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**Surplus production model in a Bayesian framework applied to witch flounder in
NAFO Div. 3NO**

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Abstract

The formulation of a surplus production model in a Bayesian framework accepted in 2015 as the basis for advice for witch flounder in NAFO Div. 3NO was updated with data to 2016. The data series included catch from 1960-2016 and three Canadian survey series. There was some indication that model performance was not as good as in 2015 with little updating of the priors for r and K and an increase in process error. Exploratory analyses indicated that there was information in the data with which to estimate r and K and that there was a trend in increasing process error with the addition of data since 2014. The production model estimated that an MSY of 3641 t can be taken from a biomass of 50 000 t at a fishing mortality of 0.07. Intrinsic rate of natural increase is estimated to be 0.14 and carrying capacity 100 000 t. These parameters are similar to the estimates from 2015. The population is estimated to have declined from a high in 1966 to low levels in the mid to late 1990s. The biomass generally increased to 2013 and has since declined (Figure 6). In 2016 there is a probability of 0.14 that the stock is below B_{lim} . Fishing mortality has been below F_{MSY} since the mid 1990s. However, F has been increasing since 2012 and in 2016 is estimated to have a probability of 0.19 of being above F_{MSY} .

Key words: Bayesian surplus production model, Div. 3NO witch flounder, assessment

Introduction

The directed witch flounder fishery in Div. 3NO was reopened in 2015 with a TAC of 1000 t. This decision was based on advice developed from an assessment based on survey trends. In 2015, Scientific Council accepted a surplus production model in a Bayesian framework as the basis for the advice for witch flounder in Div. 3NO. The model was used to evaluate the status of the stock relative to precautionary reference points and to provide catch advice for 2016 and 2017.

This paper provides an update of the assessment incorporating data for 2015 and 2016. This is the 'preferred' model in Morgan et al (2015).



Methods

The Schaefer (1954) form of a surplus production model used here is:

$$P_t = [P_{t-1} + r \cdot P_{t-1} (1 - P_{t-1}) - C_{t-1}/K] \cdot \eta_t$$

where P_{t-1} and C_{t-1} denote exploitable biomass (as a proportion of carrying capacity) and catch, respectively, for year $t-1$ (Meyer and Millar, 1999a, 1999b). Carrying capacity, K , is the level of stock biomass at equilibrium prior to commencement of a fishery, r is the intrinsic rate of population growth, and η_t is a random variable describing stochasticity in the population dynamics (process error). The model utilizes biomass proportional to an estimate of K in order to aid mixing of the Markov Chain Monte Carlo (MCMC) samples and to help minimize autocorrelation between each state and K (Meyer and Millar, 1999a, 1999b).

An observation equation is used to relate the unobserved biomass, P_t , to the research vessel survey indices:

$$I_t = q \cdot P_t \cdot \epsilon_t$$

where q is the catchability parameter, P_t is an estimate of the biomass proportional to K at time t , and ϵ_t is observation error.

Input data are given in Table 1. All priors were the same as those used in the 2015 assessment.

The prior on r was informed by that derived by Swain 2012 for witch flounder in the southern Gulf of St. Lawrence. The prior used here allowed for a higher r than derived by Swain (2012) as some of the morphometric methods indicated a higher r . Therefore the mean (0.17) derived by Swain (2012) was used as the central tendency (i.e. the median) but with a larger standard deviation.

A mean of 0.2 and standard deviation of 0.12 gives a median of 0.17 on the log normal scale. The prior used therefore was:

$$R \sim (-1.763, 3.252)$$

The prior for K was based on Ecosystem Production Potential modelling (NAFO 2014). This modelling indicated that a reasonable distribution for K would have a mean of 100 and a standard deviation of 30.

$$K \sim \text{dlnorm}(4.562, 11.6)$$

The priors on survey q and observation error were:

$$pq \sim \text{dgamma}(1, 1)$$

$$q \sim 1/pq$$

$$\tau \sim \text{dgamma}(1, 1)$$

$$\text{itau} \sim 1/\tau$$

For process error:

$\sigma \sim \text{dunif}(0,10)$

Results and Discussion

Posteriors for r and K are updated from their priors but much more similar to their priors than in the 2015 run of the model (Figure 1). Other posteriors were substantially updated during the model run (Figures 1, 2, 3).

There is some trend in process error, particularly in the most recent years (Figure 4). This may be related to the decline in survey biomass being larger than the catch can explain.

Model fit to the survey data was good for all surveys, although the model over estimates the level of the last two fall survey data points (Figure 5).

All convergence diagnostics (Appendix 1) indicated that there were no issues with model convergence.

The change in trend in biomass, increase in process error and the updating of the priors on r and K were explored further.

An intermediate year run, using data to the end of 2015, was conducted with the same priors as described above and the Bratio and process error compared to models using data to the end of 2014 (the last assessment) and the end of 2016 (the 2017 assessment).

The intermediate year run showed that the change in Bratio and process error increased as more years of data were added. Bratio showed the population increasing steadily from the mid 1990's to 2014 when data to the end of 2014 are used (Figure 6). With the addition of the 2015 data, the population is estimated to have declined between 2013 and 2014. When data for 2016 are added, this decline between 2013 and 2014 is greater. Process error also increases as each year of data were added (Figure 7). The amount of change in process error between 2014 and 2015 may have been impacted by the fact that the 2014 fall survey was not included in the data as coverage was incomplete. The addition of the fall 2015 and 2016 survey results indicates that the decline in this index began after 2012 and the 2014 fall survey index for 3NO witch was likely lower than the 2013 estimate (see Table 1). These analyses show that it was not simply the addition of the data in 2016 that led to the change in perception of stock size but rather perception changed with each additional year as the model tracked the change in survey indices.

The posteriors for r and K in the model run using data to the end of 2016 were similar to their priors. To explore if there was information in the data to allow the estimation of r and K , a run with a different prior on r was conducted. This run had a prior for r with mean of 0.3 and standard deviation of 0.12. Figure 8 shows a comparison of the two priors. The posterior for r was updated from the prior in the exploratory run with the posterior shifted to the left, away from the exploratory prior and towards the posterior resulting from the prior used in the assessment run (Figure 9). The use of

a different prior on r also resulted in updating of the posterior for K (Figure 10). These results indicate that there is information in the data with which to estimate r and K .

The production model estimated that an MSY of 3641 t can be taken from a biomass of 50 000 t at a fishing mortality of 0.07. Intrinsic rate of natural increase is estimated to be 0.14 and carrying capacity 100 000 t. These parameters are similar to the estimates from 2015 (Table 2).

The population is estimated to have declined from a high in 1966 to low levels in the mid to late 1990s. The biomass generally increased to 2013 and has since declined (Figure 11). In 2016 there is a probability of 0.14 that the stock is below B_{lim} .

Fishing mortality was at its highest levels (and above F_{msy}) from the mid 1980s to the mid 1990s (Figure 12). Since then fishing mortality has been below F_{msy} . However, F has been increasing since 2012 and in 2016 is estimated to have a probability of 0.19 of being above F_{msy} .

Acknowledgments

The Bayesian surplus production models are based on programs originally developed by Jason Bailey.

References

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Table 1. Data used in the Bayesian Surplus Production model. Values are in thousands of tons.

Year	Landings	Spring late	Fall	Spring early
1960	5.799			
1961	4.627			
1962	1.228			
1963	2.183			
1964	1.066			
1965	2.177			
1966	7.522			
1967	11.503			
1968	10.599			
1969	4.7			
1970	6.763			
1971	14.965			
1972	9.177			
1973	6.691			
1974	8.045			
1975	6.168			
1976	6.035			
1977	5.759			
1978	3.473			
1979	3.077			
1980	2.42			
1981	2.425			
1982	3.732			
1983	3.616			
1984	2.802			14.313
1985	8.771			24.581
1986	9.131			9.214
1987	7.596			11.199
1988	7.325			24.655
1989	3.688			8.988
1990	4.179		15.368	10.759
1991	4.847	7.07	5.477	
1992	4.96	8.217	9.118	
1993	4.414	4.226	9.474	
1994	1.119	16.279	7.821	
1995	0.3	4.057	11.743	
1996	0.358	4.085	12.278	
1997	0.512	7.133	4.691	
1998	0.612	2.688	6.689	

1999	0.763	8.936	13.33
2000	0.545	5.49	7.64
2001	0.694	9.418	7.021
2002	0.45	7.562	11.13
2003	1.544	15.855	10.315
2004	0.627	11.825	18.632
2005	0.257	6.865	18.132
2006	0.481		14.605
2007	0.222	7.189	7.715
2008	0.264	8.825	22.739
2009	0.376	9.179	37.708
2010	0.421	6.639	27.039
2011	0.351	9.746	17.939
2012	0.314	12.844	27.033
2013	0.328	24.396	17.668
2014	0.335	10.702	
2015	0.359	4.927	10.101
2016	1.062	7.134	7.869

Table 2. Parameter estimates from the 2017 surplus production model for Div. 3NO witch flounder compared to the model run in 2015. Weights are in thousands of tonnes.

	2017	2015
r	0.142 (0.063-0.372)	0.126 (0.08-0.244)
K	100.00 (59.62-164.7)	119.4 (74.3-165.3)
MSY	3.641 (1.65-7.23)	3.763 (2.42-5.83)
Bmsy	50.01 (29.81-82.35)	59.68 (37.15-82.63)
Fmsy	0.071 (0.032-0.186)	0.063 (0.039-0.122)

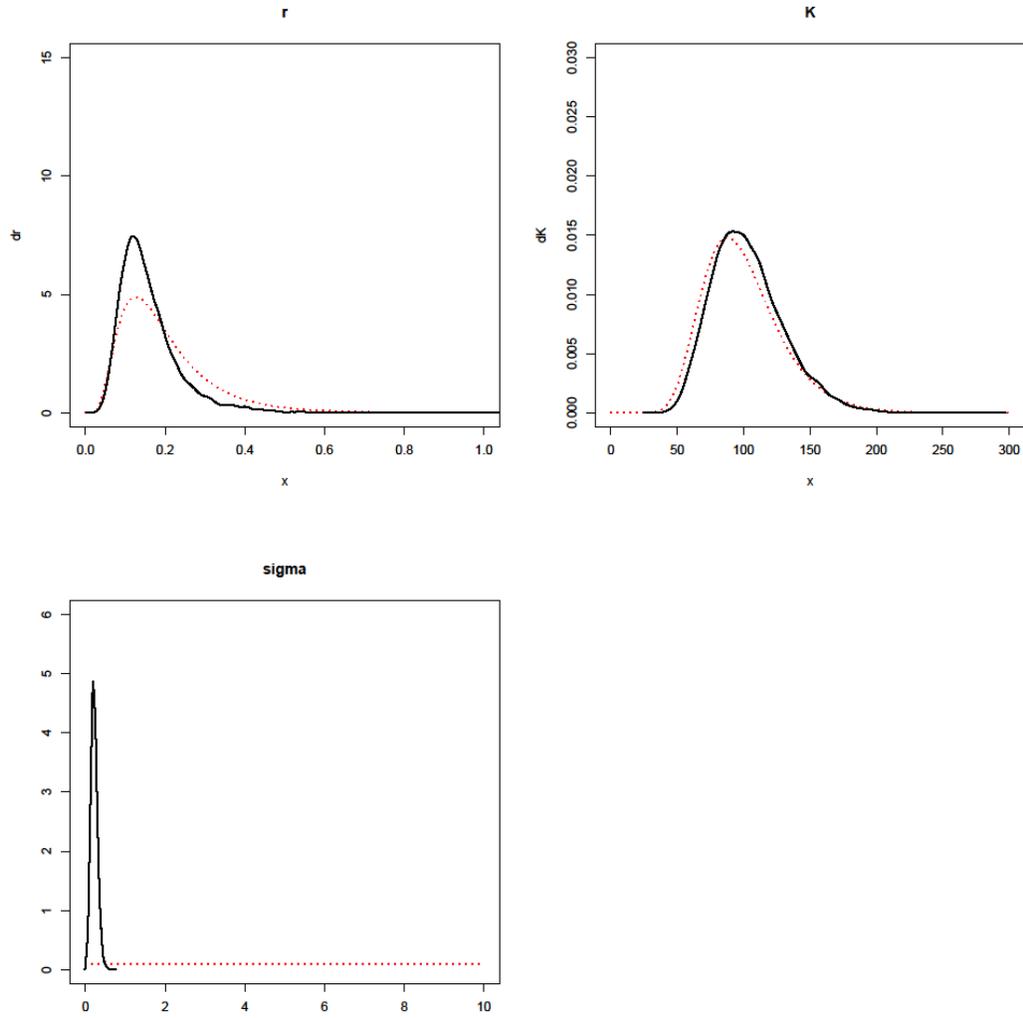


Fig. 1. Priors (red dotted line) and posteriors (black line) for r , K and σ (process error).

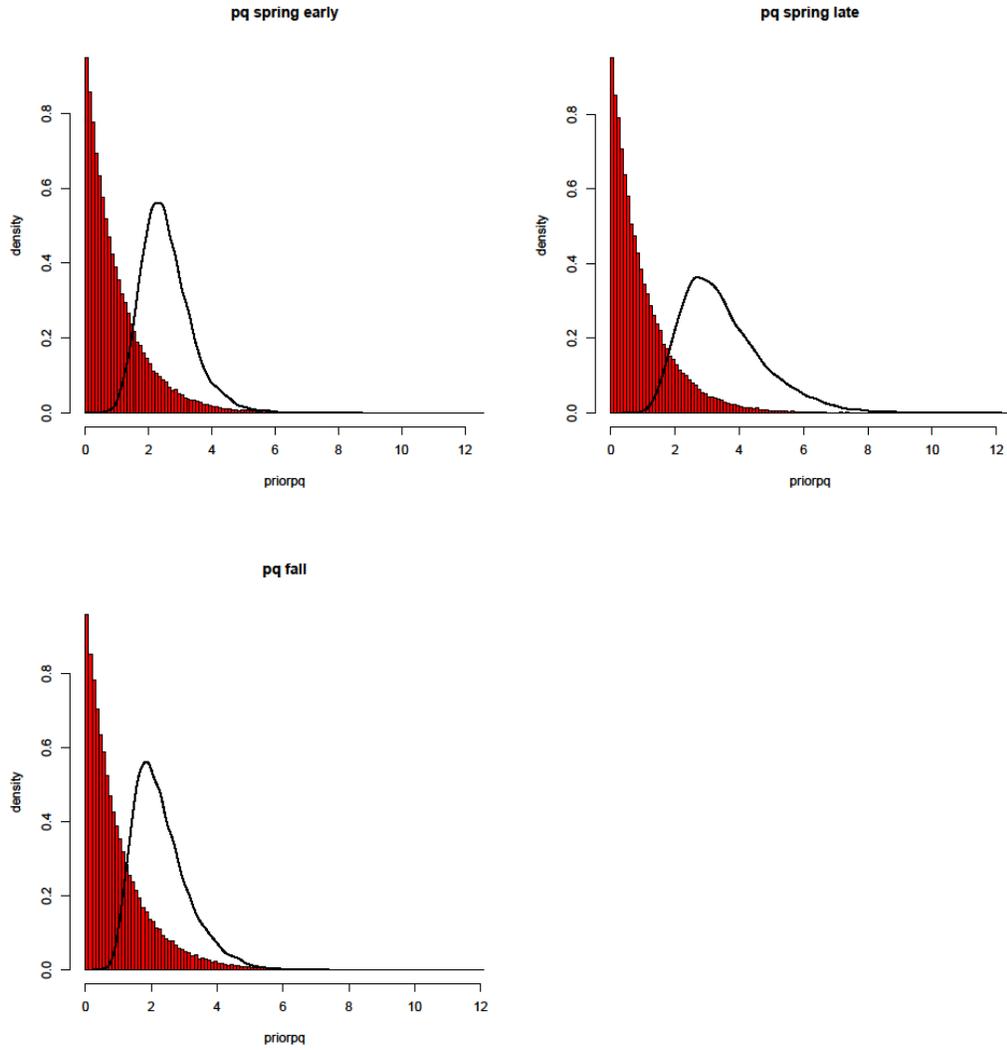


Fig. 2. Priors (red histogram) and posteriors (black lines) for pq (inverse of q) for the 3 survey indices used in the model.

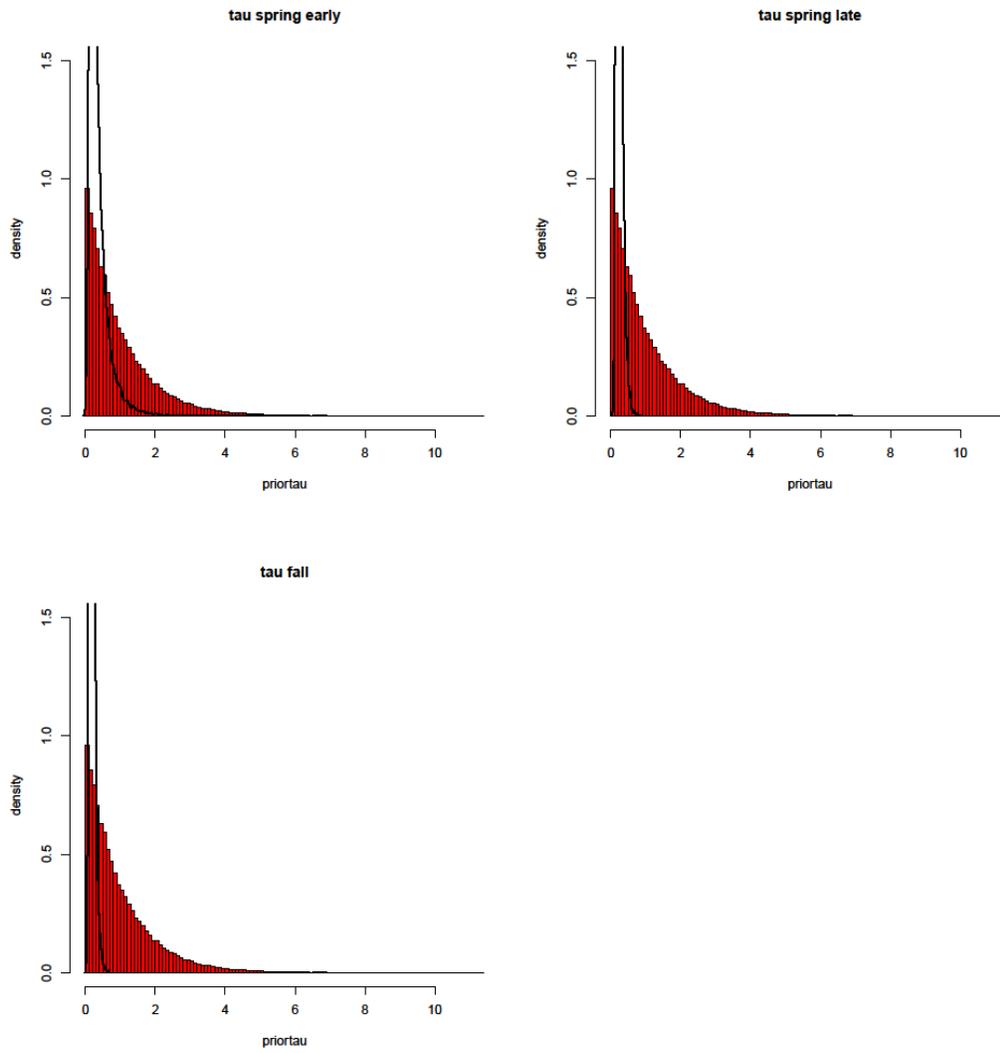


Fig. 3. Priors (red histograms) and posteriors (black lines) for observation error on surveys used in the model.

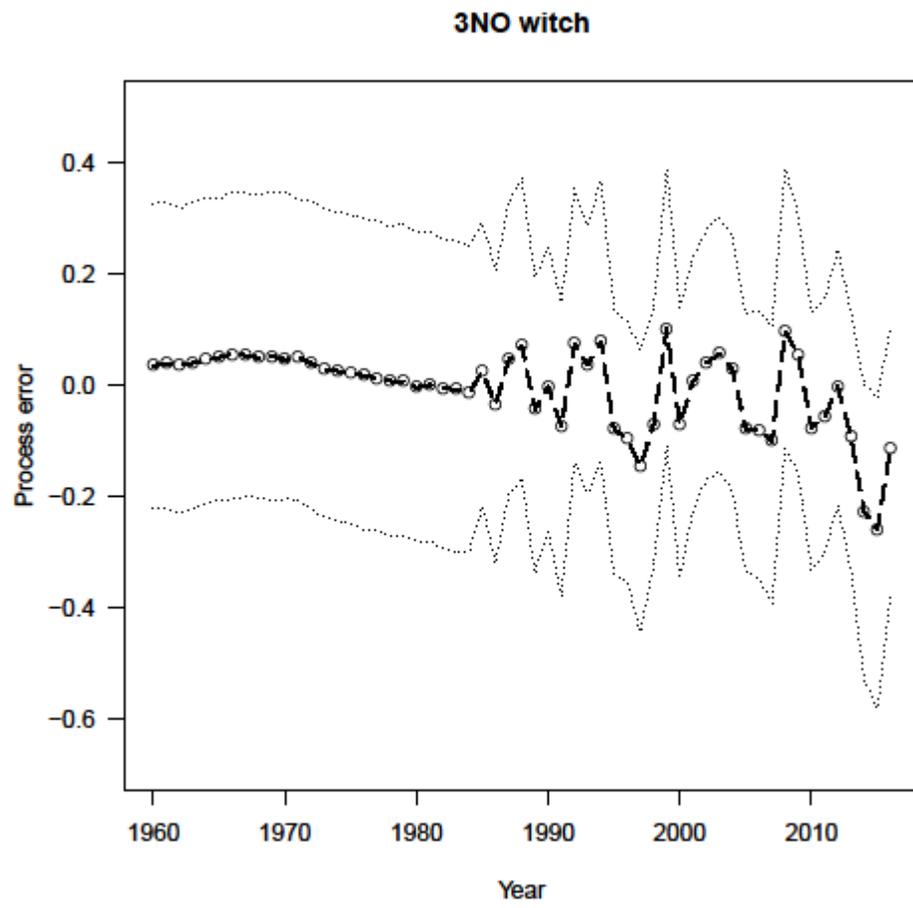


Fig. 4. Process error with 80th percent credible intervals.

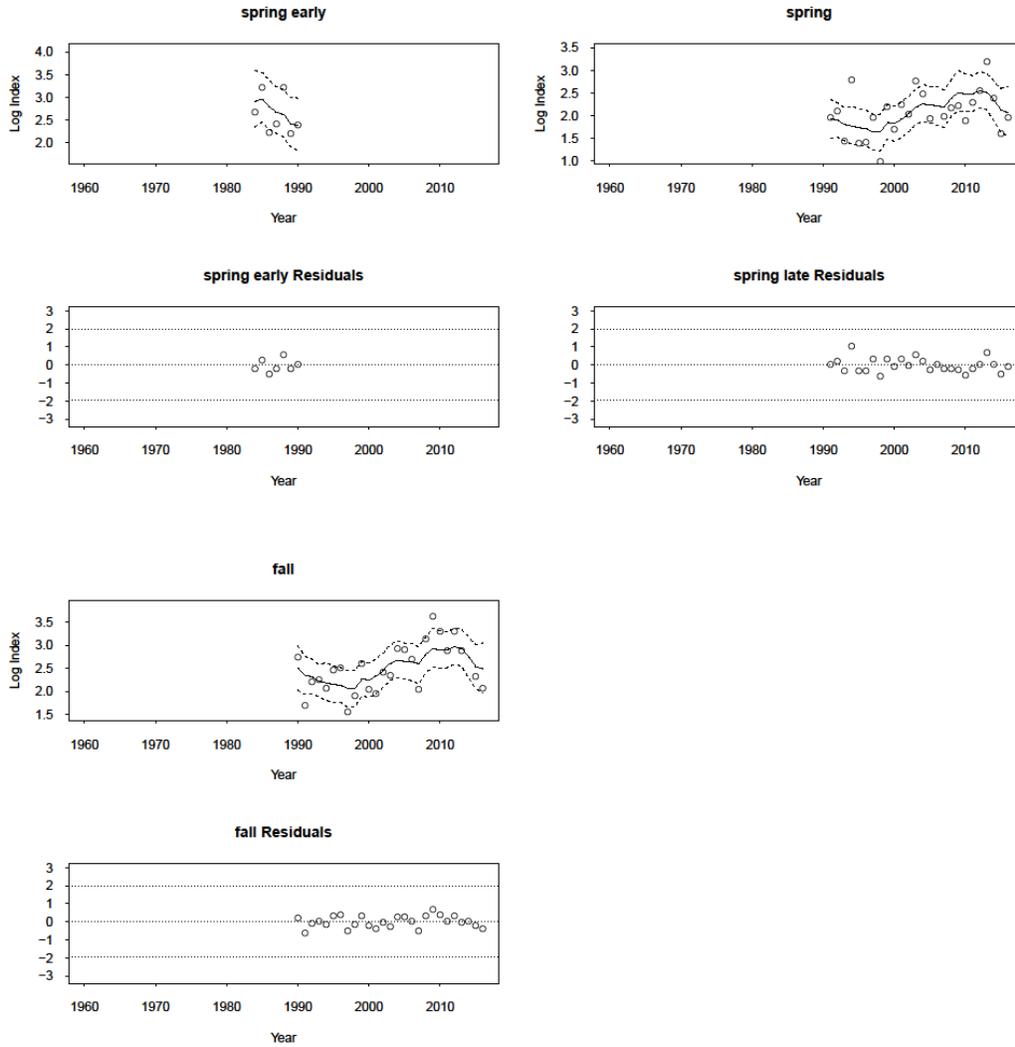


Fig. 5. Observed and predicted survey indices from each of the three surveys used in the model. For each survey the top panel gives the observed and predicted values with 95th credible intervals while the bottom panel presents standardized residuals.

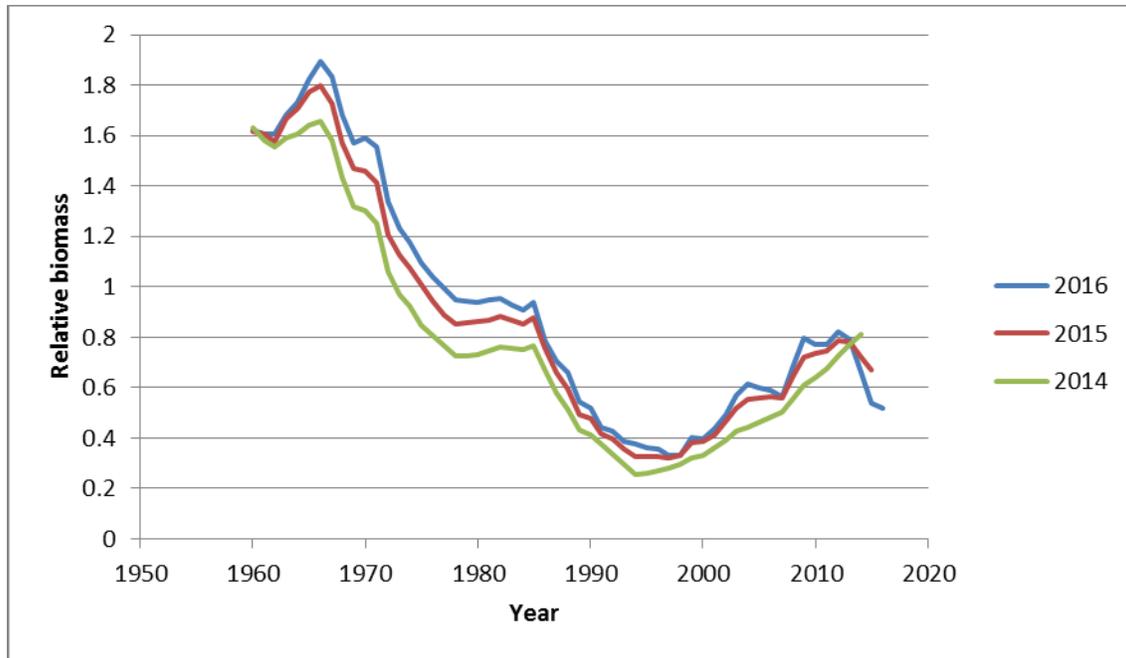


Fig. 6. Relative biomass (Biomass/Bmsy) for 3 runs of the surplus production model. All runs had the same priors but used data up to the end of 2014 (the last assessment), end of 2015 or end of 2016 (the 2017 assessment).

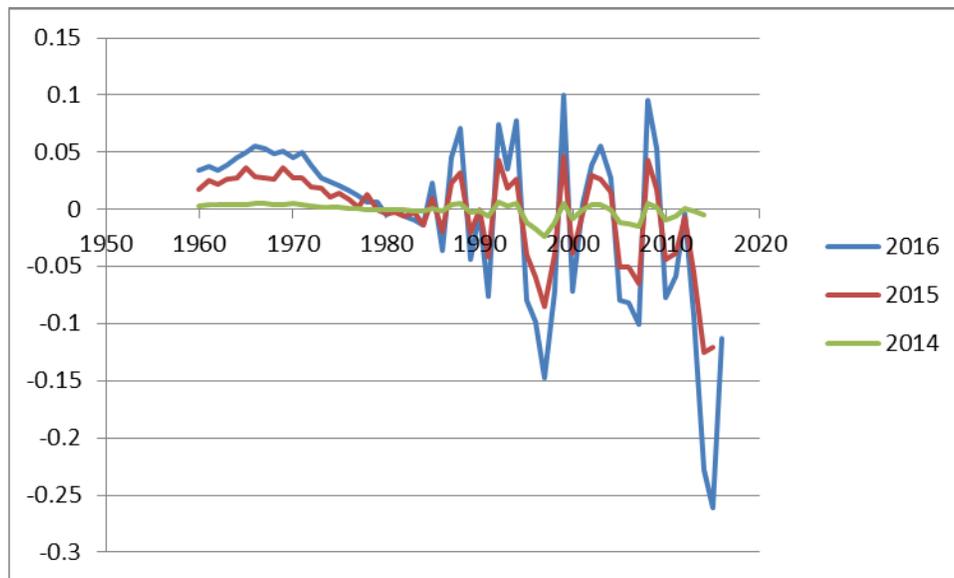


Fig. 7. Process error for 3 runs of the surplus production model. All runs had the same priors but used data up to the end of 2014 (the last assessment), end of 2015 or end of 2016 (the 2017 assessment).

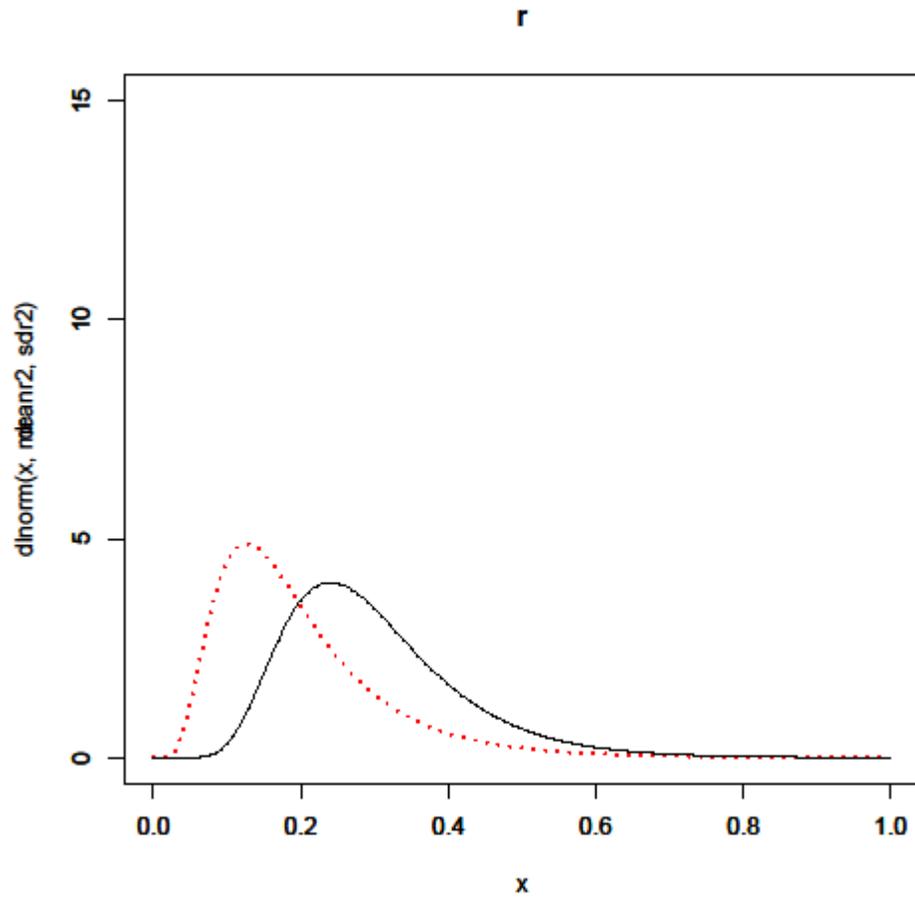


Fig. 8. Two priors on r used in the model runs. The red dotted line is the prior used in the assessment model while the black solid line is the prior used in the exploratory run.

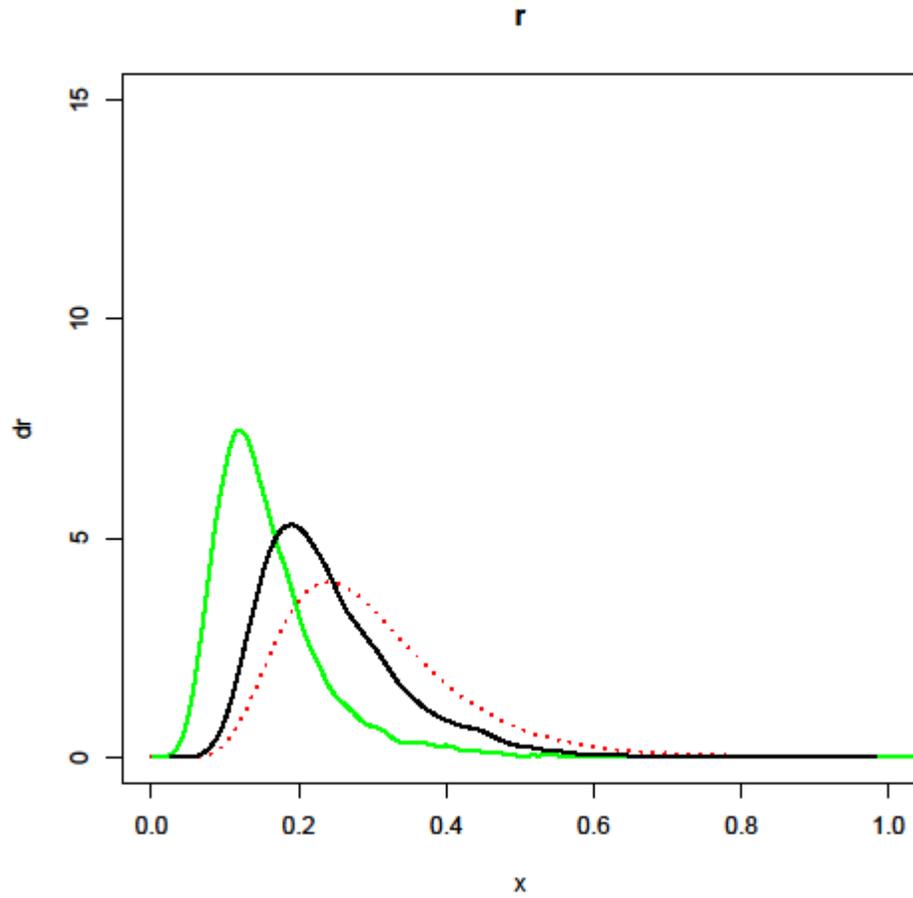


Fig.9. Priors and posteriors for r . The red dotted line is the 'test' prior used in the exploratory run. The black solid line is the posterior for r from that run and the green solid line is the posterior for r from the assessment run.

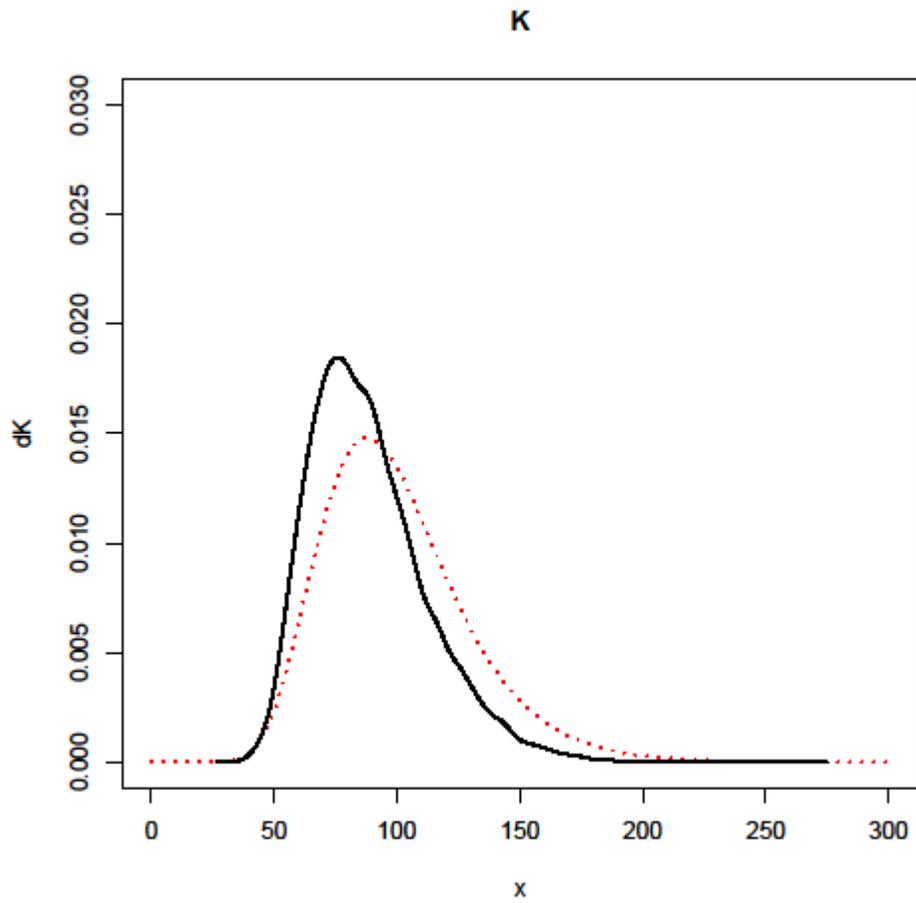


Fig. 10. Prior (dotted red line) and posterior (black solid line) for K in the exploratory run using a different prior on r than used in the assessment.

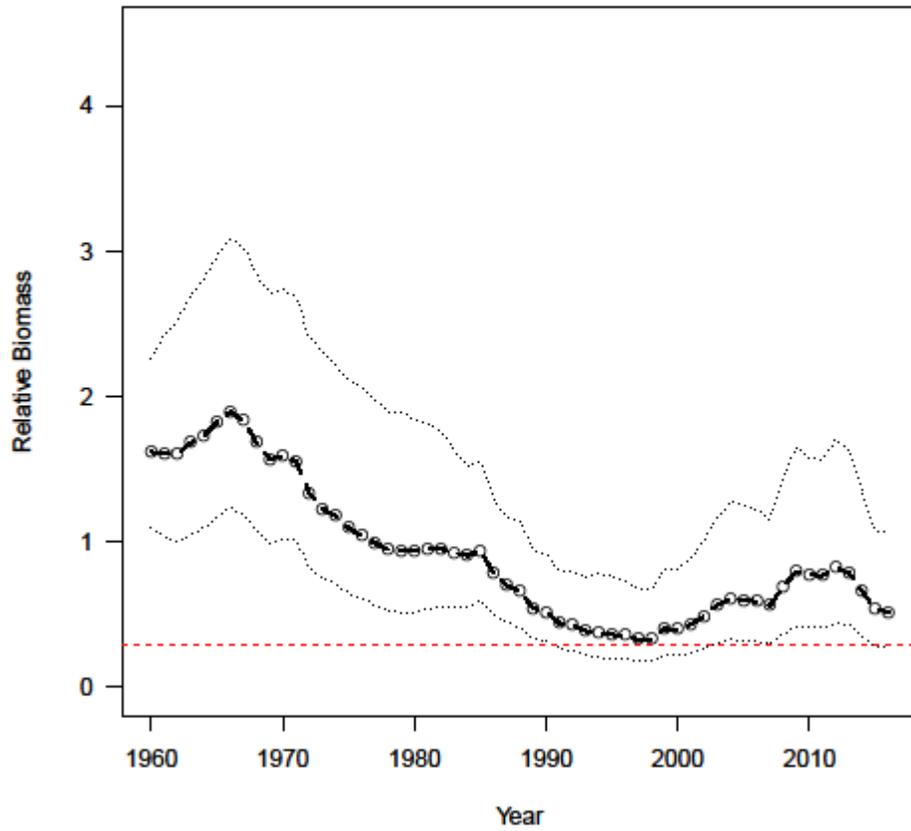


Fig. 11. Relative biomass (biomass divided by B_{MSY}) for Div. 3NO witch flounder. The median with its 80th percent credible intervals are shown. The horizontal red dashed line is B_{lim} (30% B_{MSY}).

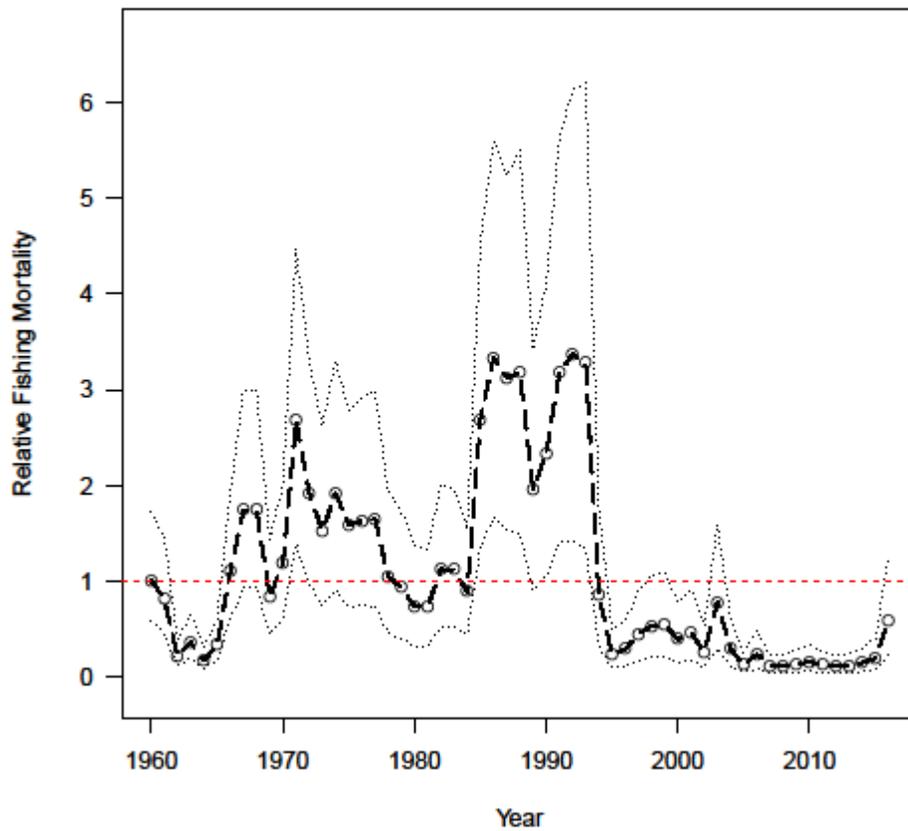


Fig. 12. Relative fishing mortality (fishing mortality divided by F_{MSY}) for Div. 3NO witch flounder. The median with its 80th percent credible intervals are shown. The horizontal red dashed line is F_{lim} (F_{MSY})

Appendix 1

Convergence Diagnostics R

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1592	0.0787	0.0011	0.00117	0.0012	0.0018	0.0622	0.1417	0.3697	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1590	0.0784	0.0011	0.0012	0.0012	-0.0016	0.0646	0.1411	0.3718	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.1615	0.0782	0.0011	0.0015	0.0015	-0.1064	0.0629	0.1432	0.3725	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000415

Multivariate Potential Scale Reduction Factor = 1.000733

Corrected Scale Reduction Factors

Estimate 0.975

x 1.001366 1.003087



GEWEKE CONVERGENCE DIAGNOSTIC:

=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.2830719

p-value 0.7771217

Chain: witchchain2

Z-Score -0.8856289

p-value 0.3758176

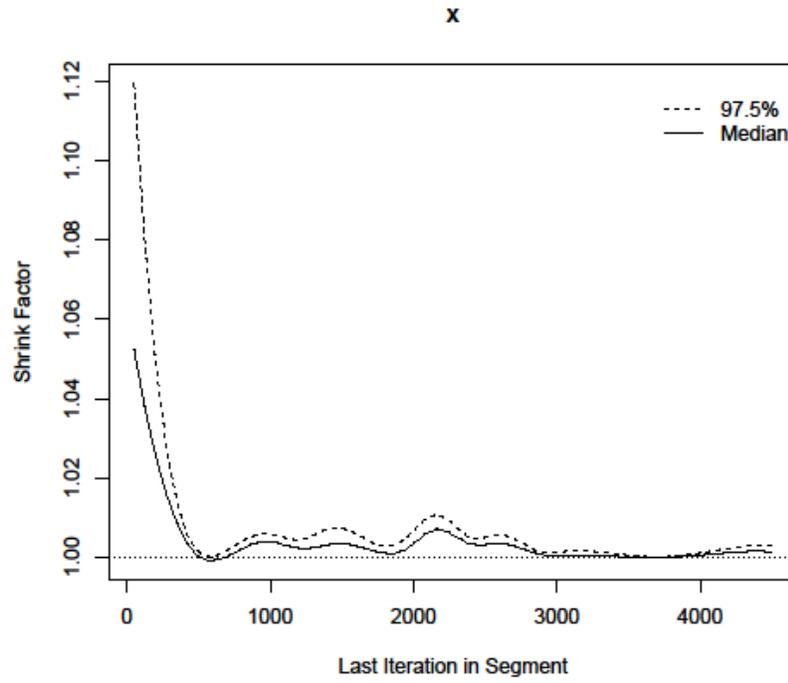
Chain: witchchain3

Z-Score 0.6186867

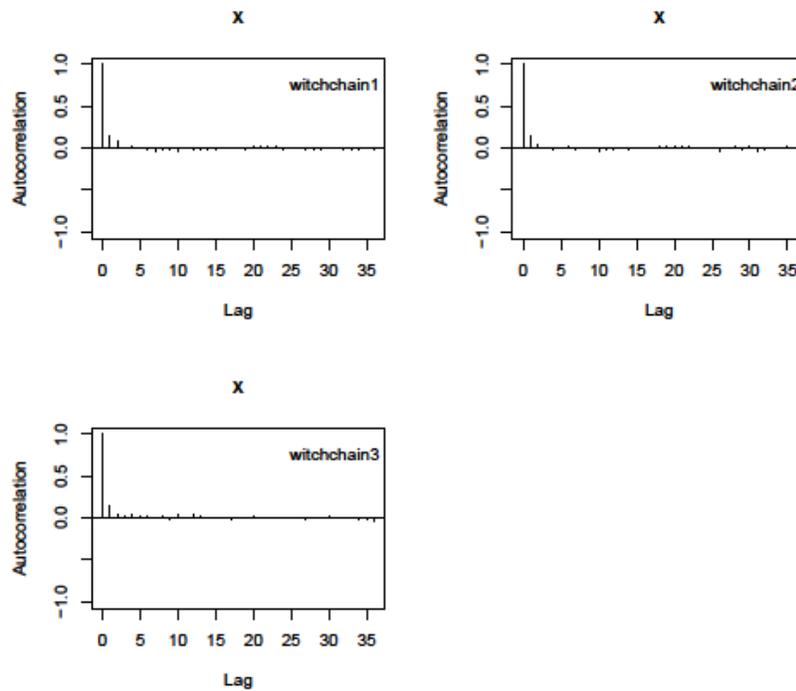
p-value 0.5361228



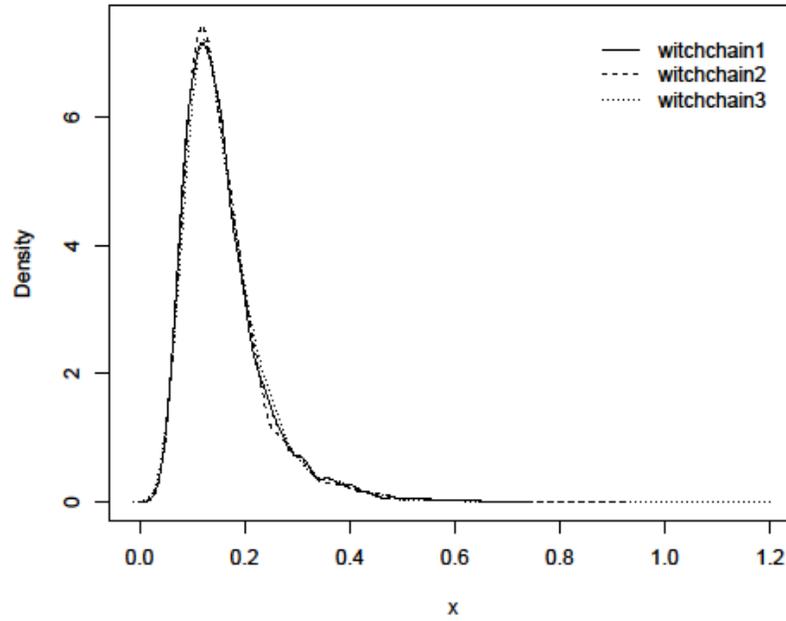
Gelman & Rubin Shrink Factors



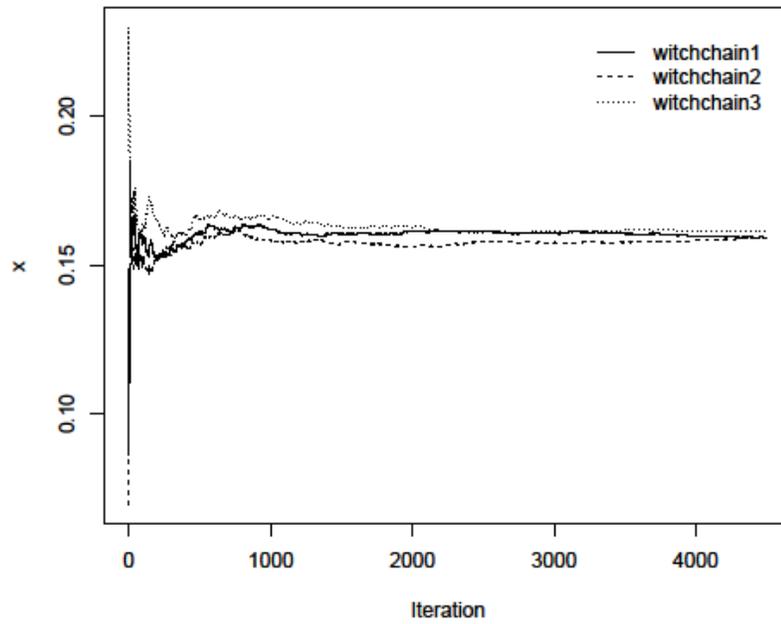
Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean



Convergence Diagnostics K

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
103.1192	27.3046	0.4070	0.3867	0.4775	-0.1392	59.3995	99.98	164.8525	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
103.6034	26.7594	0.3989	0.4170	0.4567	-0.0170	60.38	100.5	162.505	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
102.763	27.2099	0.4056	0.5198	0.5029	-0.1019	59.3847	99.58	166.1575	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

=====

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

0.9999686

Multivariate Potential Scale Reduction Factor = 1.000064

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000029 1.000542



GEWEKE CONVERGENCE DIAGNOSTIC:

=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.01764682

p-value 0.98592061

Chain: witchchain2

Z-Score 0.3674018

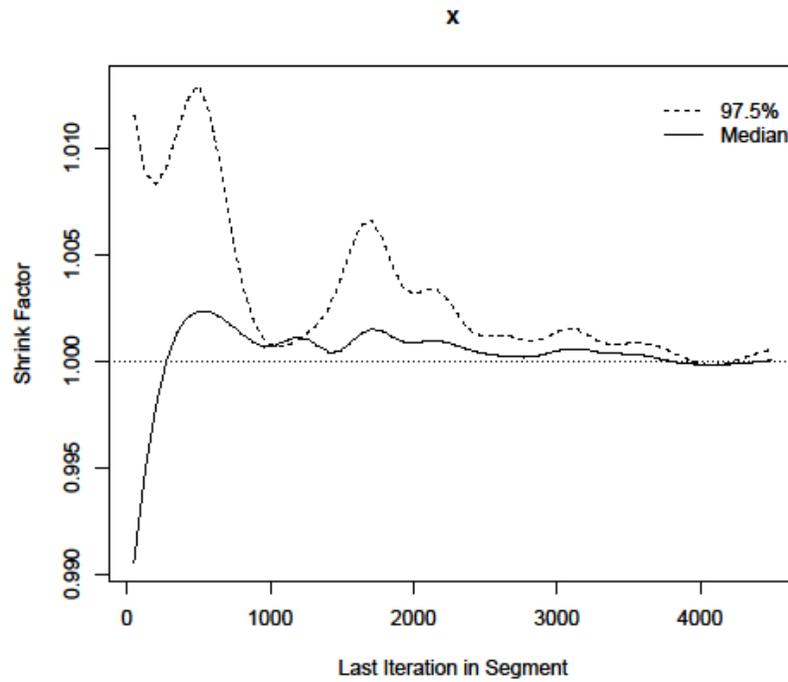
p-value 0.7133194

Chain: witchchain3

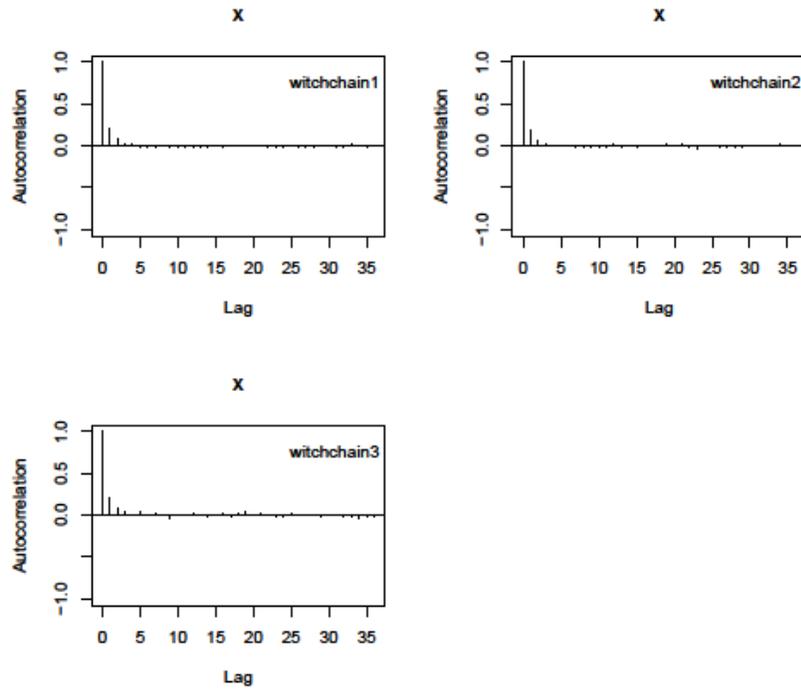
Z-Score -0.5179610

p-value 0.6044855

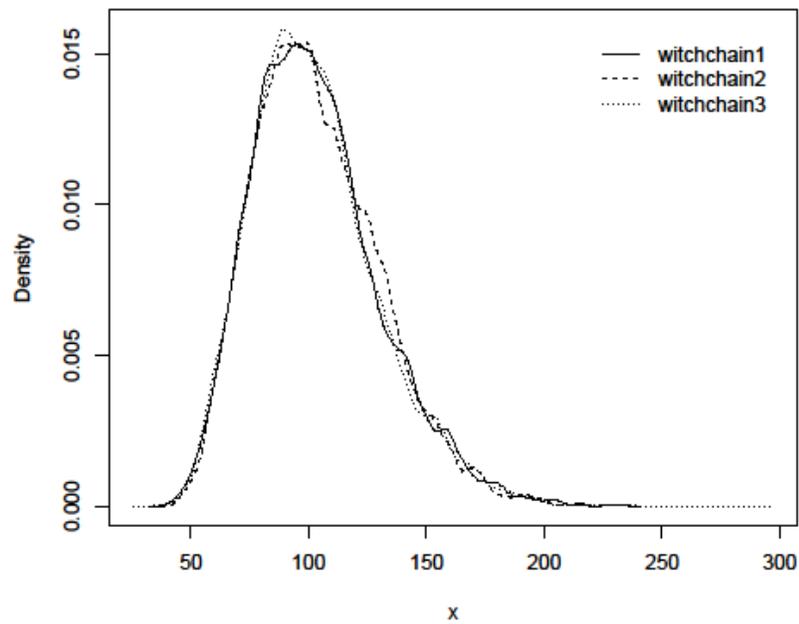
Gelman & Rubin Shrink Factors



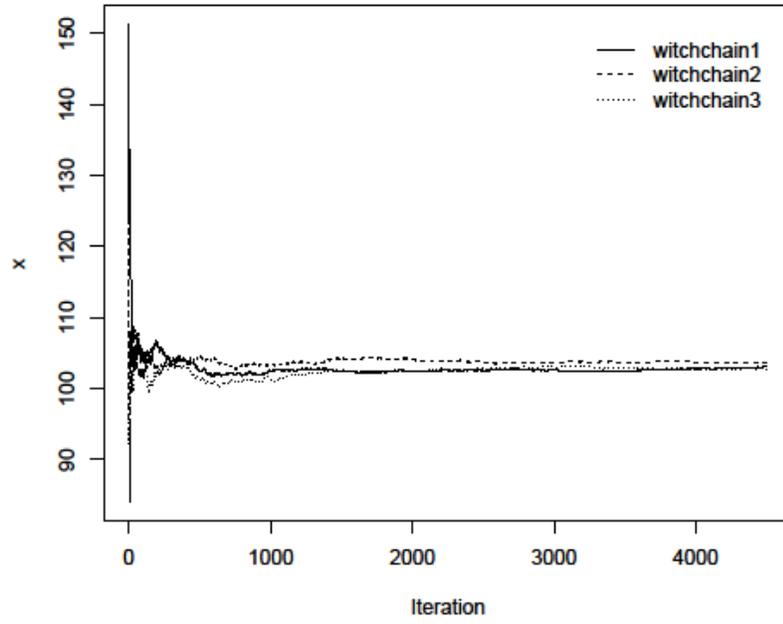
Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean



Convergence Diagnostics Sigma

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.2221545	0.08734775	0.001302103	0.001652874	0.00137509	-0.0149078	0.06645075	0.21695	0.4086525	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.218512	0.08591399	0.00128073	0.001465643	0.001410811	0.0762038	0.06753875	0.21185	0.4091	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.2218868	0.08740085	0.001302895	0.001467042	0.001359817	0.006826789	0.07119625	0.2141	0.411505	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000623

Multivariate Potential Scale Reduction Factor = 1.001045

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000947 1.003219



GEWEKE CONVERGENCE DIAGNOSTIC:

=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.3695771

p-value 0.7116976

Chain: witchchain2

Z-Score 0.8468539

p-value 0.3970765

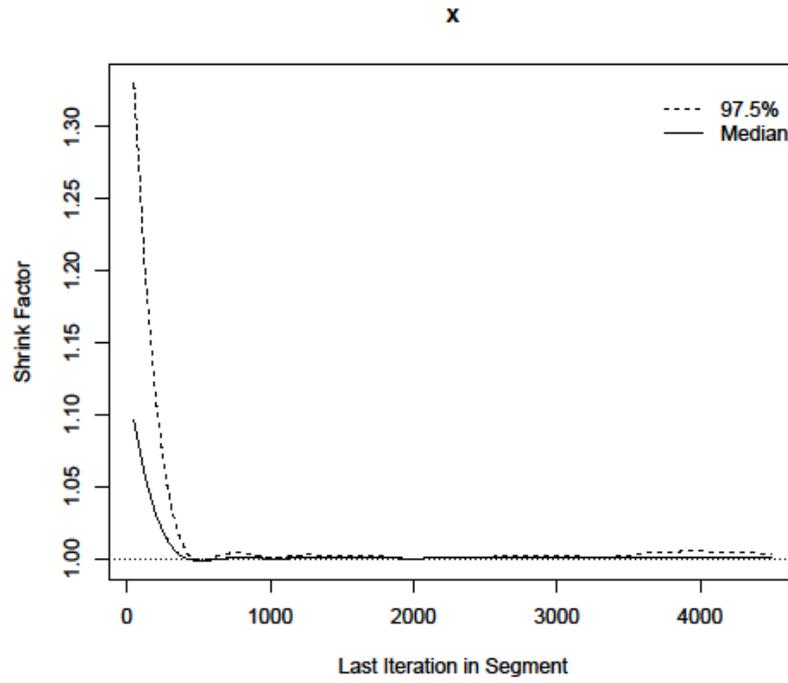
Chain: witchchain3

Z-Score 0.8474821

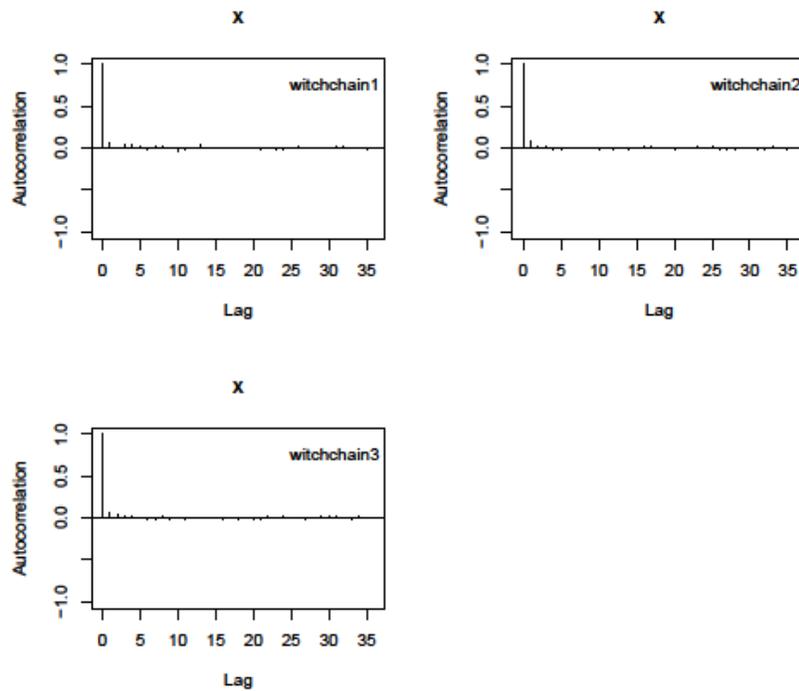
p-value 0.3967264



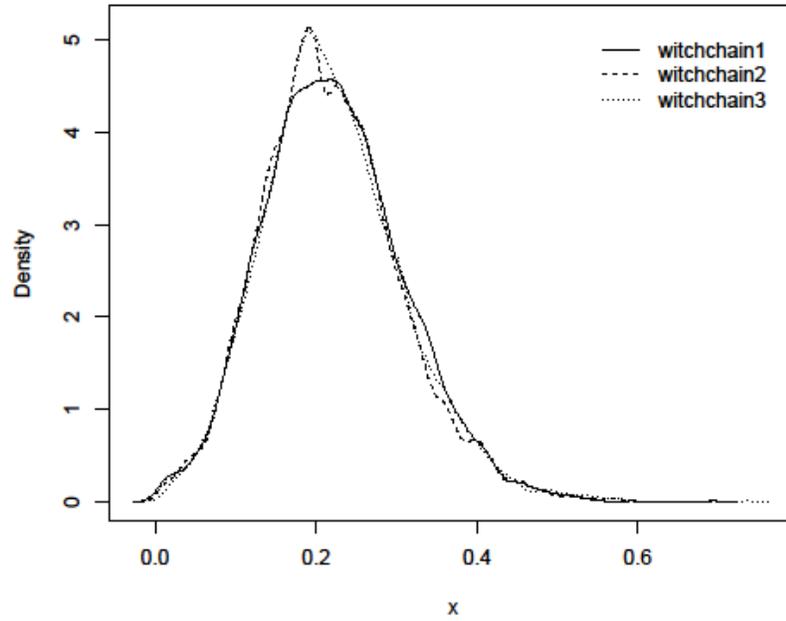
Gelman & Rubin Shrink Factors



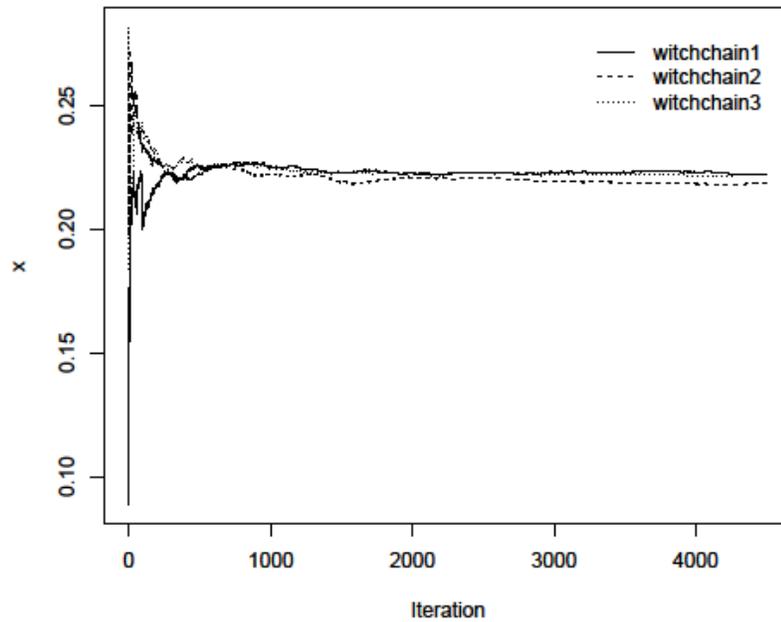
Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean



Convergence Diagnostics q spring late

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
x0.3271142	0.1168771	0.001742301	0.002228284	0.002120305	0.005584839	0.1549475	0.3089	0.609035	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3247195	0.118428	0.00176542	0.002036882	0.002099451	-0.01627699	0.152995	0.307	0.60365	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.3270732	0.1204551	0.001795638	0.002951737	0.00243572	-0.06470403	0.1518	0.3085	0.6074775	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

=====

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.001169

Multivariate Potential Scale Reduction Factor = 1.001863

Corrected Scale Reduction Factors

Estimate 0.975



x 1.001255 1.004987

GEWEKE CONVERGENCE DIAGNOSTIC:

=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.5657671

p-value 0.5715521

Chain: witchchain2

Z-Score 1.81988475

p-value 0.06877656

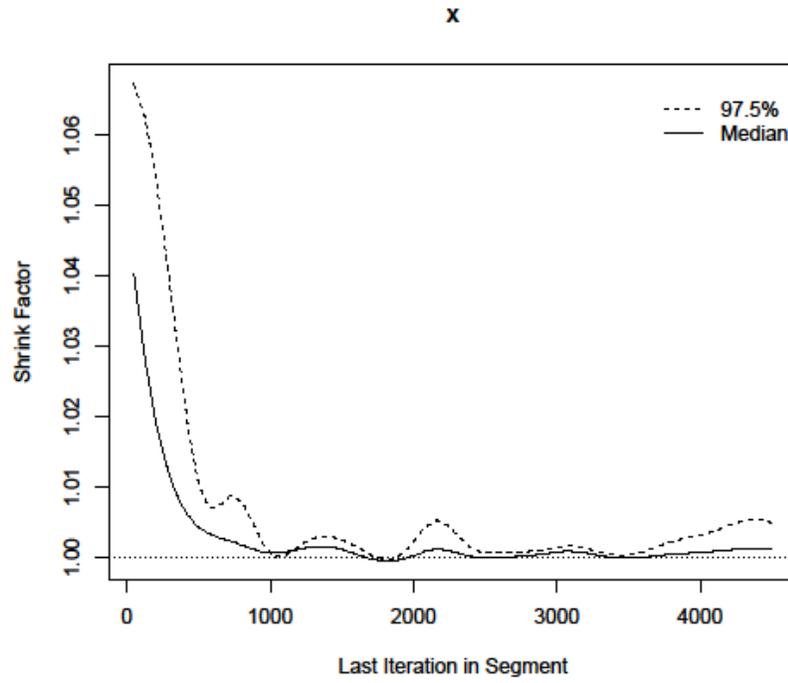
Chain: witchchain3

Z-Score 1.1193533

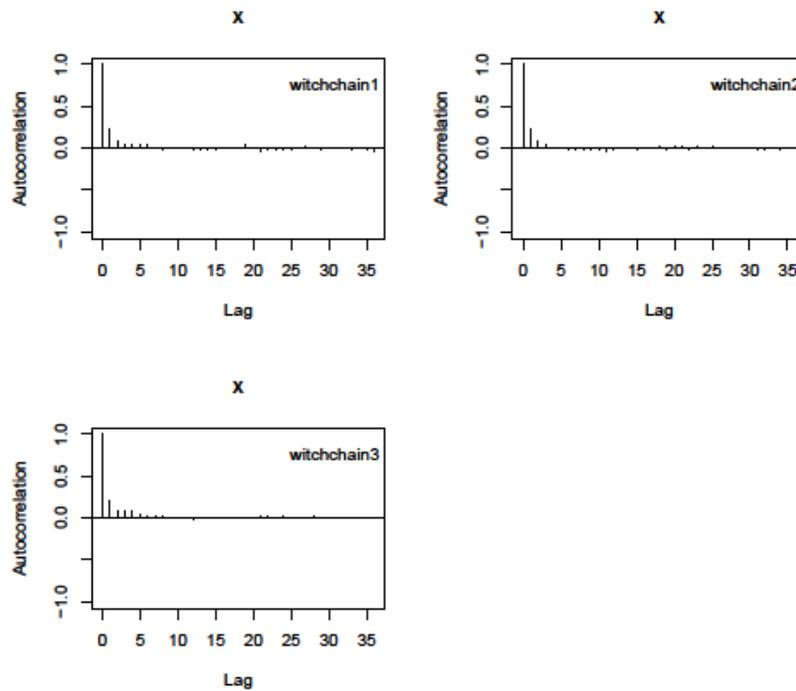
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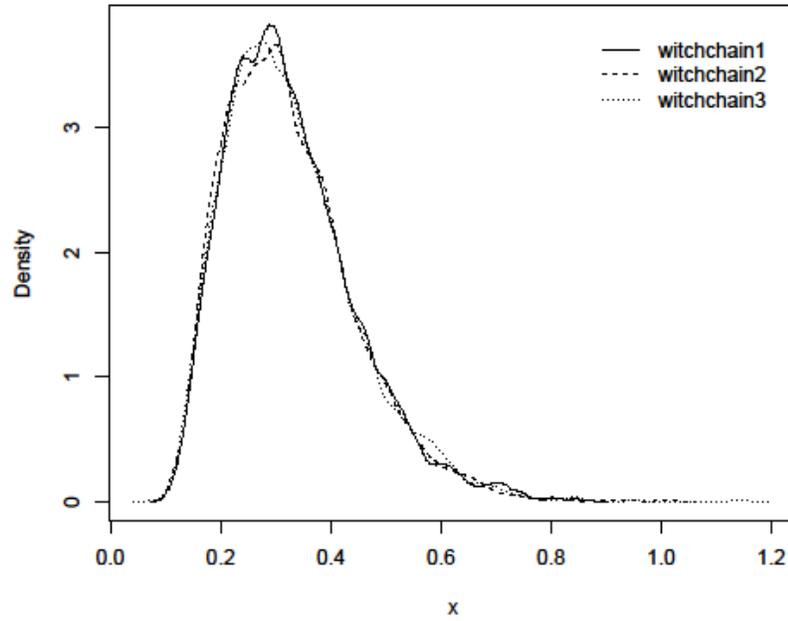
Gelman & Rubin Shrink Factors



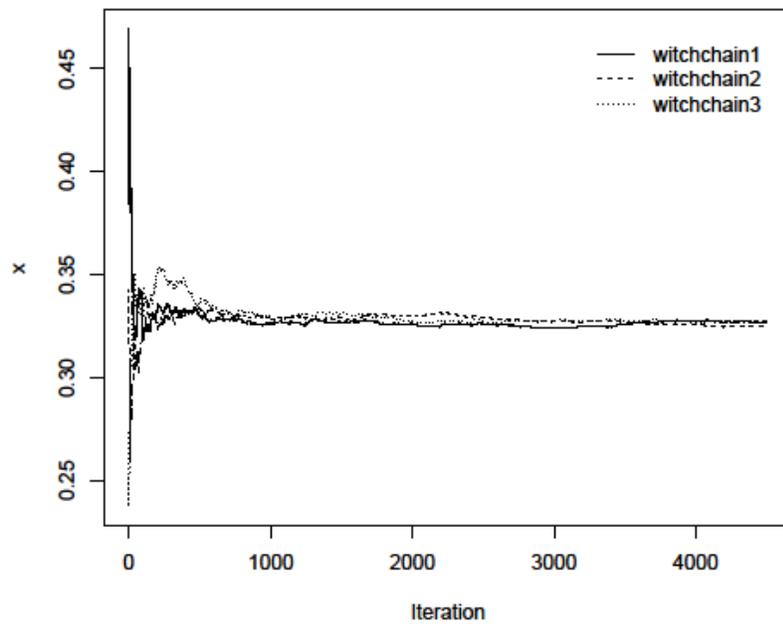
Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean



Convergence Diagnostics q spring early

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.431606	0.1489472	0.002220374	0.002451124	0.002435754	0.006499718	0.2219475	0.409	0.7645575	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.4306661	0.1449903	0.002161388	0.002354234	0.002116227	0.0605492	0.2233	0.4067	0.7781675	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.4300649	0.1399443	0.002086166	0.002214821	0.002285665	-0.03194471	0.2283475	0.4073	0.7823325	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

=====

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.0001

Multivariate Potential Scale Reduction Factor = 1.000261

Corrected Scale Reduction Factors

Estimate 0.975

x 1.002095 1.002972



GEWEKE CONVERGENCE DIAGNOSTIC:

=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 1.0100475

p-value 0.3124725

Chain: witchchain2

Z-Score 1.0554567

p-value 0.2912165

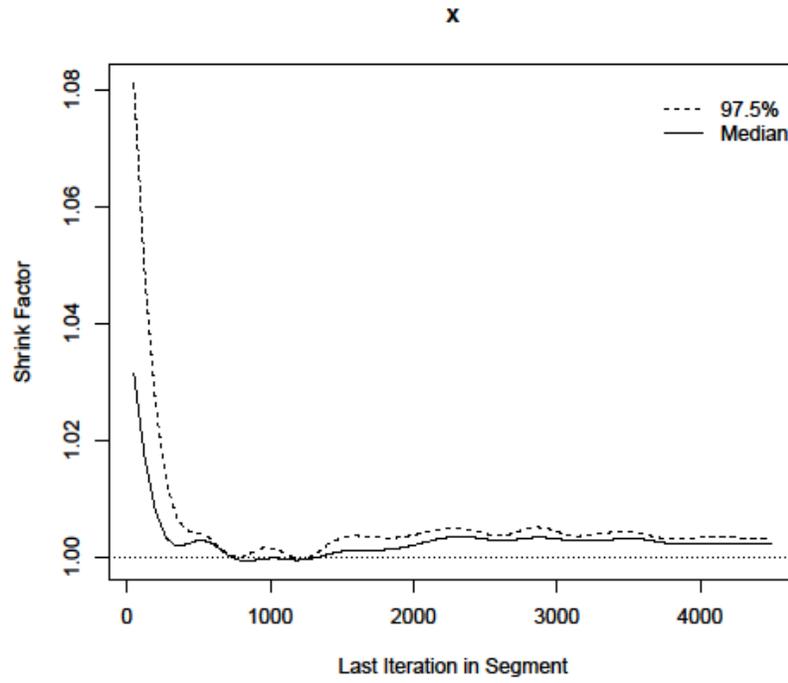
Chain: witchchain3

Z-Score 1.71365551

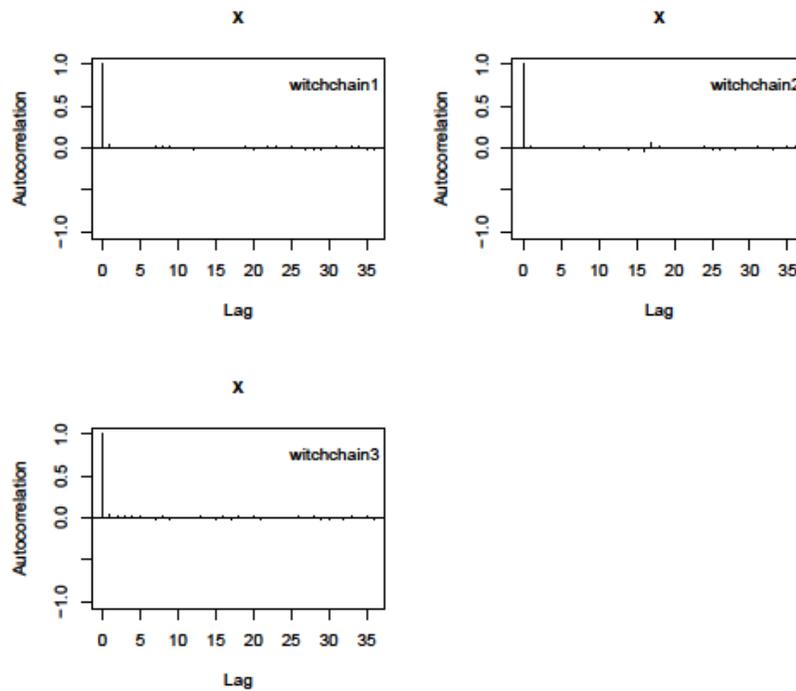
p-value 0.08659201



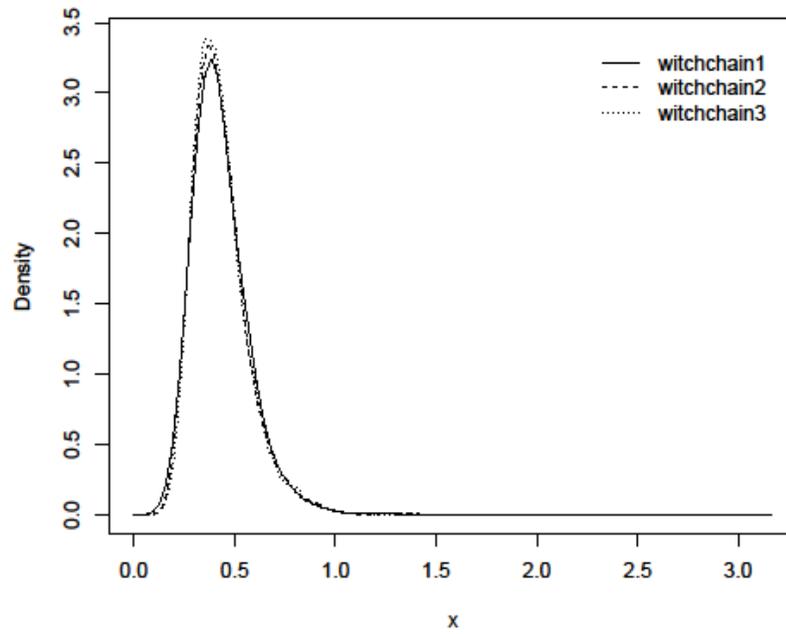
Gelman & Rubin Shrink Factors



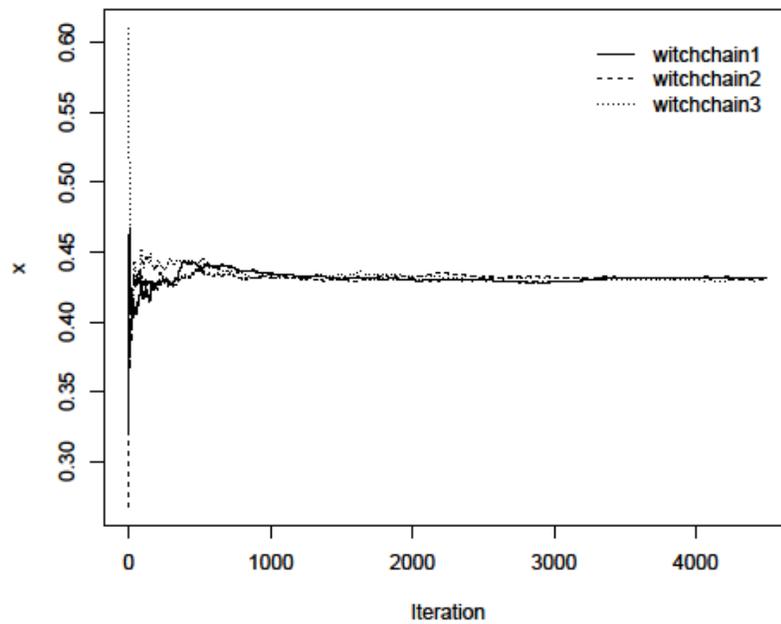
Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean



Convergence Diagnostics q fall

SUMMARY STATISTICS:

=====

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.4948816	0.1757207	0.00261949	0.003315976	0.003210778	0.02330467	0.2334475	0.4671	0.914625	1	4500	4500

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.49142	0.1767507	0.002634844	0.003205253	0.003254874	0.001975328	0.23099	0.4661	0.91203	1	4500	4500

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975	MinIter	MaxIter	Sample
0.4948337	0.1790703	0.002669422	0.004405959	0.003749702	-0.06154052	0.2297475	0.46765	0.9216575	1	4500	4500

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

=====

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000909

Multivariate Potential Scale Reduction Factor = 1.001474

Corrected Scale Reduction Factors

Estimate 0.975

x 1.001007 1.004044

GEWEKE CONVERGENCE DIAGNOSTIC:



=====

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.6482523

p-value 0.5168218

Chain: witchchain2

Z-Score 1.7545950

p-value 0.0793286

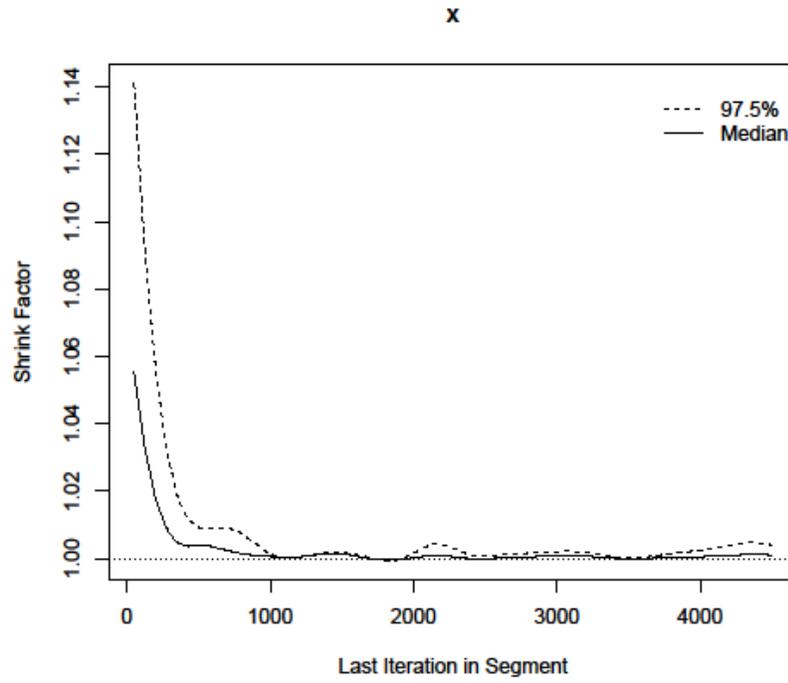
Chain: witchchain3

Z-Score 1.2635469

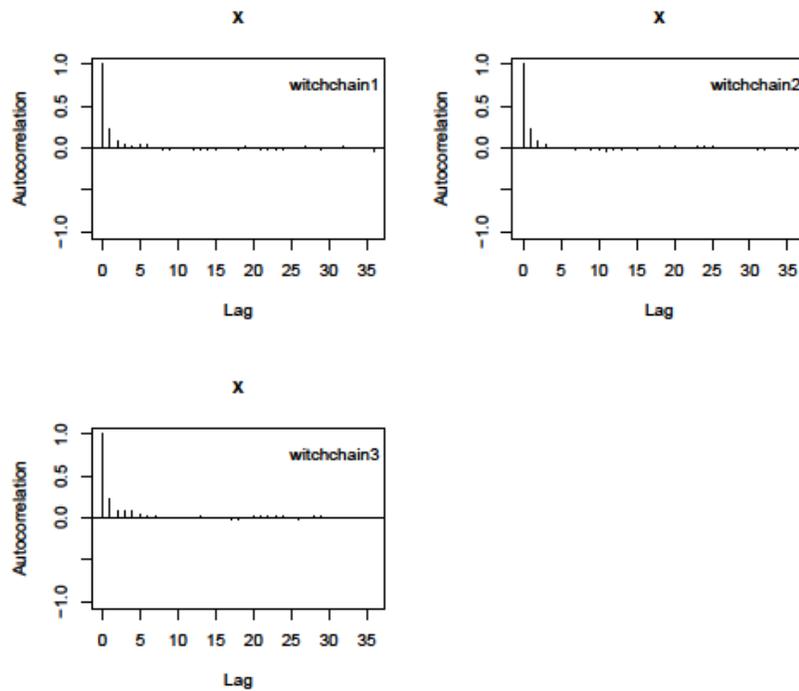
p-value 0.2063927



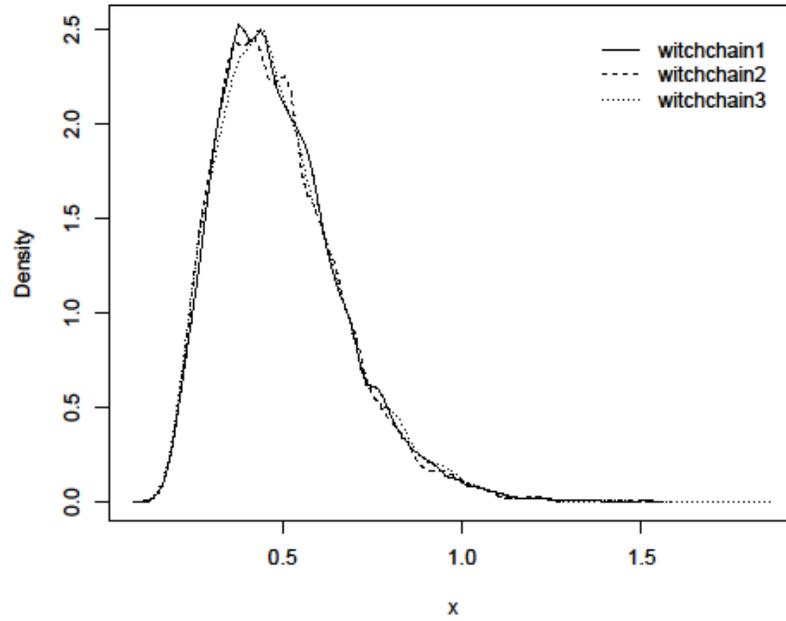
Gelman & Rubin Shrink Factors



Sampler Lag-Autocorrelations



Estimated Posterior Density



Sampler Running Mean

