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**A stochastic length-based assessment model for the *Pandalus* stock in Skagerrak and the Norwegian Deep**

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**Abstract**

This working document describes a length based stochastic assessment model of *Pandalus* in ICES areas IIIA and IVA. The model describing stock development is age based, but the model also partly estimates the relation between age and length assuming a von Bertalanffy growth curve. The model presented in this document is based on the assessment data with catches from 2014, but with updated survey information from 2015.

**Input data**

Data for this assessment covers the period from 1988 to 2015. Total commercial catch-at-length in numbers  $C_{l,y}$  are available only in the years from 1988 to 2014. The total catches are accumulated in 1mm length classes starting with 8mm.

In addition to the total catches a (scientific) survey-index  $I_{l,y}$  is available in all years except 2003. The survey length classes start at 7mm.

The natural mortality is assumed to be  $M_{a,y} = 0.75$  for all ages in all years. This figure has been used in previous assessments of this stock.

**The model**

The model is a full parametric statistical model. In this model the stock development from year to year is an age based model, but the data is given in lengths.

The stock dynamics is similar to most fully parametric statistical age based stock assessment models. The stock sizes in the first age class  $N_{a=0,y=1988..2015}$  are assumed to be model parameters to be estimated. For the stock sizes in the first year  $N_{a=0..4,y=1988}$ ,  $N_{a=1,y=1988}$ ,  $N_{a=2,y=1988}$ , and  $N_{a=3,y=1988}$  are model parameters and the last is set by assuming that the total mortality and stock sizes were similar the year before, such that  $N_{a=4,y=1988} = N_{a=3,y=1988}e^{Z_{a=3,y=1988}}$ .

A multiplicative model is assumed for the age specific fishing mortalities  $F_{a,y} = F_a F_y$ . Further, to ensure that the model is identifiable, it is assumed that  $F_{a=3} = F_{a=4}$ . The total mortality is defined as  $Z_{a,y} = M_{a,y} + F_{a,y}$ .

Having defined the total mortality  $Z$  and the marginal stock sizes, all other stock sizes  $N_{a,y}$  can be calculated successively by the stock equation  $N_{a+1,y+1} = N_{a,y}e^{-Z_{a,y}}$ .

The relationship between age and length is assumed to follow the Von Bertalanffy growth function

$$L(a) = L_{\infty} - (L_{\infty} - L_0)e^{-ka}$$

Where  $k$  is model parameter to be estimated describing the growth rate.  $L_0$  and  $L_{\infty}$  are a model

parameters describing individual length at age zero and asymptotic length respectively. These are fixed to zero and 29 in this model. The length distribution of  $a$  year old fish are assume to be normally distributed with a mean given by the Von Bertalanffy growth function and a standard deviation estimated by the model. This normal distribution would more logically be assumed on the log-scale, this was attempted, but found too unstable in this model.

The data are observed commercial catch-at-length  $C_{l,y}$  and survey catch-at-length  $I_{l,y}$ , for a number of length groups  $g_1, \dots, g_n$ .

The expected number of fish caught by the survey in a length group  $g_i$  can be computed by:

$$\widehat{I}_{g_i,y} = \sum_{a=0}^4 \left( \Phi \left( \frac{\text{upper}(g_i) - L(a + \tau)}{\sigma_L} \right) - \Phi \left( \frac{\text{lower}(g_i) - L(a + \tau)}{\sigma_L} \right) \right) Q(a) N_{a,y} e^{-Z_{a,y} \tau}$$

In this equation the term  $Q(a)N_{a,y}e^{-Z_{a,y} \tau}$  is recognized as the expected number caught from age group  $a$ , if  $\tau$  is the time of year where the survey is conducted, and  $Q(a)$  is the survey catchability for age group  $a$ . The remaining large parenthesis computes what fraction of lengths at age  $a + \tau$  that are within length group  $g_i$ . Here  $\Phi$  is the distribution function for a standard normal distribution,  $\text{lower}(g_i)$  and  $\text{upper}(g_i)$  are the lower and upper limits defining the length group  $g_i$ .  $L(a + \tau)$  is, according to the von Bertalanffy growth function the mean length at age  $a + \tau$  and  $\sigma_L$  is the corresponding standard deviation. Finally, the lengths in  $g_i$  from all age groups are summed.

It is slightly more difficult to compute the expected commercial catch in each specified length group, since this is not taken at a specific time, but throughout the entire year, while the individuals are growing. Here it is assumed that fishing occurs uniformly in time within each year.

Computing the expected total catch in each length group is done by stepping through the year in sufficiently small steps (here 10 per year). In each step the fraction with lengths in the length group  $g_i$  of the total catch from that step is added up. With 10 steps define  $\Delta t = 1/10$ , then:

$$\widehat{C}_{g_i,y} = \sum_{s=1}^{10} \sum_{a=0}^4 \left( \Phi \left( \frac{\text{upper}(g_i) - L(a + (s - 0.5)\Delta t)}{\sigma_L} \right) - \Phi \left( \frac{\text{lower}(g_i) - L(a + (s - 0.5)\Delta t)}{\sigma_L} \right) \right) \cdot \frac{F_{a,y}}{Z_{a,y}} (1 - e^{-Z_{a,y} \Delta t}) N_{a+(s-1)\Delta t,y}$$

Here the last term is recognized as the catch equation applied in each step, and the first term is computing what fraction has lengths within length group  $g_i$ .

The remaining part of the model is the observation error which is assumed to be independent normally distributed for the catches with a separate variance parameter. The surveys indices are also assumed normally distributed, but correlated within each year. Survey indices are computed from relatively few catches from relatively few days at sea, and as such the catch is more influenced by local fluctuating conditions (like weather). The measurement noise is described by two model parameters. A correlation parameter describing the within year correlation and a separate variance parameter for the survey catch.

### Example calculating reference points

The model is able to produce full stock numbers at age, fishing mortality at age, and as such normal calculations of reference points can also be conducted. As an example the calculation of  $F_{0.1}$  is illustrated here.

The yield per recruit curve must be calculated, which should not be based on the plus group selected for the assessment model, as the cohort need to be followed to the end. The maximum age was set to 20, which

should be more than enough. The fishing selectivity at age  $F_a$  was set as estimated by the model and extended to maximum age by setting the selectivity equal to selectivity at age 4 for all ages above 4. The average proportion mature over the last 15 years was used, and set to 1 for ages above 4. The natural mortality was extended by setting it equal age 4 for all ages above 4.

The mean weight by length group recorded from the samples (by quarter) are used for estimating an allometric length weight relationship  $W = aL^b$  ( $W$  = weight,  $L$  = length). This relationship is calculated both on a quarterly and annual basis and is available for the period 1988-2014. However, as the year to year variations in parameter values are small an average for the period 1998 -2014 is used in the model ( $a = 0.0016$ ,  $b = 2.7532$ ). This length-weight relationship is then used in the von Bertalanffy growth equation for calculating the weight at age figures used in the estimated SSB.

Having extended all the variables that determinate the cohort development, a yield is computed for a number of different levels of  $F_{1-3}$ , here the levels 0,0.01,0.02,...,2.0 were chosen, and the yield was plotted against  $F_{1-3}$ . Finally, the point on the yield per recruit curve, where the slope is 10% of its initial slope (at  $F_{1-3} = 0$ ), is identified.

### **Example calculating forecasts**

Following the estimated stock numbers at age and fishing mortality at age it is possible to carry out normal short term forecasts. An example the calculation is shown here. The stock status at the end of the final assessment year (here 2014) is used as the starting point.

To forecast different stock options assumptions about the level of the future recruitment is needed. For this example the median of the last 7 years was used, since there is no obvious stock-recruitment relationship. For a future stock weights, catch weights, proportion mature and natural mortality, an average of the last three years (2012-2014) was used.

Consider now the scenario, where fishing is set to be the same in the three following years. For the first forecast year (2015) the recruitment is set to the median described above, and following the fishing and natural mortalities each N-at-age is updated one step ahead from the previous year. The catch and SSB is calculated. This is repeated three times.

The total catch weight for the three forecast years are computed by converting the forecasted catch-at-ages  $C_{a,y}$  into catch-at-lengths  $C_{l,y}$  via the estimated von Bertalanffy growth curve, and

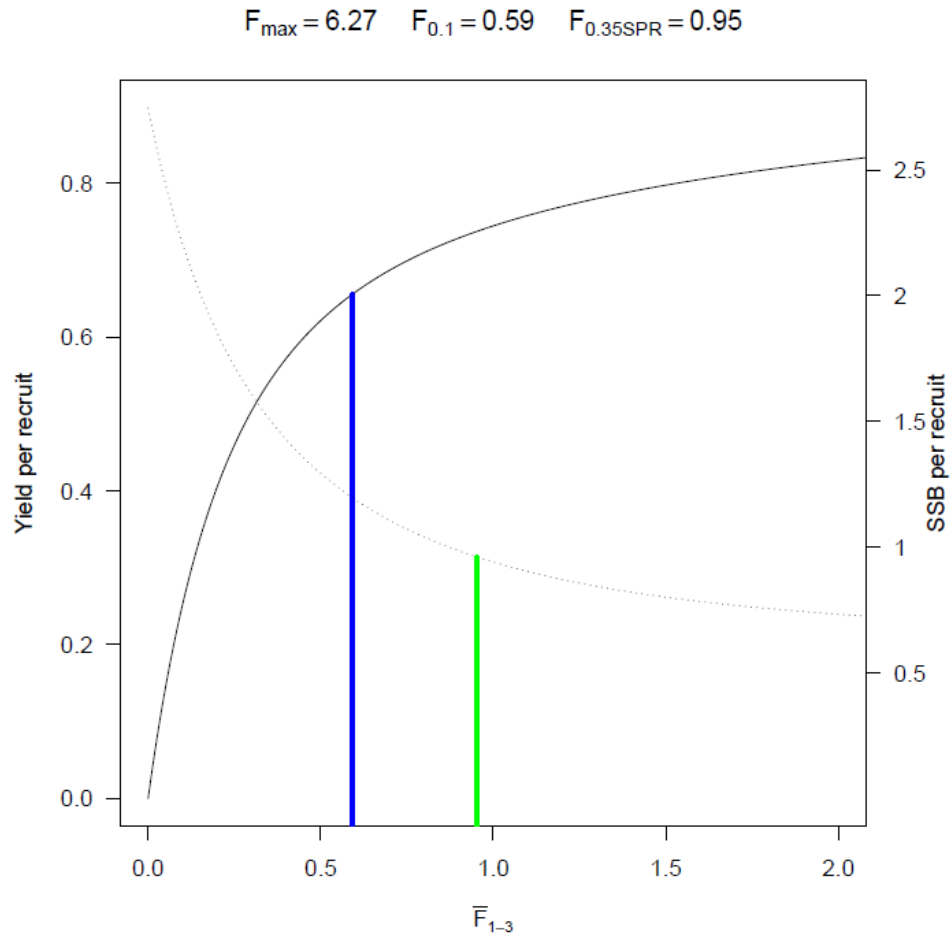


Fig. 1. Yield per recruit and reference point calculation example

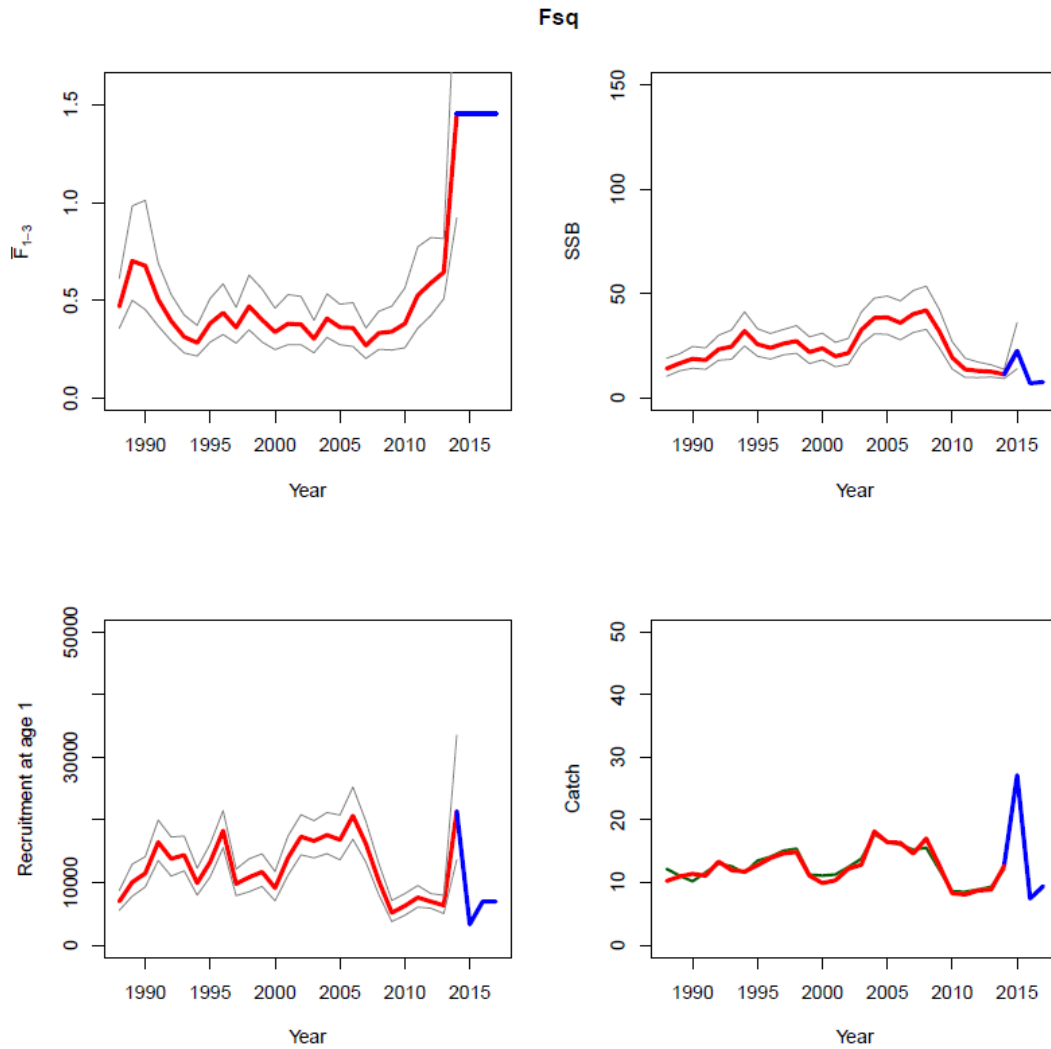


Fig. 2. Short term forecast example calculation

adding the weight-at-lengths  $W_{l,y}$  to get the total catch weight  $CW_y = \sum_l C_{l,y} W_{l,y}$ . As a simplifying approximation the lengths for each cohort is computed one third into the year, which approximates to the point where half of the catch in numbers is taking, when fishing mortality is assumed uniform and within the observed levels.

### 1—Stockassessment.org

All details about the model, its code, and the results can be found at: <http://www.stockassessment.org>

## Results

**Table 1.** Yearly estimates of  $\overline{F}_{1-3}$  and Spawning stock biomass

<b>Year</b>	<b><math>\overline{F}_{1-3}</math></b>	<b>SSB</b>
1988	0.47	14.2
1989	0.7	16.7
1990	0.68	18.9
1991	0.51	18.3
1992	0.39	23.4
1993	0.31	24.7
1994	0.28	32.2
1995	0.38	25.9
1996	0.44	24.1
1997	0.36	26.2
1998	0.47	27.4
1999	0.4	22.1
2000	0.34	23.9
2001	0.38	20.1
2002	0.38	21.6
2003	0.3	32.8
2004	0.41	38.5
2005	0.36	38.7
2006	0.36	36.1
2007	0.27	40.3
2008	0.33	42.1
2009	0.34	32.1
2010	0.38	19.5
2011	0.53	13.8
2012	0.59	13.1
2013	0.64	12.8
2014	1.45	11.4
2015	NA	22.6

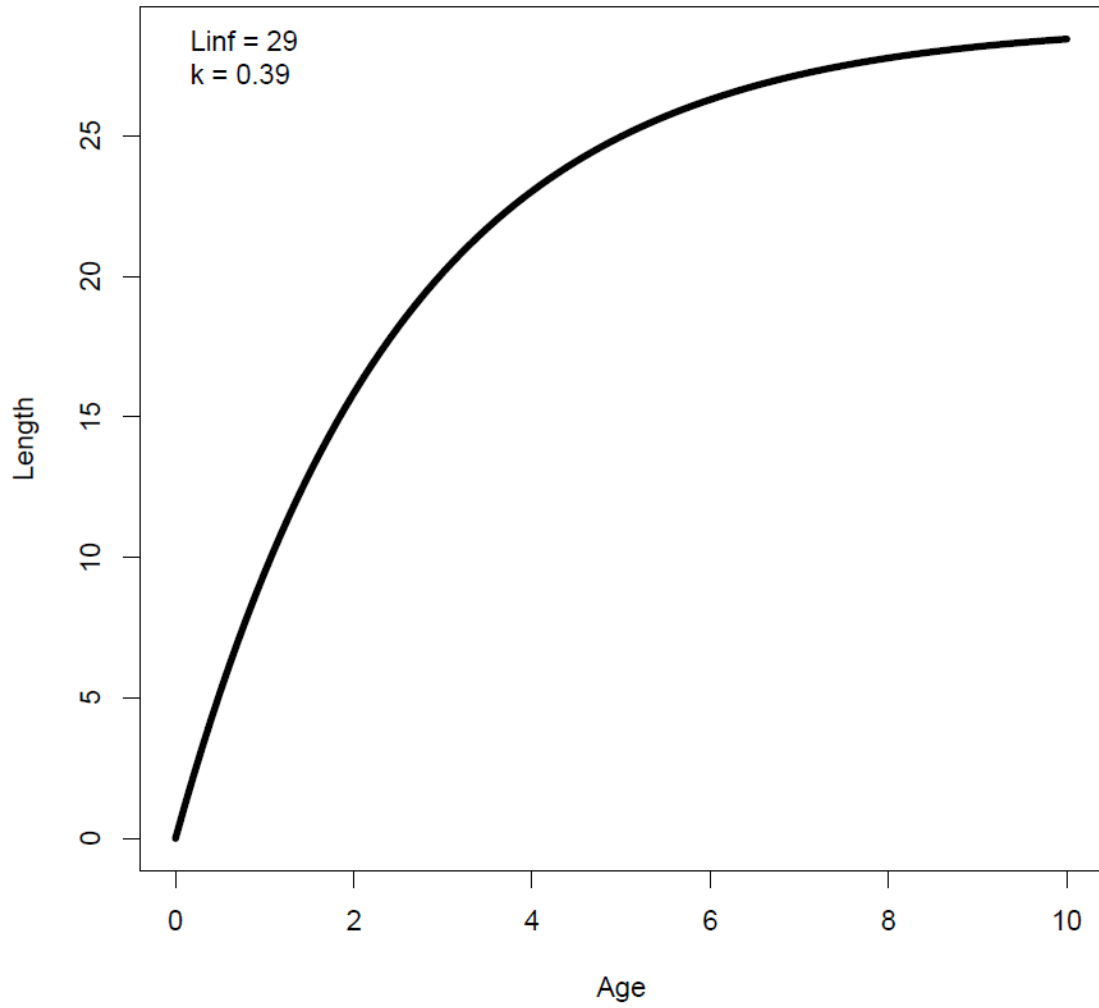


Fig. 3. The estimated relation between age and length.

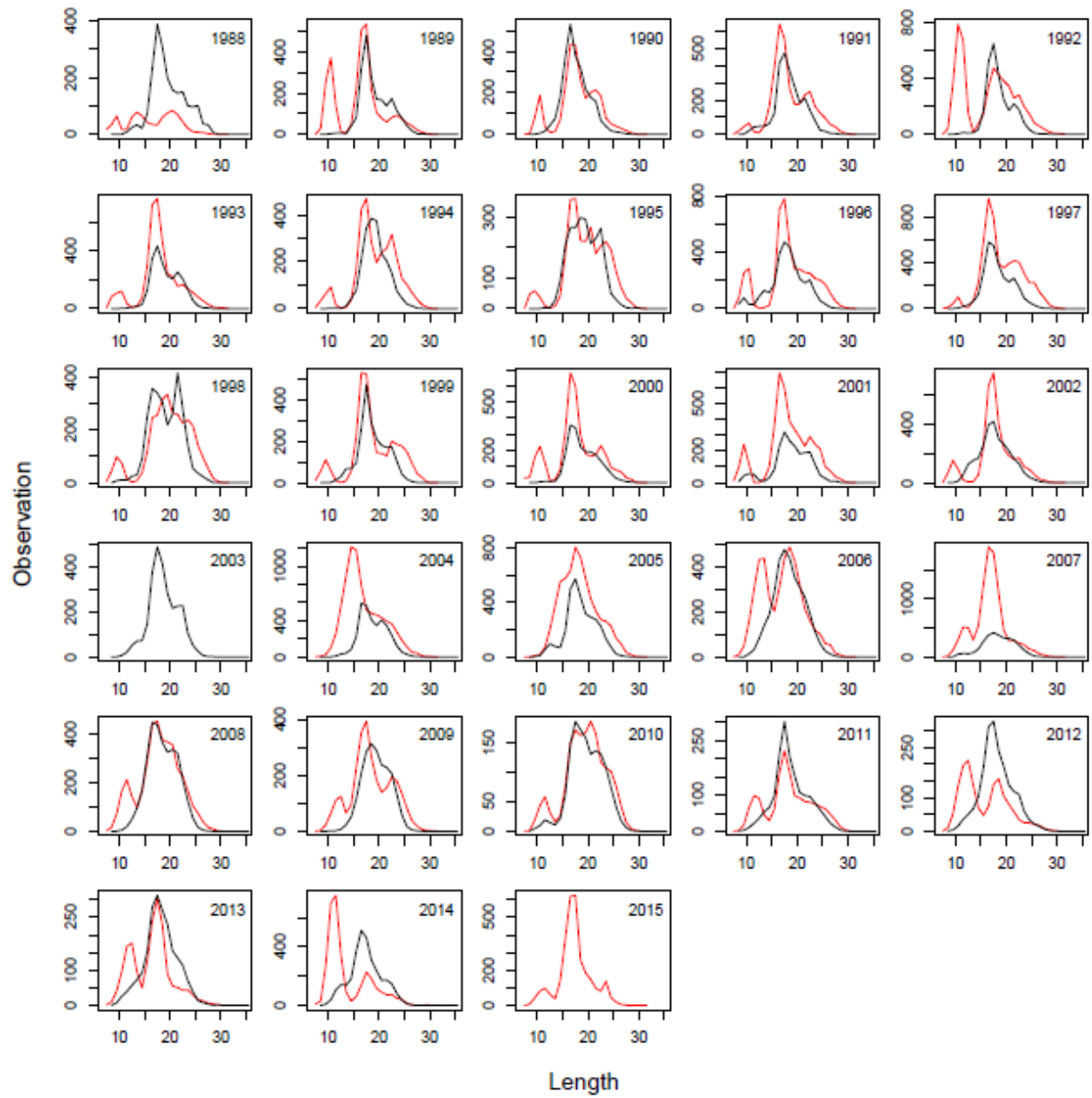


Fig.4. The length observations from surveys (red) and from total catches (black).



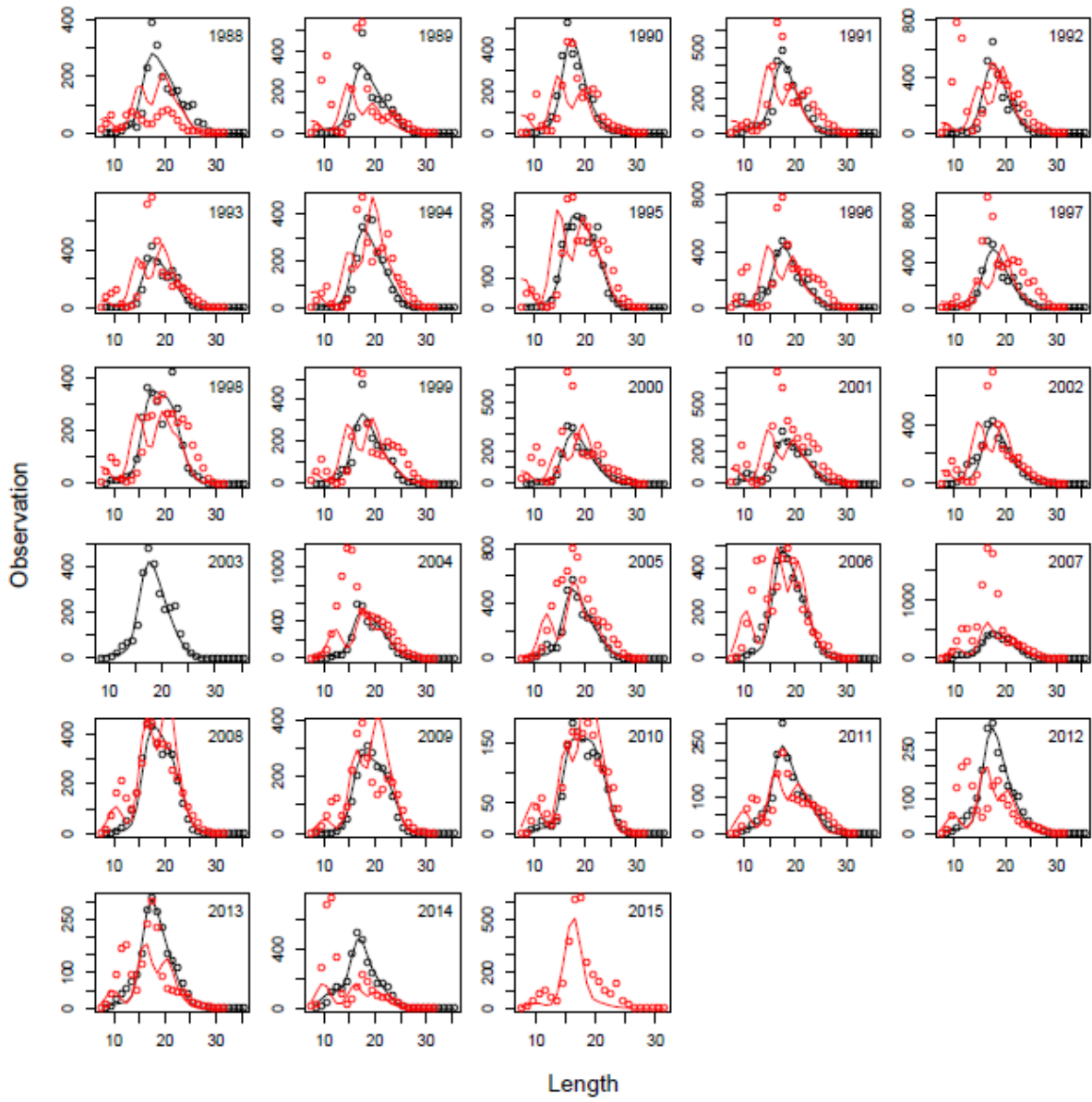


Fig. 5. The length observations from survey (red circles) and total catches (black circles) on top of the predicted length distributions from survey (red lines) and total catches (black lines).

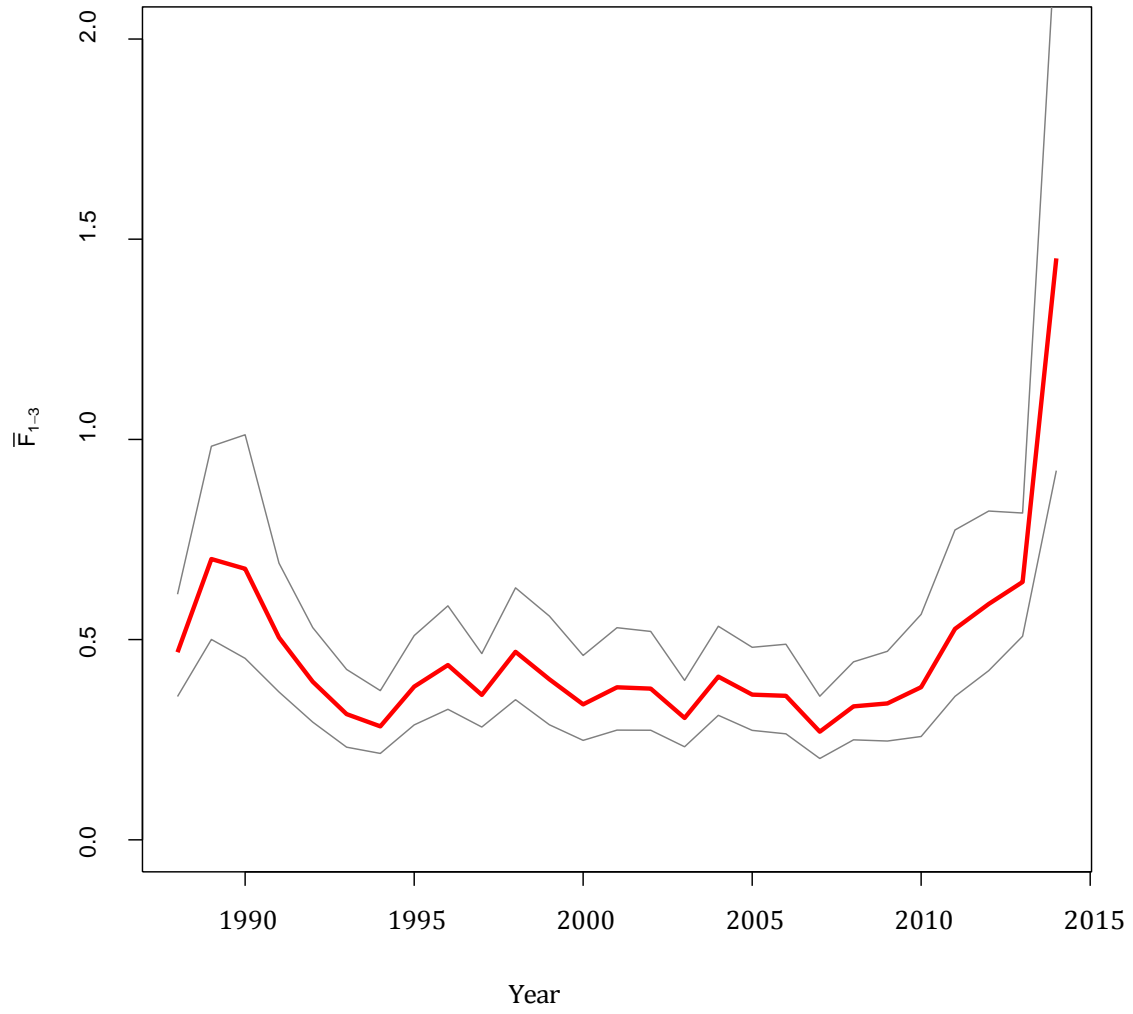


Fig.6. The estimated fishing mortalities (red) compared to results from production model (green)

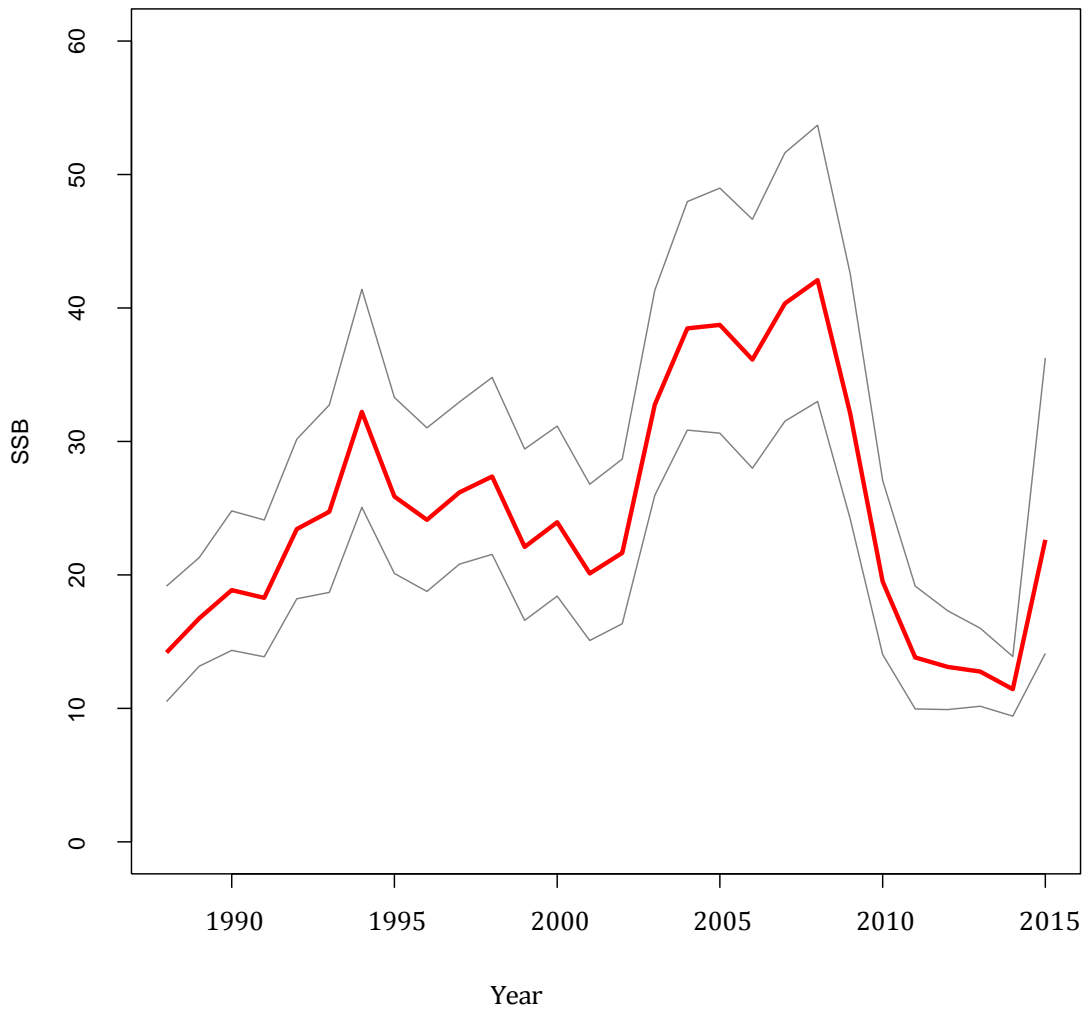


Fig.7. The estimated spawning stock biomass (red) compared to results from production model (green)

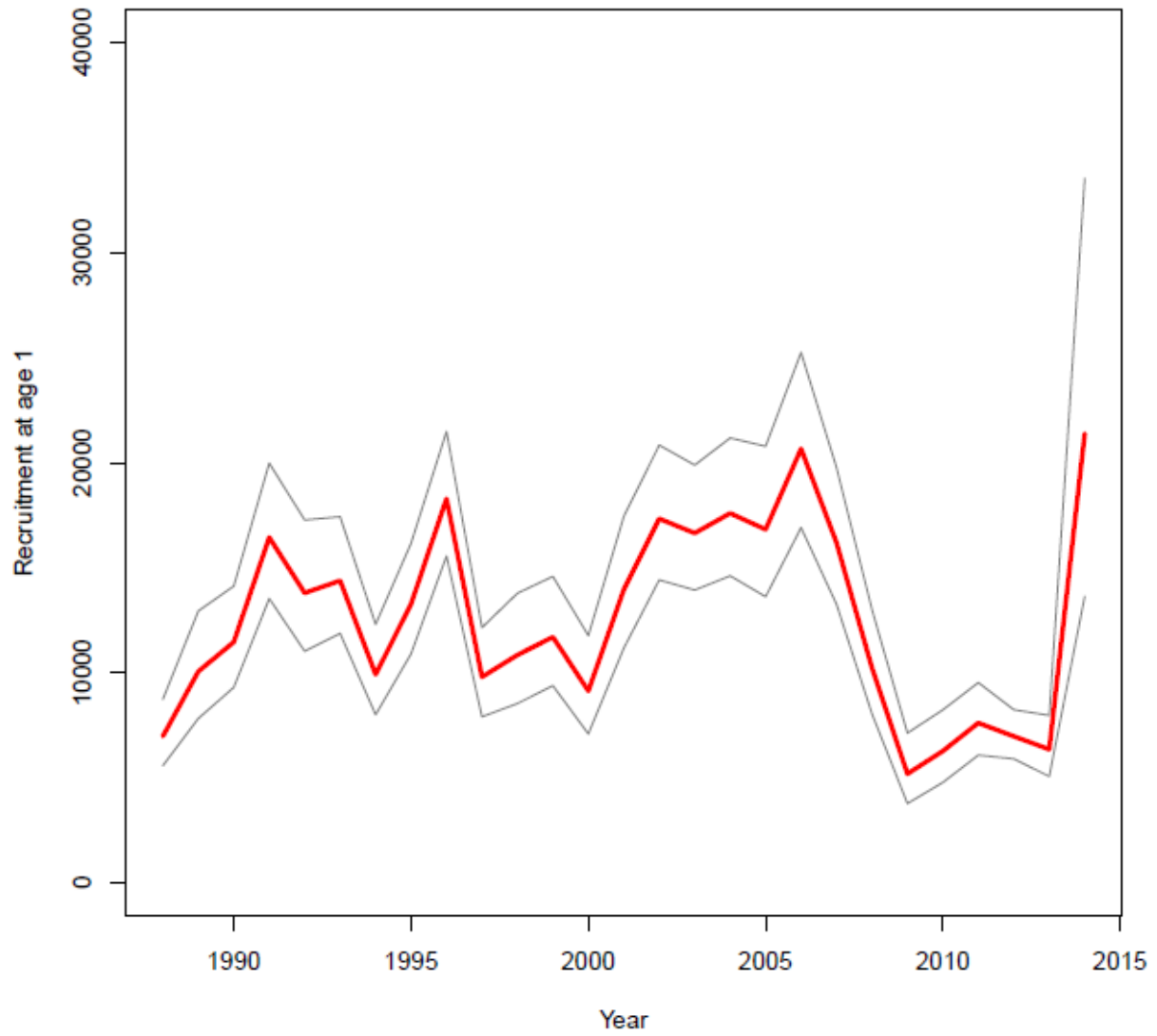


Fig. 8. The estimated recruitment (age class 1)

**Comments**

Growth rate estimated within.

It is noticed that the survey fit is still rather poor. However, this has improved, and the mismatch is somewhat accounted for by the correlation structure.

The catch is fitted well.

Easy to try out due to the web interface.

**Acknowledgement**

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