261

Diagnostics K

SUMMARY STATISTICS:

==================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

116.843 21.91943 0.6931533 1.448868 1.073645 -0.1376279 74.48925 116.7 159.

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

117.4727 22.99421 0.7271407 1.163924 1.29789 -0.1331056 72.86725 116.95 165.405

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

116.8816 24.78545 0.7837846 1.848439 1.274178 -0.2328701 70.4495 116.85 171.6325

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 501:1000

Potential Scale Reduction Factors

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x

0.9993121

Multivariate Potential Scale Reduction Factor = 0.9994683

Corrected Scale Reduction Factors

Estimate 0.975

x 1.004211 1.005077

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score 0.5440047

p-value 0.5864383

Chain: witchchain2

----

x

Z-Score -1.2570025

p-value 0.2087527

Chain: witchchain3

Z-Score 0.2591829

p-value 0.7954941

DIAGNOSTICS Sigma

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.0988	0.0783186	0.002476652	0.005669452	0.005750504	-0.05332953	0.004298925	0.08115	0.294005

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.100 0.07806125 0.002468514 0.005277202 0.004555375 -0.2517043 0.003983625 0.08466 0.3030475

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.105 0.07974159 0.002521651 0.005664608 0.005160324 -0.3092862 0.003654525 0.09112 0.289015

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 501:1000

Potential Scale Reduction Factors

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x

1.000905

Multivariate Potential Scale Reduction Factor = 1.001857

Corrected Scale Reduction Factors

Estimate 0.975

x 1.002253 1.007401

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

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x

Z-Score -0.1977201

p-value 0.8432640

Chain: witchchain2

----

x

Z-Score 0.4520637

p-value 0.6512231

Chain: witchchain3

----

x

Z-Score 0.3893007

p-value 0.6970537

Diagnostics q spring early

SUMMARY STATISTICS:

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Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.429	0.1176532	0.00372052	0.004112245	0.003237042	0.178179	0.254225	0.41265	0.71259

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.4375	0.1241872	0.003927143	0.004646036	0.004945815	-0.3122723	0.2566625	0.42235	0.70131

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.444	0.1204435	0.003808758	0.005358805	0.005289481	-0.1639334	0.2665825	0.4255	0.7373025

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

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x

1.001327

Multivariate Potential Scale Reduction Factor = 1.002489

Corrected Scale Reduction Factors

Estimate 0.975

x 1.002351 1.008621

GEWEKE CONVERGENCE DIAGNOSTIC:

============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -0.5676327

p-value 0.5702844

Chain: witchchain2

----

x

Z-Score 0.02068146

p-value 0.98349976

Chain: witchchain3

----

x

Z-Score 0.5708556

p-value 0.5680975

DIAGNOSTICS Q SPRING LATE

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.334	0.1043877	0.00330103	0.008170589	0.007702128	0.09219642	0.169575	0.32205	0.58236

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.346	0.1115445	0.003527346	0.008184179	0.007298505	0.09831343	0.1815975	0.32905	0.596845

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.347	0.1161859	0.00367412	0.008949705	0.009531398	-0.1790226	0.16669	0.32985	0.6177

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.004417

Multivariate Potential Scale Reduction Factor = 1.007116

Corrected Scale Reduction Factors

Estimate 0.975

x 1.00449 1.018917

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -0.02315583

p-value 0.98152597

Chain: witchchain2

----

x

Z-Score -0.7846138

p-value 0.4326800

Chain: witchchain3

----

x

Z-Score 1.3014929

p-value 0.1930898

DIAGNOSTICS Q FALL

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.523	0.1604986	0.00507541	0.01275729	0.01094546	0.2016087	0.27885	0.5023	0.889935

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.543	0.1724698	0.005453974	0.01321371	0.01125988	0.08557092	0.281075	0.5185	0.93917

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.545	0.1862986	0.005891279	0.01468672	0.01480828	-0.1197015	0.26429	0.51565	0.963615

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

===

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.005042

Multivariate Potential Scale Reduction Factor = 1.00805

Corrected Scale Reduction Factors

Estimate 0.975

x 1.006593 1.022831

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score 0.1388382

p-value 0.8895780

Chain: witchchain2

----

x

Z-Score -0.8766368

p-value 0.3806840

Chain: witchchain3

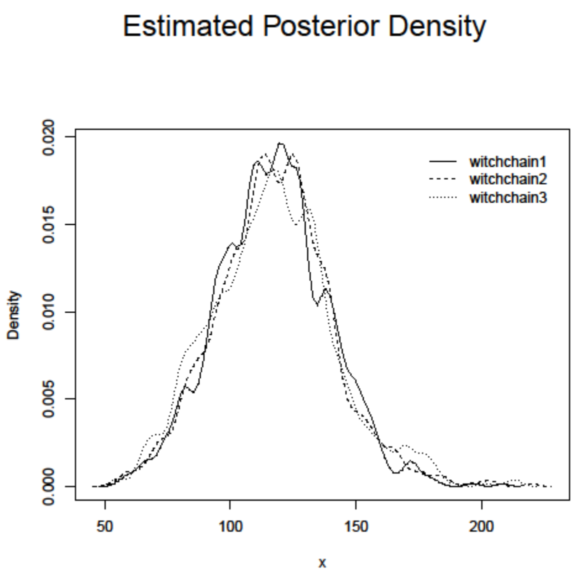
----

x

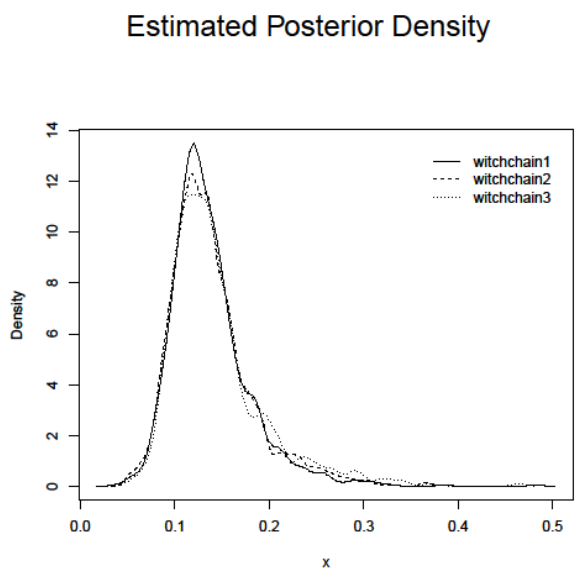
Z-Score 1.1318959

p-value 0.2576782

K



r



Sigma

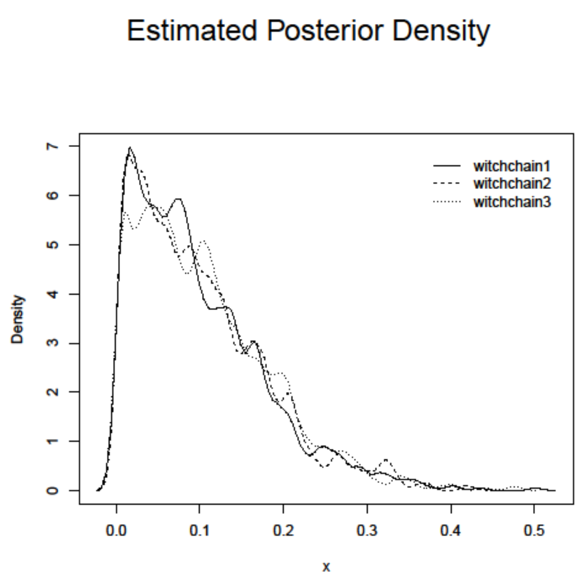
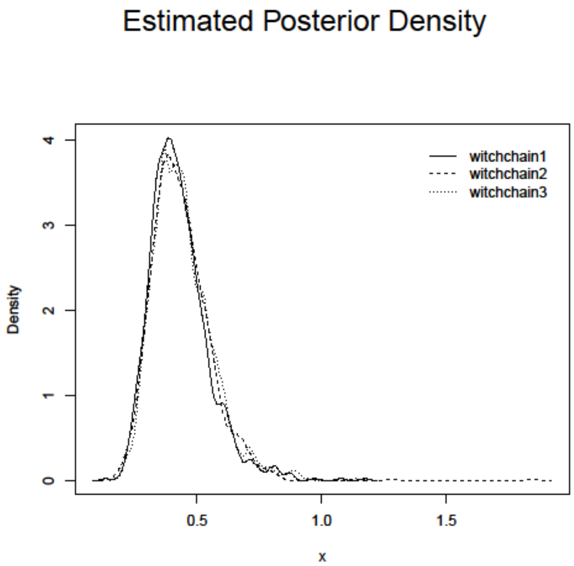
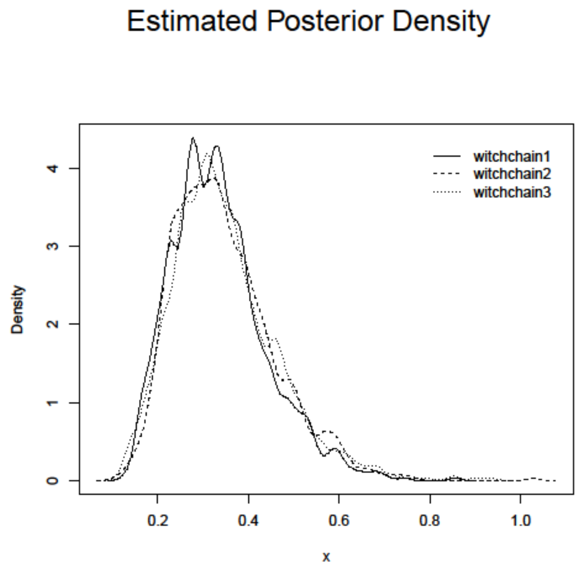


Figure A1.1. Posterior density for K, r, and sigma for each chain for RUN 10.

Spring early



Spring late



Fall

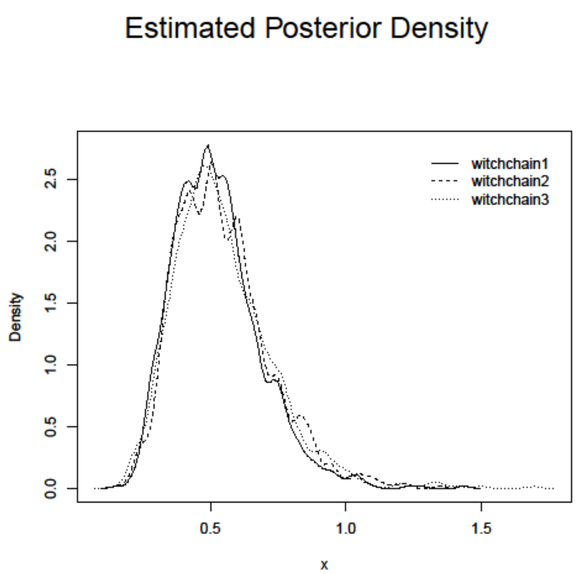
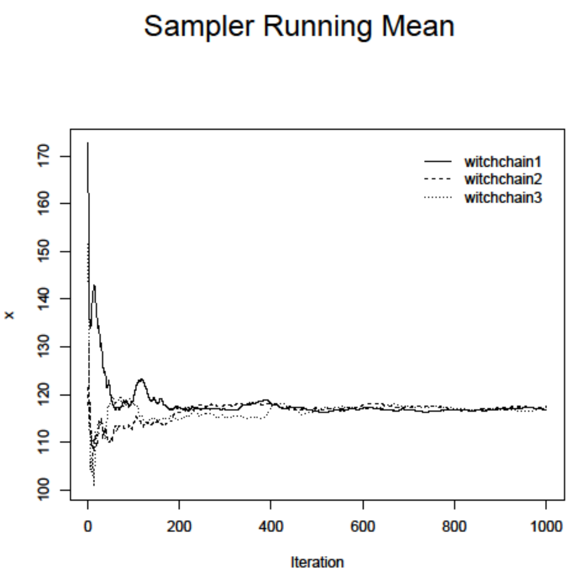
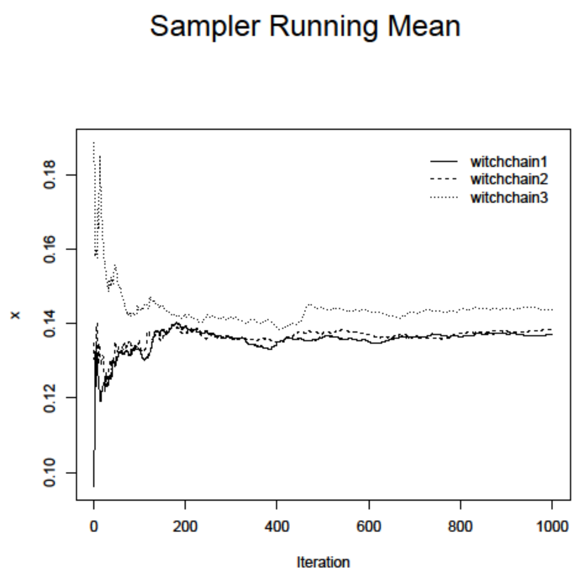


Figure A1.2. Posterior density for q for each survey series for each chain for RUN 10.

K



r



Sigma

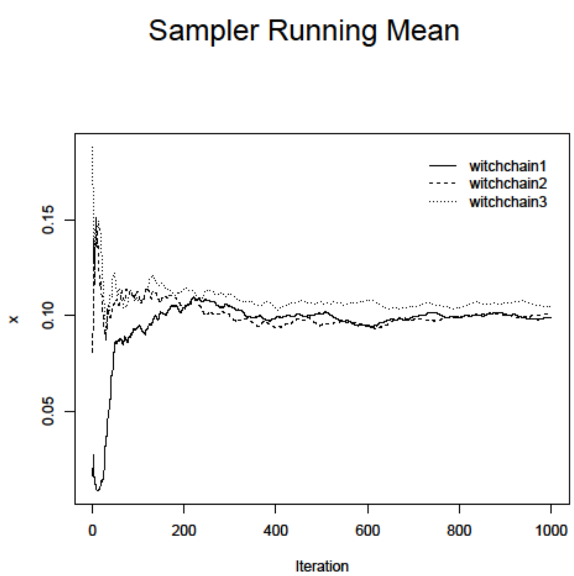
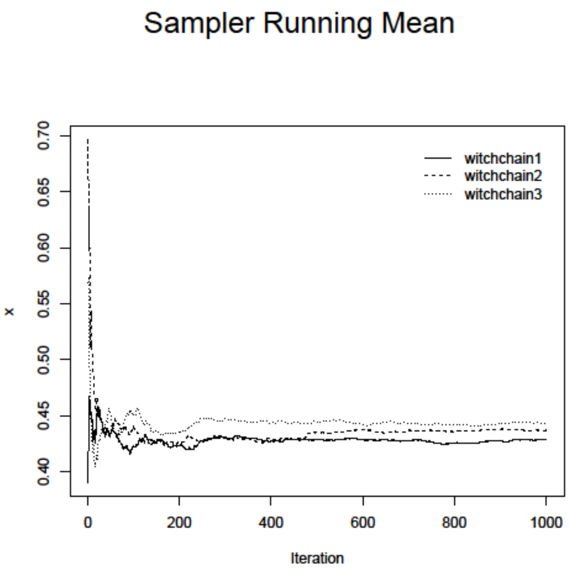
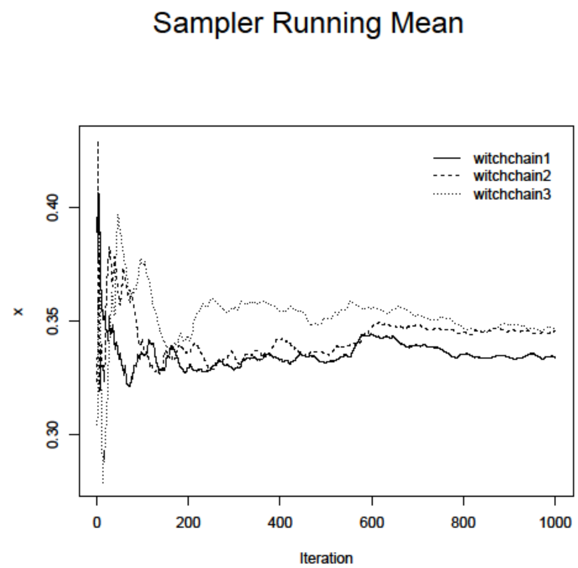


Figure A1.3. Running mean from three chains for K, r and sigma from RUN 10.

Spring early



Spring late



Fall

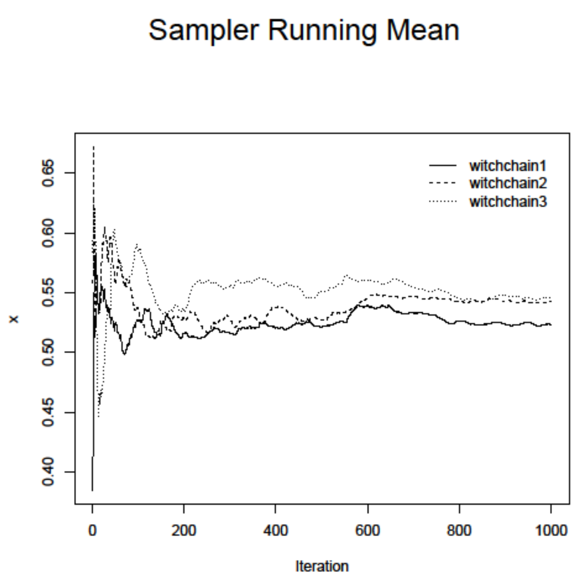


Figure A1.4. Running mean from three chains for q from each survey series from RUN 10.

Appendix II Convergence diagnostics for RUN 11

DIAGNOSTICS r

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.173	0.08822076	0.002789785	0.005839283	0.005519755	0.03811593	0.081479	0.1485	0.3845525

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.1858	0.09373802	0.002964256	0.005726324	0.005323687	-0.1100361	0.077399	0.16185	0.458115

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.176	0.08540728	0.002700815	0.005747791	0.006033245	0.2864733	0.073754	0.15495	0.3995025

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

===

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.005046

Multivariate Potential Scale Reduction Factor = 1.008056

Corrected Scale Reduction Factors

Estimate 0.975

x 1.010389 1.027034

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score 1.6753361

p-value 0.0938683

Chain: witchchain2

----

x

Z-Score 1.84175502

p-value 0.06551099

Chain: witchchain3

----

x

Z-Score -0.1973210

p-value 0.8435763

DIAGNOSTICS K

SUMMARY STATISTICS:

==================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

118.1122 25.65425 0.8112586 1.288315 1.187927 -0.008886133 73.723 117.05 172.505

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

116.0266 26.0212 0.8228626 1.182475 1.187046 -0.1756614 71.187 113.45 171.415

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

118.4506 25.25048 0.7984903 1.67879 1.556823 0.1762963 75.399 117.2 170.1

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.00193

Multivariate Potential Scale Reduction Factor = 1.003392

Corrected Scale Reduction Factors

Estimate 0.975

x 1.003751 1.01168

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -1.75048596

p-value 0.08003449

Chain: witchchain2

----

x

Z-Score -1.75066155

p-value 0.08000423

Chain: witchchain3

----

x

Z-Score 0.2022152

p-value 0.8397485

DIAGNOSTICS Sigma

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.116 0.07928254 0.002507134 0.005830743 0.005660676 0.005208483 0.0087738 0.09973 0.29675

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.128 0.0797529 0.002522008 0.004208324 0.004472125 -0.1483878 0.01824975 0.1128 0.325625

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.116 0.0823155 0.002603045 0.006145939 0.00560156 -0.07386171 0.002193025 0.10415 0.30982

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.002872

Multivariate Potential Scale Reduction Factor = 1.004803

Corrected Scale Reduction Factors

Estimate   0.975

x 1.004716 1.01518

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score 0.8179462

p-value 0.4133879

Chain: witchchain2

----

x

Z-Score 0.4906773

p-value 0.6236547

Chain: witchchain3

----

x

Z-Score -0.2335020

p-value 0.8153716

DIAGNOSTICS Q SPRING EARLY

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.2790957 0.105778 0.003344995 0.006467145 0.005955857 -0.2821016 0.11457 0.2663 0.520125

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.2614762 0.09593047 0.003033588 0.004842437 0.005161193 0.1768417 0.113495 0.25065 0.47259

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.2634 0.09494318 0.003002367 0.005750754 0.006341032 0.4311507 0.1186875 0.25415 0.4730275

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

===

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.007502

Multivariate Potential Scale Reduction Factor = 1.011727

Corrected Scale Reduction Factors

Estimate 0.975

x 1.013814 1.037241

GEWEKE CONVERGENCE DIAGNOSTIC:

===============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -0.5431681

p-value 0.5870141

Chain: witchchain2

----

x

Z-Score -2.41941913

p-value 0.01554532

Chain: witchchain3

----

x

Z-Score -0.4422093

p-value 0.6583377

DIAGNOSTICS Q SPRING LATE

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.1583959	0.0753619	0.002383153	0.006879415	0.005179381	-0.3464903	0.06940475	0.13645	0.37334

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.1458048 0.06762826 0.002138593 0.005153378 0.00459862 0.006335384 0.064975 0.1251 0.319165

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.1486 0.06591235 0.002084331 0.005495384 0.006111668 0.3549448 0.0693285 0.13265 0.318915

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.005089

Multivariate Potential Scale Reduction Factor = 1.00812

Corrected Scale Reduction Factors

Estimate 0.975

x 1.012847 1.029835

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -0.6752723

p-value 0.4995028

Chain: witchchain2

----

x

Z-Score -2.86165751

p-value 0.00421432

Chain: witchchain3

----

x

Z-Score -1.1863825

p-value 0.2354713

DIAGNOSTICS Q FALL

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.2515381	0.1177187	0.003722592	0.01056609	0.008353312	-0.3515673	0.1106675	0.2217	0.5712525

Chain: witchchain2

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.2330288 0.1042914 0.003297983 0.008302613 0.007270266 -0.02101828 0.1075325 0.20465 0.49362

Chain: witchchain3

Mean SD Naive SE MC Error Batch SE Batch ACF 0.025 0.5 0.975

0.2359976 0.1025658 0.003243417 0.008605521 0.009448168 0.3440917 0.112795 0.2097 0.4994525

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 501:1000

Potential Scale Reduction Factors

----

x

1.006149

Multivariate Potential Scale Reduction Factor = 1.009705

Corrected Scale Reduction Factors

Estimate 0.975

x 1.015001 1.035049

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

----

x

Z-Score -0.8087519

p-value 0.4186579

Chain: witchchain2

----

x

Z-Score -3.059761113

p-value 0.002215136

Chain: witchchain3

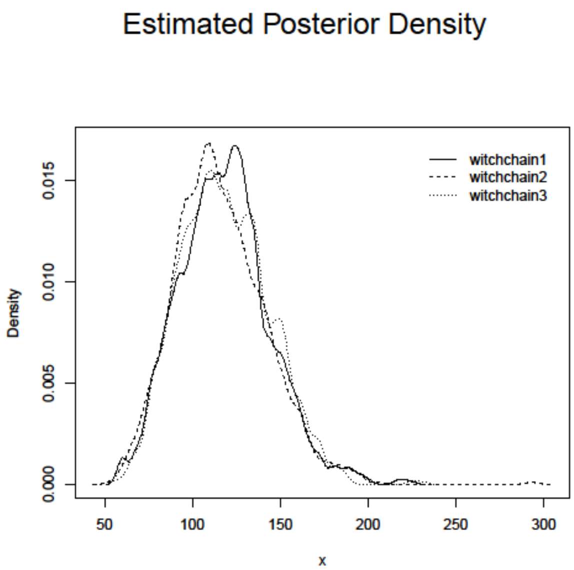
----

x

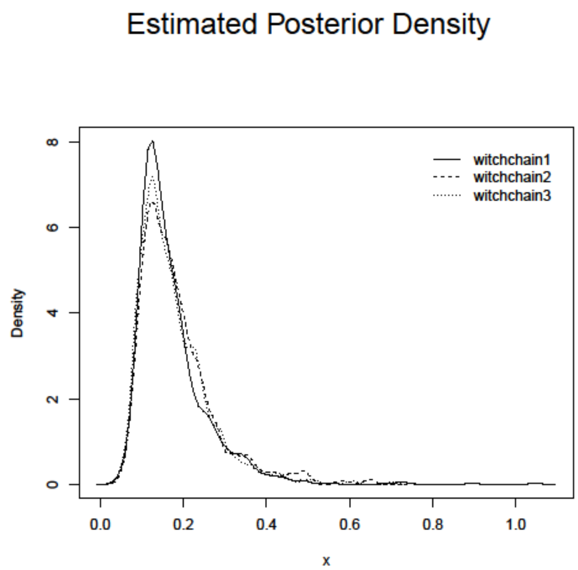
Z-Score -0.9976007

p-value 0.3184730

K



r



Sigma

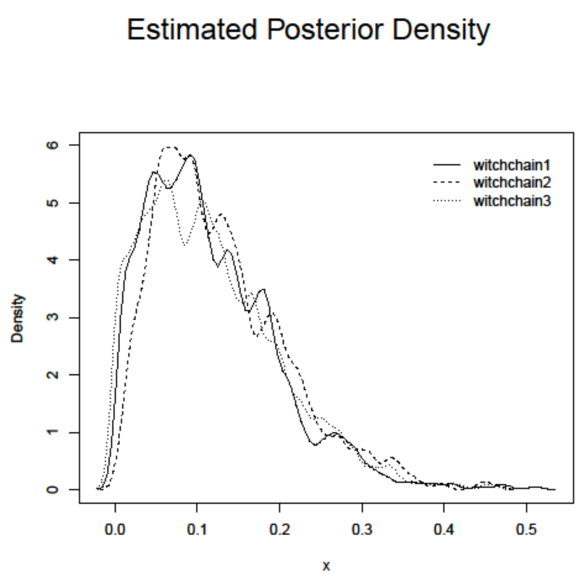
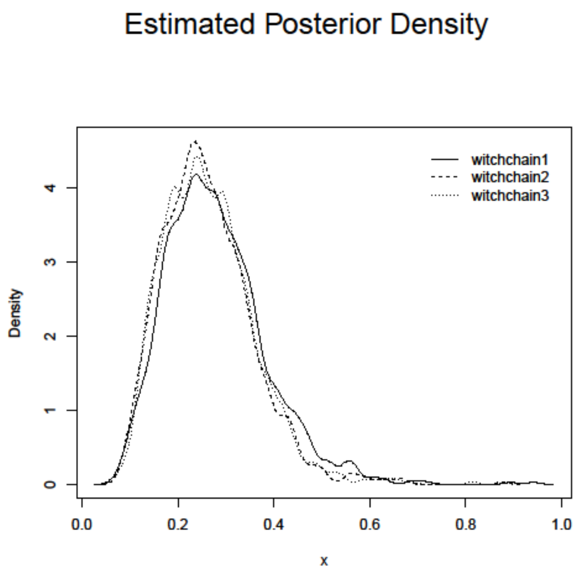
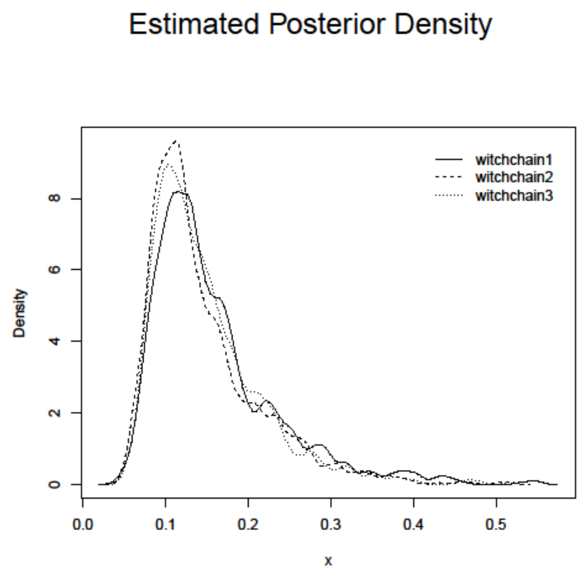


Figure A2.1. Posterior density for K, r, and sigma for each chain for RUN 11.

Spring early



Spring late



Fall

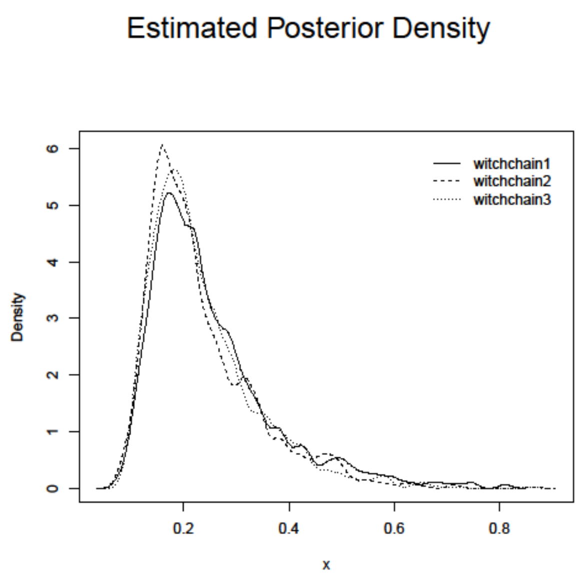
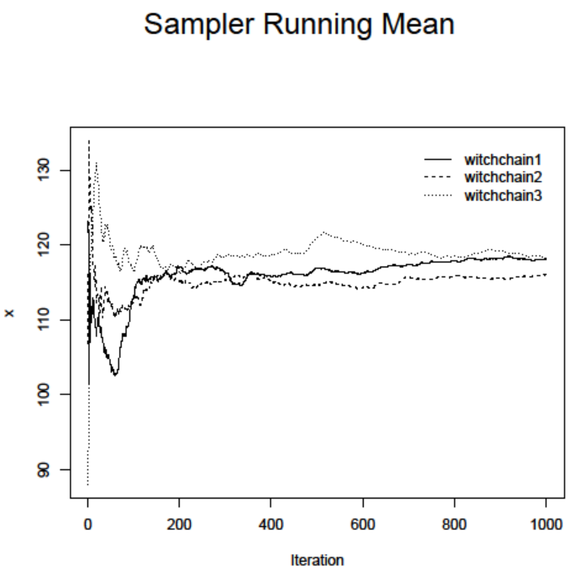
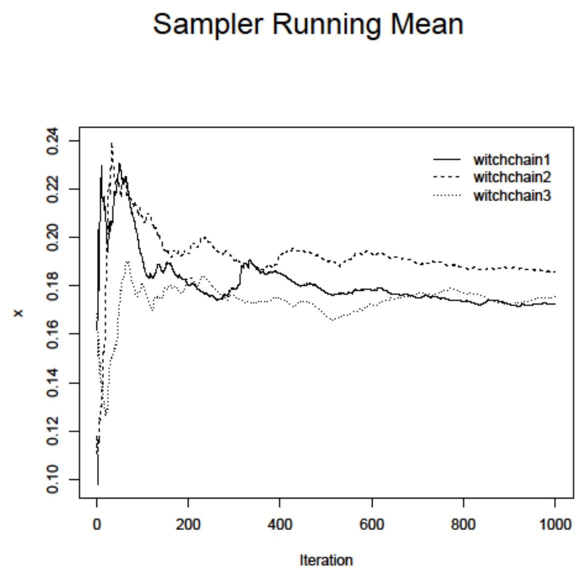


Figure A2.2. Posterior density for q for each survey series for each chain for RUN 11.

K



r



Sigma

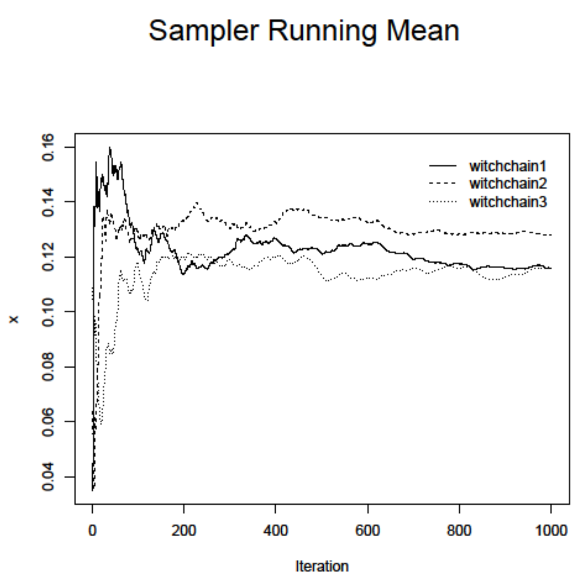
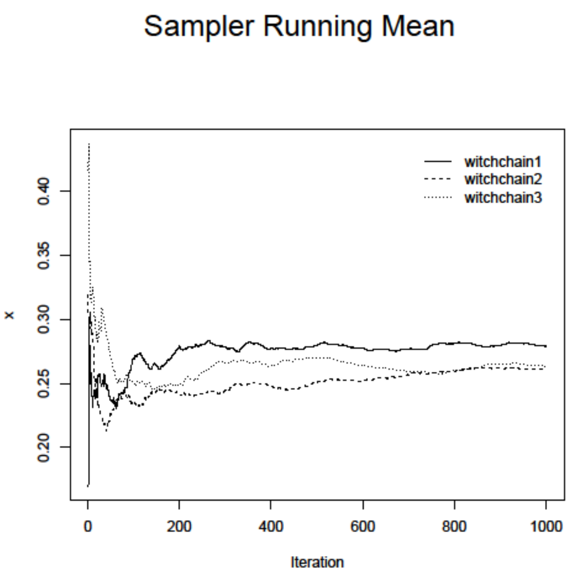
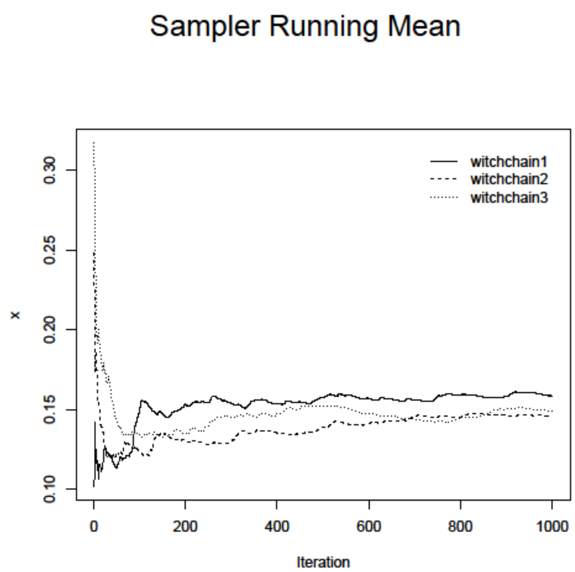


Figure A2.3. Running mean from three chains for K, r and sigma from RUN 11.

Spring early



Spring late



Fall

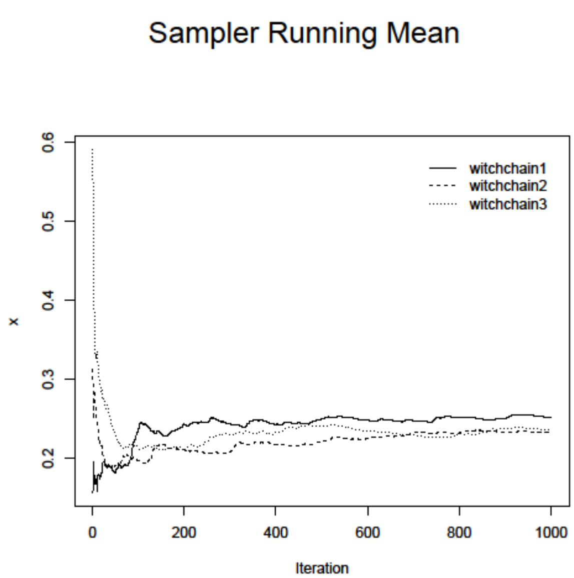


Figure A2.4. Running mean from three chains for q from each survey series from RUN 11.