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Surplus production model in a Bayesian framework applied to witch flounder in NAFO Div. 3NO

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Abstract

The formulation of a surplus production model in a Bayesian framework accepted in 2015 as the basis for advice for witch flounder in NAFO Div. 3NO was updated with data to 2017. The data series included catch from 1960-2017 and three Canadian survey series. Model results indicated that over 2014-2016 the survey indices were declining faster than can be explained by the process being modelled. To account for this a change was made to allow the process error to increase in 2014, 2015 and 2016 compared to the rest of the years (the sigma parameter was increased by 1 in those years). The production model estimated that an MSY of 3774 t can be taken from a biomass of 59 910 t at a fishing mortality of 0.063. Intrinsic rate of natural increase is estimated to be 0.126 and carrying capacity 119 800 t. After declining to below Blim in the mid 1990s, biomass generally increased to 2013. The current formulation of the model estimates a large drop in biomass from 2013 to 2015 to just above Blim, followed by an increase over the last 3 years to 37% of Bmsy in 2018. There is a probability of 0.29 that the stock is below Blim. Fishing mortality has been below F_{MSY} since the mid 1990s. However, F increased from 2014-2016 before declining again in 2017. In 2017 the probability of being above Fmsy is estimated to be 0.04.

Key words: Bayesian surplus production model, Div. 3NO witch flounder, assessment

Introduction

The directed witch flounder fishery in Div. 3NO was reopened in 2015 with a TAC of 1000 t. This decision was based on advice developed from an assessment based on survey trends. In 2015, Scientific Council accepted a surplus production model in a Bayesian framework as the basis for the advice for witch flounder in Div. 3NO. The model was used to evaluate the status of the stock relative to precautionary reference points and to provide catch advice.

In the 2017 assessment (Morgan and Lee, 2017) an increase in process error was detected. Process error increased as data from each year from 2014 to 2016 were added. This indicated that the surplus production model was not modelling part of the process over this time period. A change to the model is described here and some of the results compared to the 2017 assessment model updated with data to the end of 2017. This revised model formulation is the basis for the advice for 2019 for this stock. Its full results and diagnostics are also given.

Methods

The Schaefer (1954) form of a surplus production model used here is:

$$P_t=[P_{t-1}+ r\bullet P_{t-1} (1 - P_{t-1})- C_{t-1}/K]\bullet\eta_t$$

where P_{t-1} and C_{t-1} denote exploitable biomass (as a proportion of carrying capacity) and catch, respectively, for year t-1 (Meyer and Millar, 1999a, 1999b). Carrying capacity, K, is the level of stock biomass at equilibrium prior to commencement of a fishery, r is the intrinsic rate of population growth, and η_t is a random variable describing stochasticity in the population dynamics (process error). The model utilizes biomass proportional to an estimate of K in order to aid mixing of the Markov Chain Monte Carlo (MCMC) samples and to help minimize autocorrelation between each state and K (Meyer and Millar, 1999a, 1999b).

An observation equation is used to relate the unobserved biomass, P_t, to the research vessel survey indices:

$$I_t=q\bullet P_t \bullet\varepsilon_t$$

where q is the catchability parameter, P_t is an estimate of the biomass proportional to K at time t, and ε_t is observation error.

Input data are given in Table 1. All priors were the same as those used in the 2015 and 2017 assessments.

The prior on r was informed by that derived by Swain 2012 for witch flounder in the southern Gulf of St. Lawrence. The prior used here allowed for a higher r than derived by Swain (2012) as some of the morphometric methods indicated a higher r. Therefore the mean (0.17) derived by Swain (2012) was used as the central tendency (i.e. the median) but with a larger standard deviation.

A mean of 0.2 and standard deviation of 0.12 gives a median of 0.17 on the log normal scale. The prior used therefore was:

R~(-1.763,3.252)

The prior for K was based on Ecosystem Production Potential modelling (NAFO 2014). This modelling indicated that a reasonable distribution for K would have a mean of 100 and a standard deviation of 30.

K~dlnorm(4.562,11.6)

The priors on survey q and observation error were:

pq ~dgamma(1,1)
q <-1/pq

tau ~dgamma(1,1)
itau2 <- 1/tau

For process error:

sigma ~ dunif(0,10)

The results of the 2017 assessment indicated that over 2014-2016 the survey indices were declining faster than can be explained by the process being modelled. To account for this a change was made to allow the process error to increase in 2014, 2015 and 2016 compared to the rest of the years (the sigma parameter was increased by 1 in those years). A formulation with process error allowed to increase in only 2015 was included for comparison.

Results and Discussion

The process error from various formulations of the surplus production model was compared as the main diagnostic in model choice. The process error from the 2015 assessment (with data up to and including 2014) shows little process error that varied without trend (Figure 1). The median for the process error from this formulation was 0.075. Process error for the same model formulation but with data up to and including 2017 showed a large increase in process error (median value 0.22) with a trend to larger (negative) process error in recent years. When process error was allowed to increase in 2015, the overall process error declined (median 0.072) and there was a very large negative process error in 2015. The change in population size from 2014 to 2015 in this formulation was very large and abrupt (not shown). This did not seem reasonable. The formulation which allowed process error to increase from 2014-2016 also resulted in a decrease in process error (median 0.067). It removed the overall trend and put this increased error into 2014 and 2015 (Figure 1 and 2) with very little additional error in 2016. It also produced a step wise decrease in biomass over the period (see Bratio figure 8). Model fit to the fall survey data was much improved by allowing process error to increase over the 2014-2016 period (Figure 3). When process error is allowed to increase the estimates of K and r are more similar to the model using data only to 2014 i.e. before the apparent change in process (Table 2).

This change to the formulation results in a lower process error that varies without trend and a better fit to the fall survey indices. It does not explain the reasons for the change in process but is a simply way to account for an apparent change in state of the population that is not captured in the process being modelled. The decline in biomass from 2014 to 2016 estimated using the present formulation is consistent with declines in other fish species on the Grand Bank and with changes in other components of the ecosystem. The remainder of the text in this document discusses the results of this formulation.

All posteriors were updated from their priors (Figures 4, 5 and 6). Model fit to the survey data was relatively good for all surveys (Figure 7).

All convergence diagnostics (Appendix 1) indicated that there were no issues with model convergence.

The production model estimated that an MSY of 3774 t can be taken from a biomass of 59 910 t at a fishing mortality of 0.063. Intrinsic rate of natural increase is estimated to be 0.126 and carrying capacity 119 800 t (Table 2).

The population is estimated to have declined from a high in 1966 to below Blim in the mid to late 1990s (Figure 8). The biomass generally increased to 2013. The current formulation of the model estimates a large drop in biomass from 2013 to 2015 to just above Blim, followed by an increase over the last 3 years to 37% of Bmsy in 2018. There is a probability of 0.29 that the stock is below Blim.

Fishing mortality was at its highest levels (and above Fmsy) from the mid 1980s to the mid 1990s (Figure 9). Since then fishing mortality has been below Fmsy. However, F increased from 2014-2016 before declining again in 2017. In 2017 the probability of being above Fmsy is estimated to be 0.04.

Acknowledgments

We thank all of the people involved in the collection of these data.

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Table 1. Data used in the Bayesian Surplus Production model. Values are in thousands of tons.

Year	Landing s	spring late	fall	spring early
1960	5.799			
1961	4.627			
1962	1.228			
1963	2.183			
1964	1.066			
1965	2.177			
1966	7.522			
1967	11.503			
1968	10.599			
1969	4.7			
1970	6.763			
1971	14.965			
1972	9.177			
1973	6.691			
1974	8.045			
1975	6.168			
1976	6.035			
1977	5.759			
1978	3.473			
1979	3.077			
1980	2.42			
1981	2.425			
1982	3.732			
1983	3.616			
1984	2.802			14.313
1985	8.771			24.581
1986	9.131			9.214
1987	7.596			11.199
1988	7.325			24.655
1989	3.688			8.988
1990	4.179		15.368	10.759
1991	4.847	7.07	5.477	
1992	4.96	8.217	9.118	
1993	4.414	4.226	9.474	
1994	1.119	16.279	7.821	
1995	0.3	4.057	11.743	
1996	0.358	4.085	12.278	
1997	0.512	7.133	4.691	
1998	0.612	2.688	6.689	
1999	0.763	8.936	13.33	
2000	0.545	5.49	7.64	

2001	0.694	9.418	7.021
2002	0.45	7.562	11.13
2003	1.544	15.855	10.315
2004	0.627	11.825	18.632
2005	0.257	6.865	18.132
2006	0.481		14.605
2007	0.222	7.189	7.715
2008	0.264	8.825	22.739
2009	0.376	9.179	37.708
2010	0.421	6.639	27.039
2011	0.351	9.746	17.939
2012	0.314	12.844	27.033
2013	0.328	24.396	17.668
2014	0.335	10.702	
2015	0.359	4.927	10.101
2016	1.062	7.134	7.869
2017	0.656	9.054	9.478

Table 2. Parameter estimates from various formulations of a surplus production model for Div. 3NO witch flounder. Weights are in thousands of tonnes. 95% Credible Intervals are also given.

	Data to 2014	Data to 2017	process error 2015	Process error 2014-2016
r	0.126 (0.078-0.244)	0.143 (0.06-0.371)	0.124 (0.077-0.239)	0.126 (0.081-0.235)
K	119.4 (74.3-165.3)	99.2 (59.67-164.7)	120.1 (75.1-167.3)	119.8 (75.82-163.8)
MSY	3.763 (2.43-5.83)	3.632 (1.63-7.24)	3.717 (2.41-5.74)	3.774 (2.52-5.69)
Bmsy	59.68 (37.15-82.63)	49.61 (29.84-82.33)	60.07 (37.55-83.64)	59.91 (37.91-81.91)
Fmsy	0.06 (0.039-0.122)	0.072 (0.031-0.186)	0.062 (0.039-0.120)	0.063 (0.041-0.117)

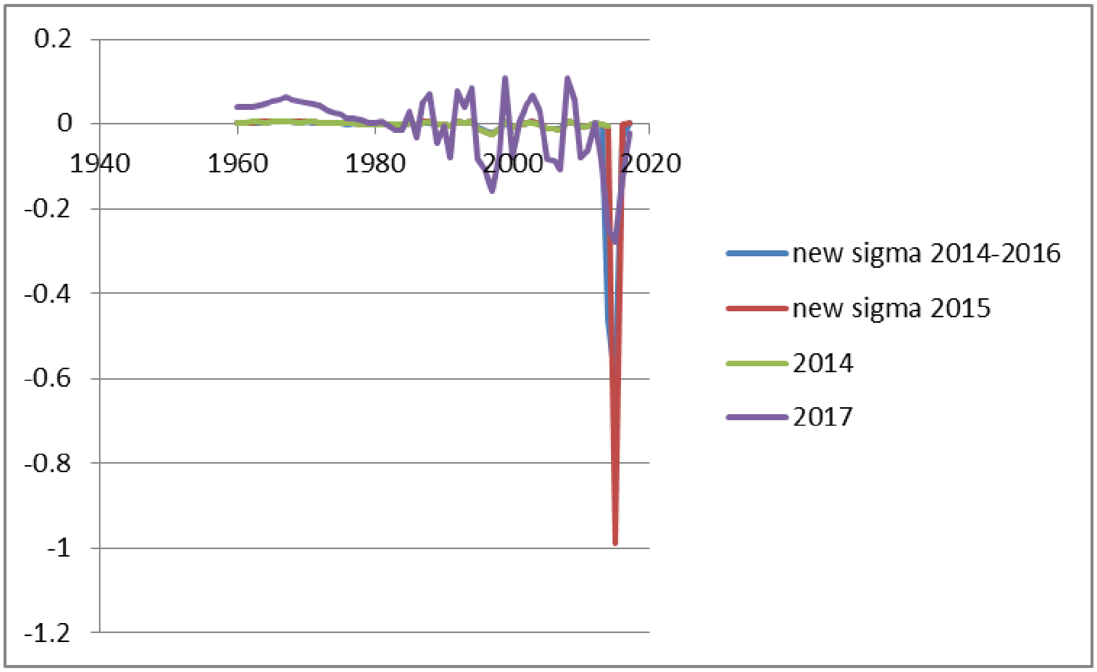

Fig. 1. Process error from four formulations of the surplus production model. 2014 and 2017 are the formulation used in the last assessment with data up to including 2014 and 2017 respectively. The two 'new sigma' runs allow sigma to increase in only 2015 or in 2014-2016 respectively.

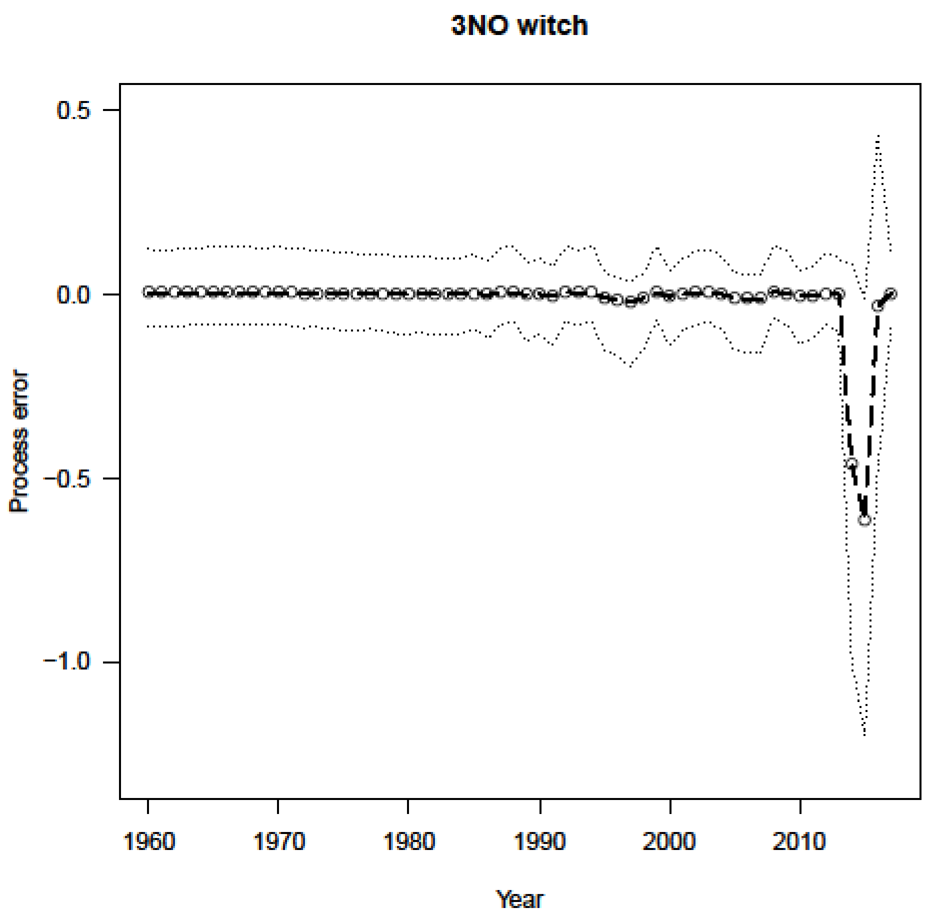

Fig. 2. Process error from the surplus production model fit to 3NO witch flounder with process error allowed to increase in 2014-2016.

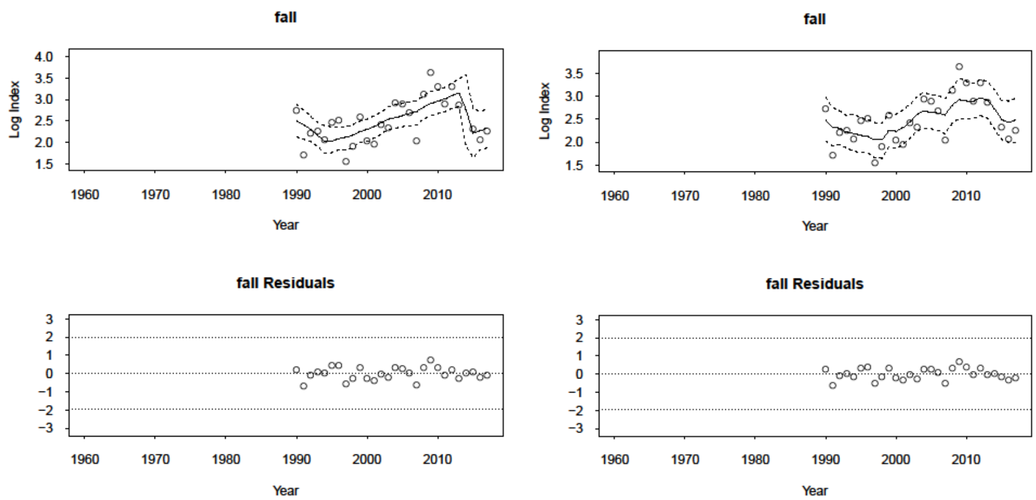

Fig. 3. Observed and predicted survey indices from the fall surveys from two surplus production model formulations. For each survey the top panel gives the observed and predicted values with 95th credible intervals while the bottom panel presents standardized residuals. The left hand panels show the results from the model with process error allowed to increase in 2014-2016 while the right hand panels show the results for the model with the same process error throughout.

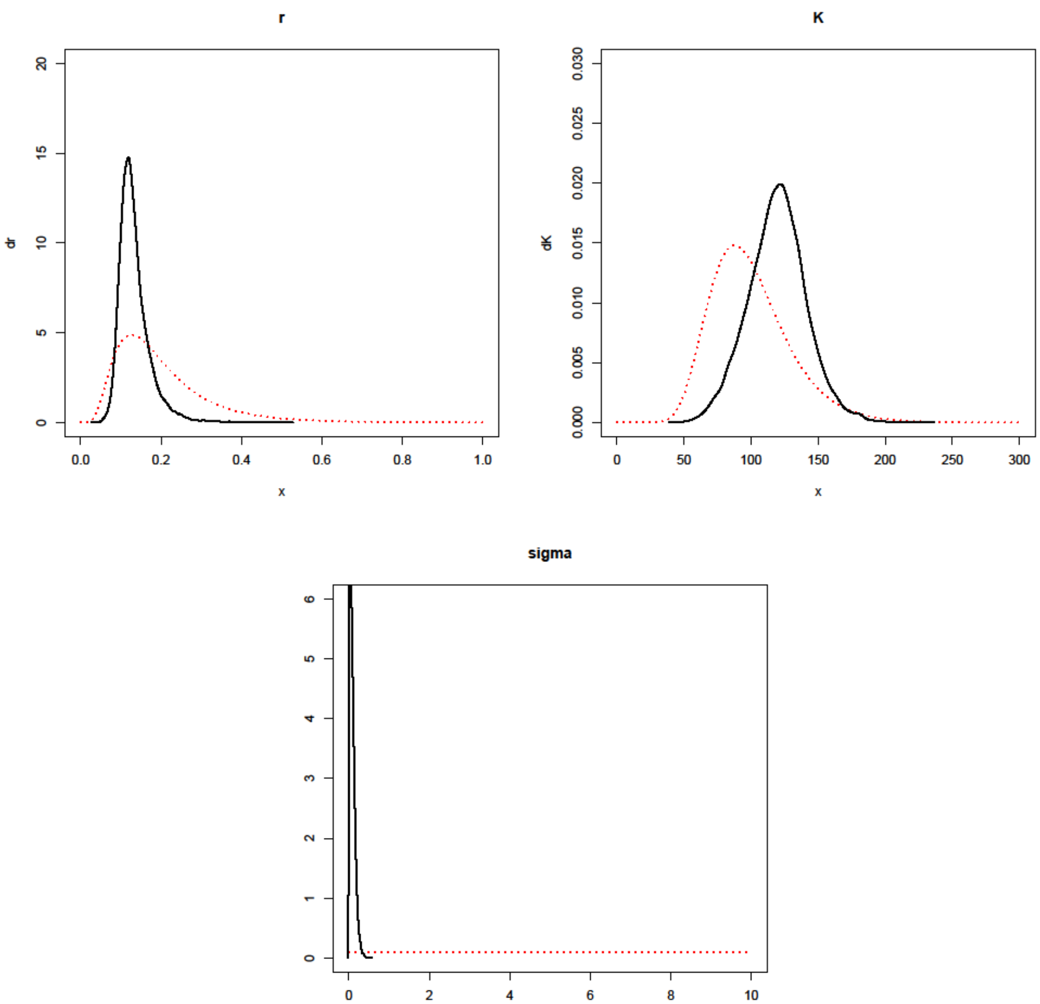

Fig. 4. Priors (red dotted line) and posteriors (black line) for r, K and sigma (process error).

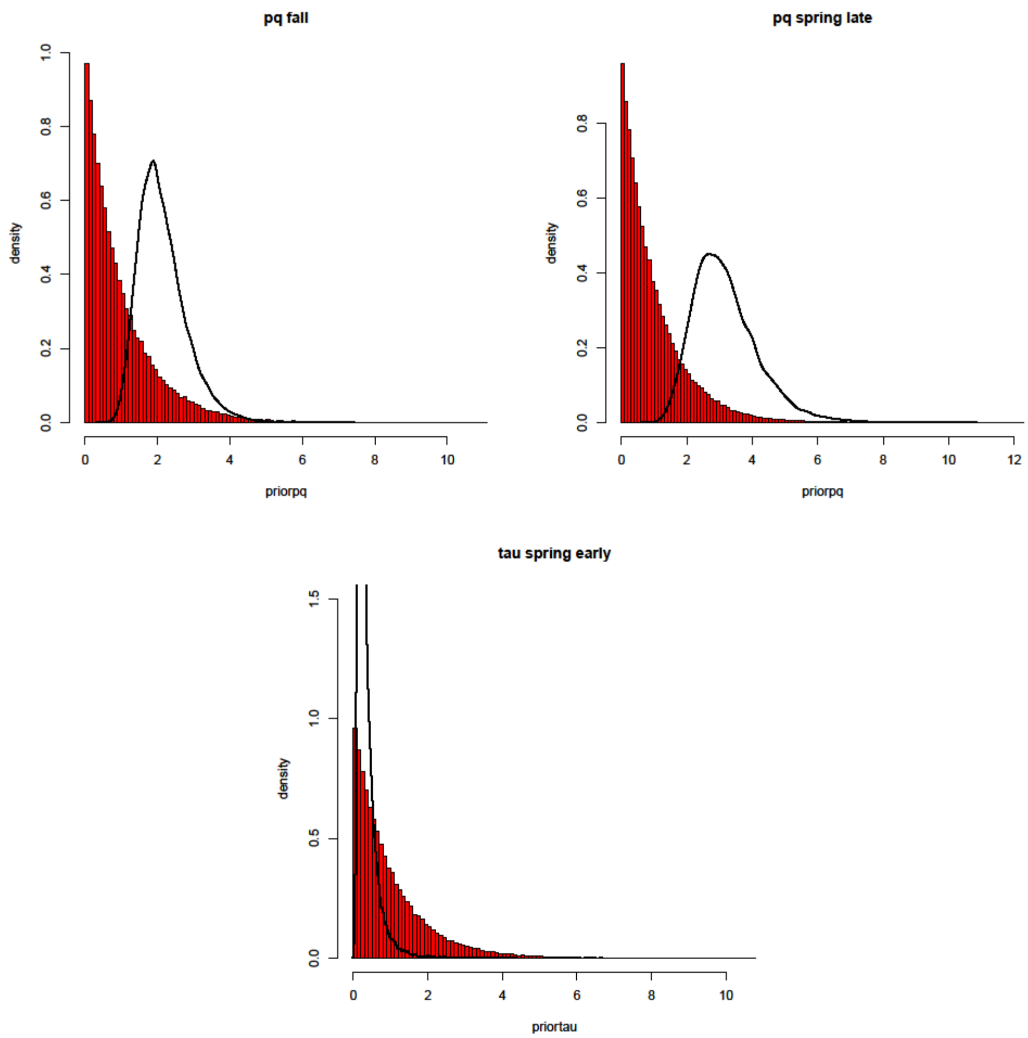

Fig. 5. Priors (red histogram) and posteriors (black lines) for pq (inverse of q) for the 3 survey indices used in the model.

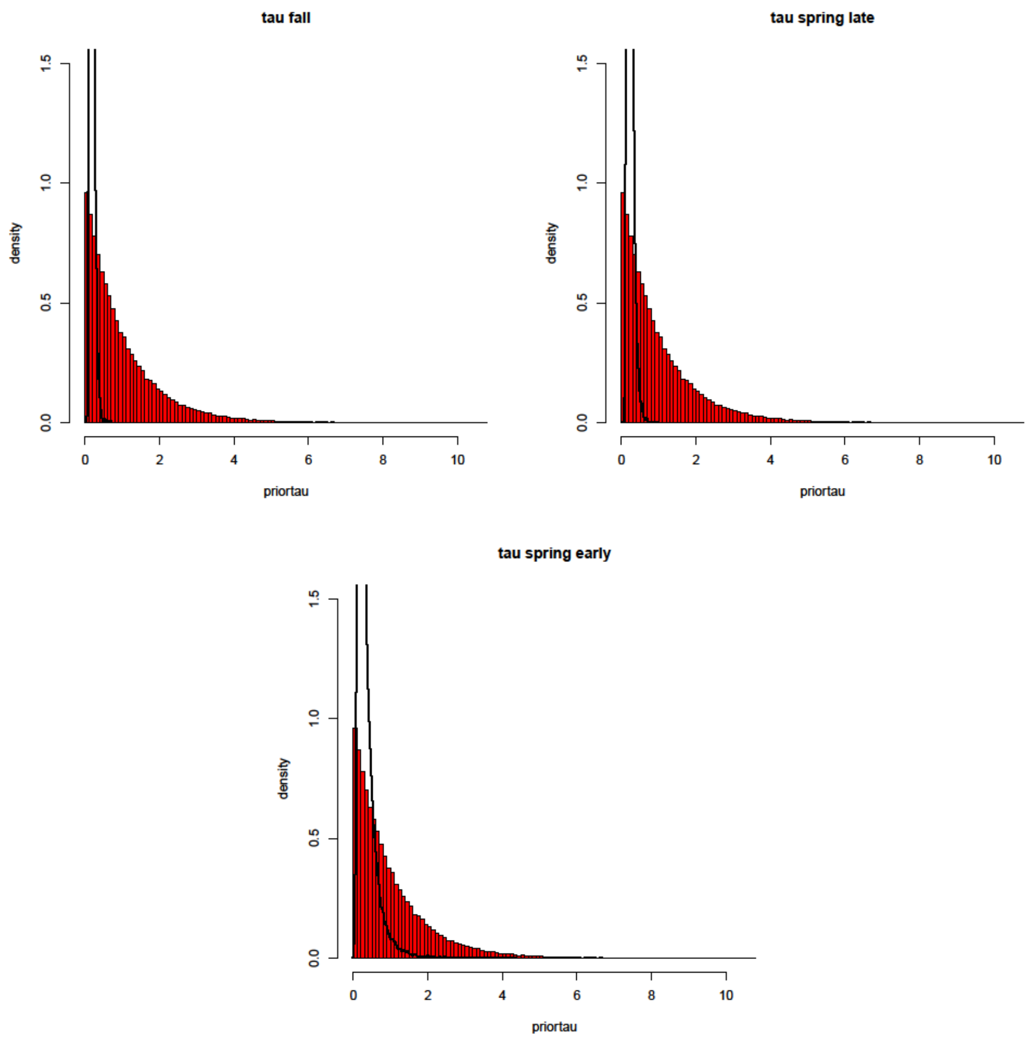

Fig. 6. Priors (red histograms) and posteriors (black lines) for observation error on surveys used in the model.

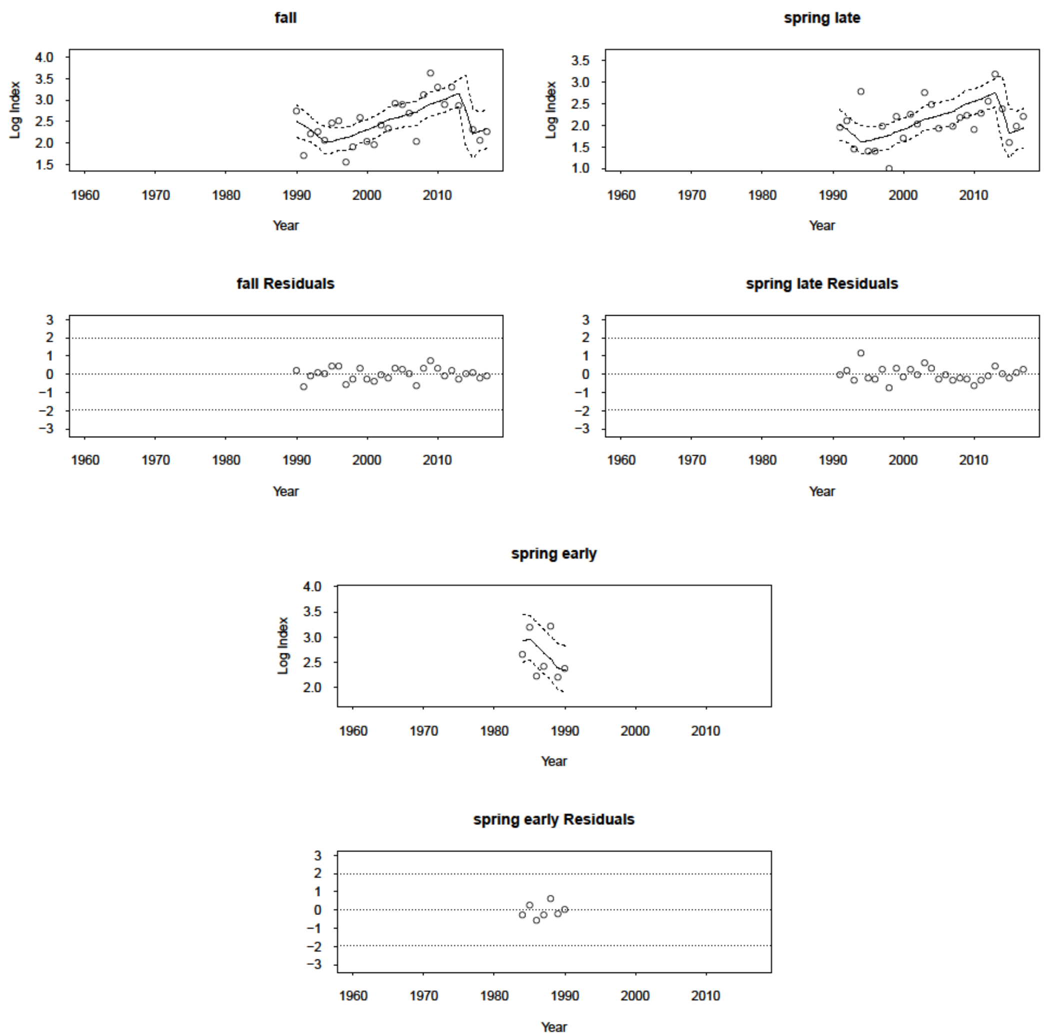

Fig. 7. Observed and predicted survey indices from each of the three surveys used in the model. For each survey the top panel gives the observed and predicted values with 95th credible intervals while the bottom panel presents standardized residuals.

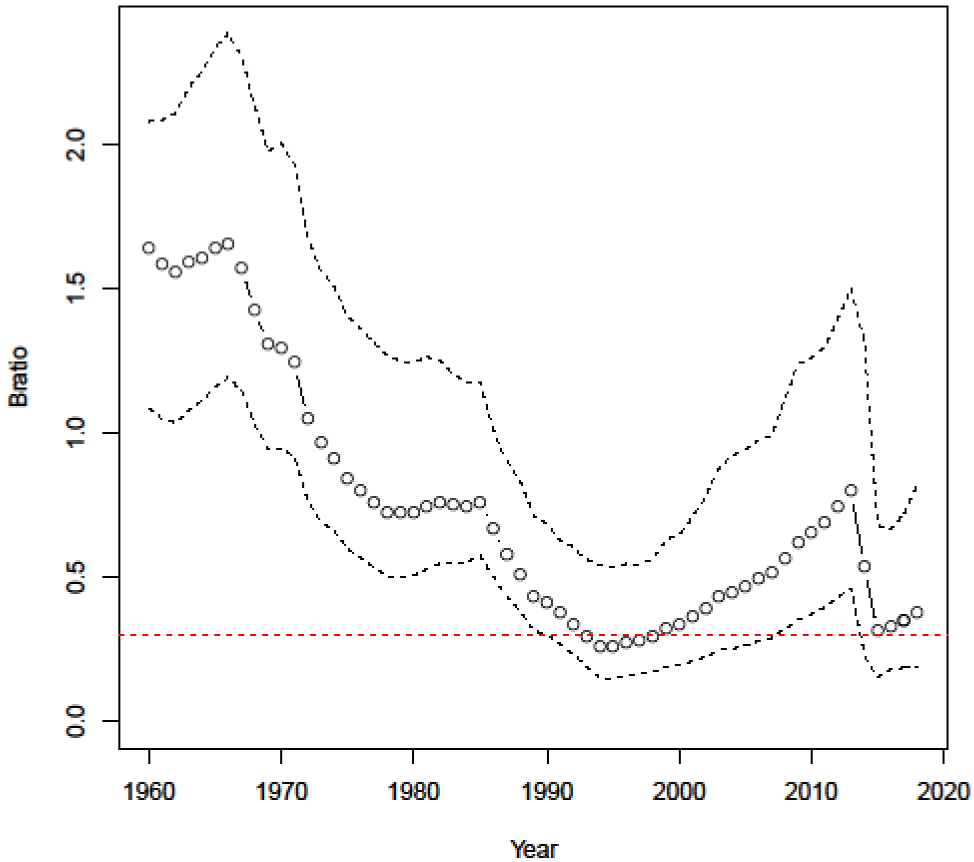

Fig. 8. Relative biomass (biomass divided by B_{MSY}) for Div. 3NO witch flounder. The median with its 90th percent credible intervals are shown. The horizontal dashed line is B_{lim} (30% B_{MSY}).

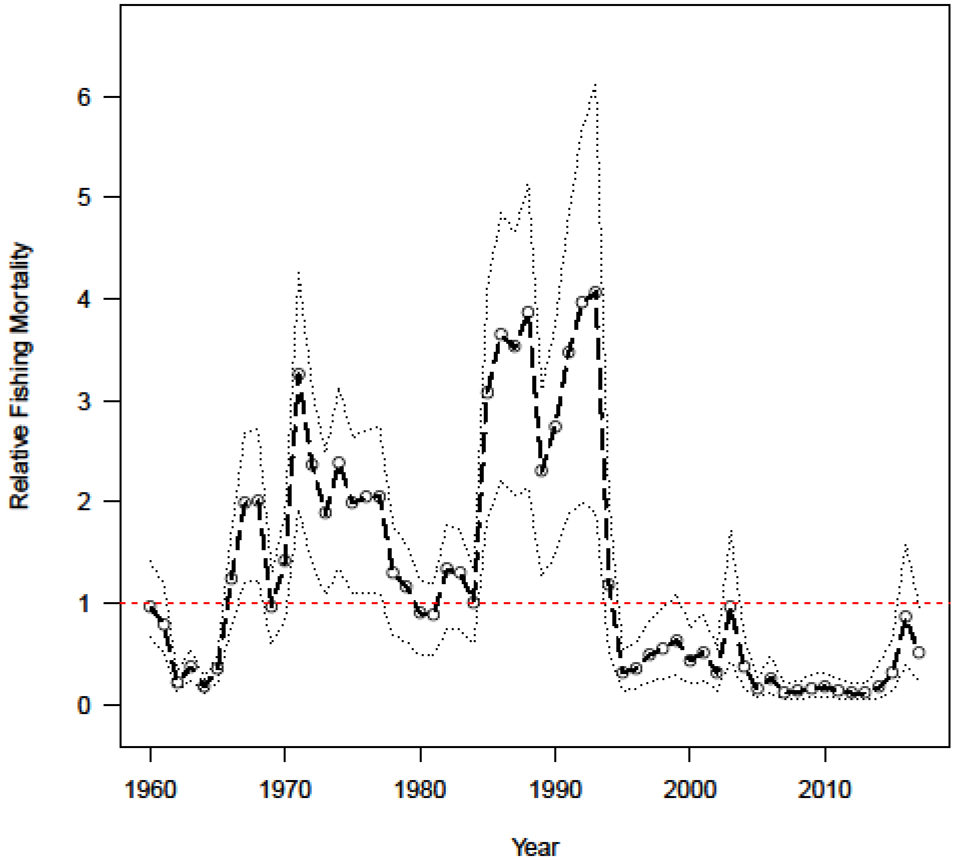

Fig. 9. Relative fishing mortality (fishing mortality divided by F_{MSY}) for Div. 3NO witch flounder. The median with its 90th percent credible intervals are shown. The horizontal red dashed line is F_{lim} (F_{MSY})

Appendix 1

Convergence diagnostics R

SUMMARY STATISTICS:

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Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.1340	0.0400	0.0005	0.0007	0.0007	-0.0521	0.0810	0.1259	0.2396

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.1341	0.0411	0.0006	0.0008	0.0008	-0.0432	0.0824	0.1254	0.2359

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.1329	0.0382	0.0005	0.0007	0.0007	0.1138	0.0808	0.1257	0.2270

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

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x

1.000612

Multivariate Potential Scale Reduction Factor = 1.001029

Corrected Scale Reduction Factors

Estimate 0.975

x 1.001585 1.003838

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score -0.1754256

p-value 0.8607452

Chain: witchchain2

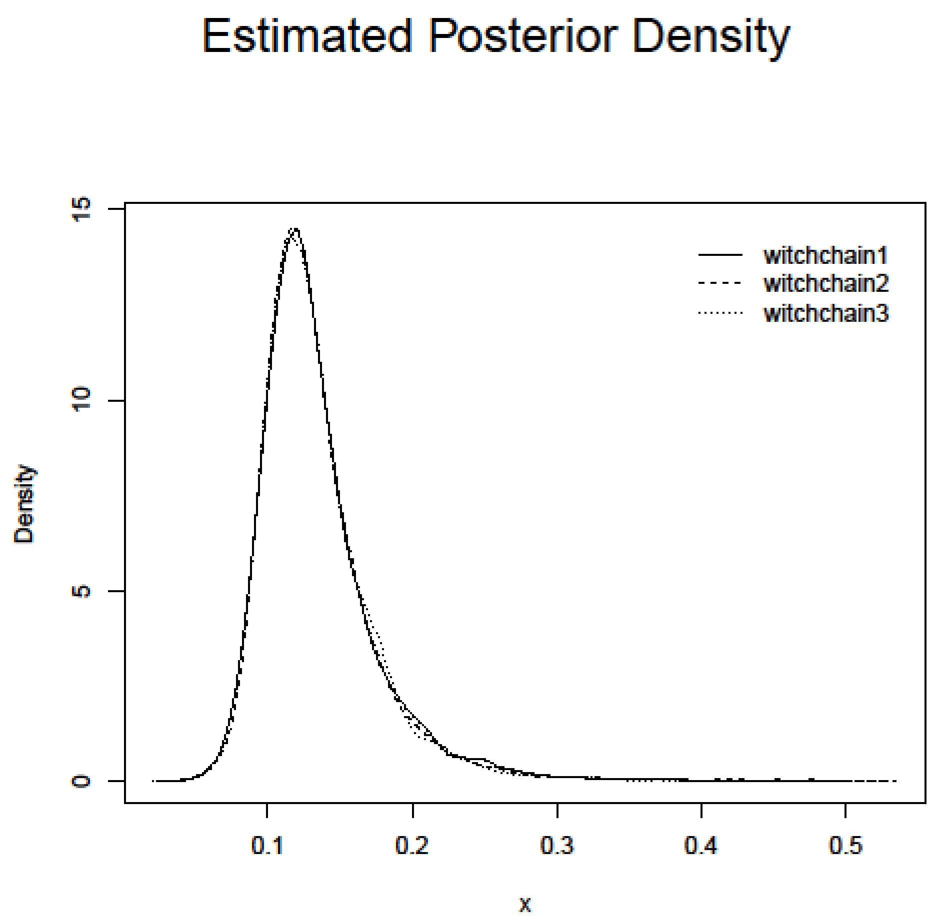
Z-Score 0.9216767

p-value 0.3566973

Chain: witchchain3

Z-Score 1.1967657

p-value 0.2313979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witchchain2
witchchain3
Density
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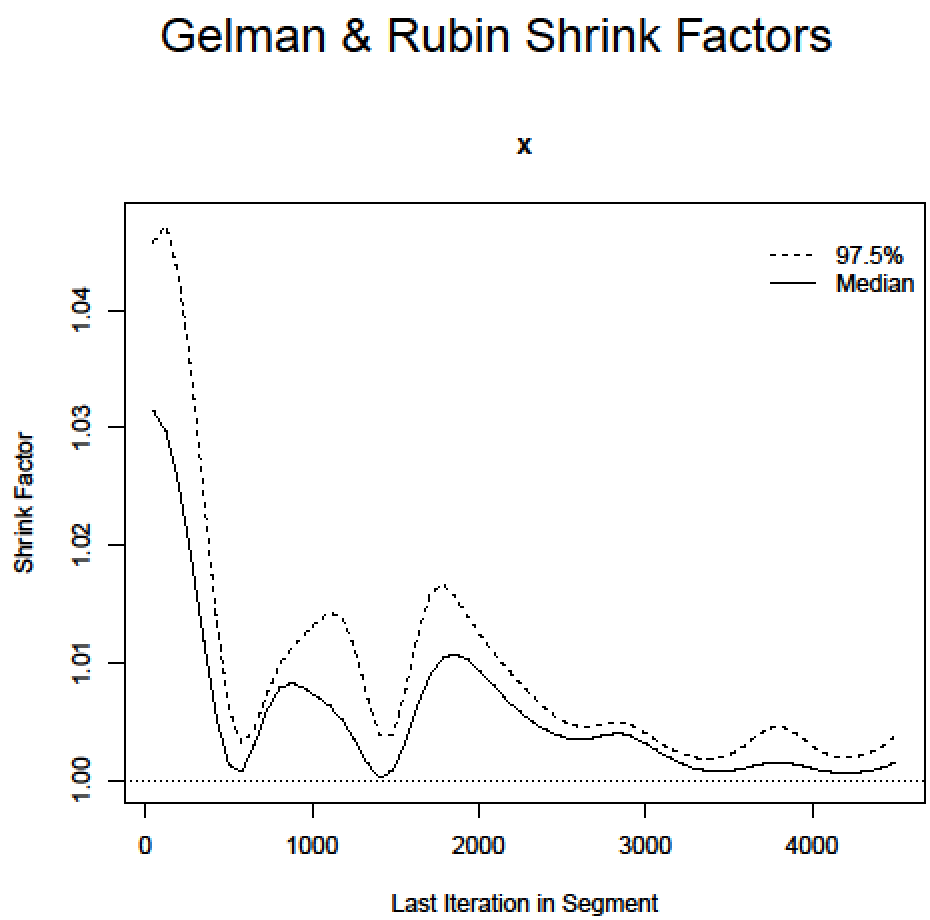
0.2
0.3
0.4
0.5


Gelman & Rubin Shrink Factors
x
97.5%
Median
Shrink Factor
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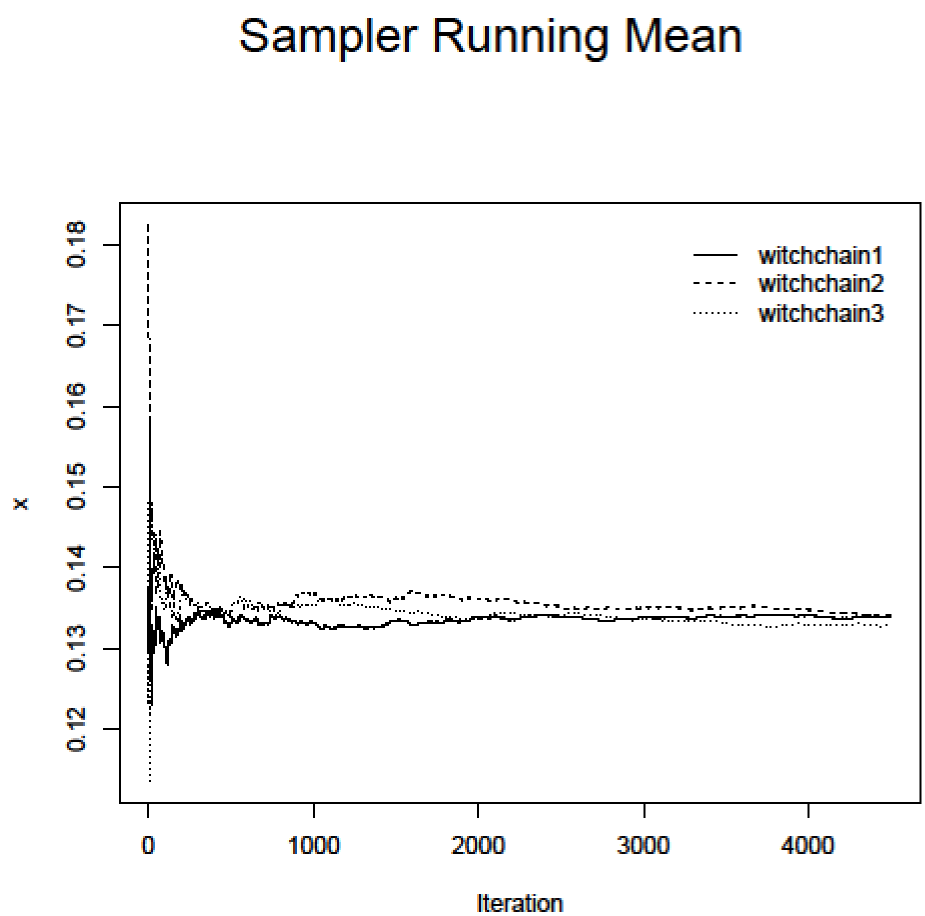
1.01
1.02
1.03
1.04
0
1000
2000
3000
4000
Last Iteration in Segment


Sampler Running Mean
witchchain1
witchchain2
witchchain3
x
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0.13
0.14
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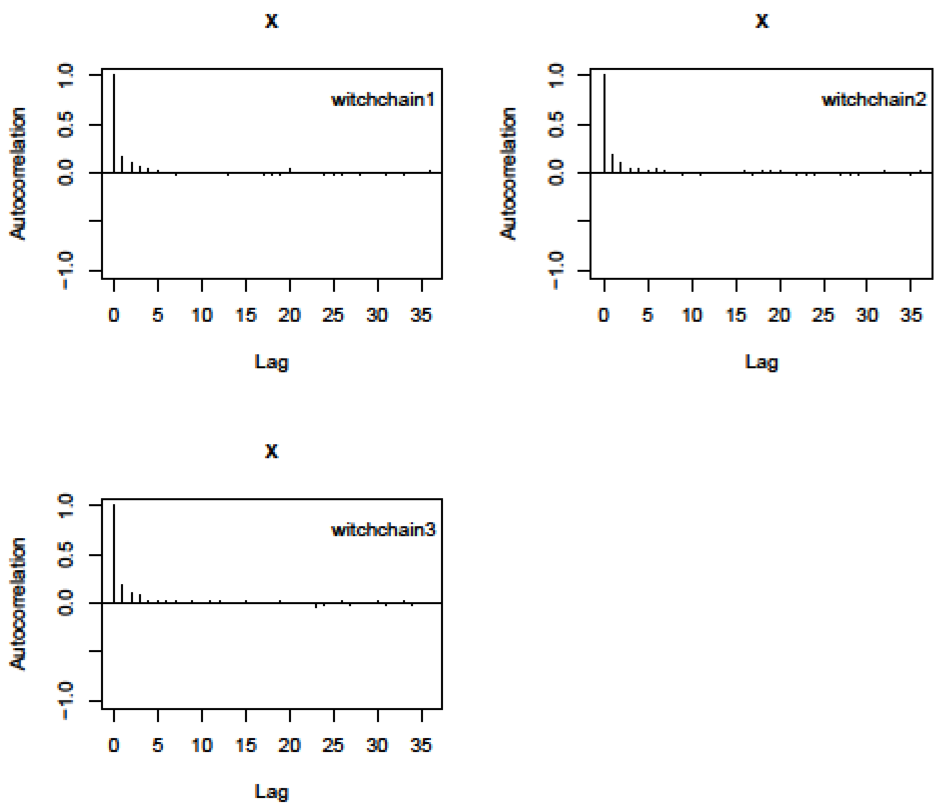
0.16
0.17
0.18
0
1000
2000
3000
4000
Iteration

Sampler Lag−Autocorrelations



Convergence Diagnostics K

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
119.4	21.91	0.3266	0.5910	0.5284	0.0226	77.0	119.7	164.0

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
119.2	21.6	0.3234	0.5582	0.5670	-0.0272	75.2	119.4	163.3

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
119.8	21.7	0.3235	0.6074	0.5697	0.1080	76.0	120.4	163.9

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000152

Multivariate Potential Scale Reduction Factor = 1.000339

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000416 1.001423

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.3132464

p-value 0.7540934

Chain: witchchain2

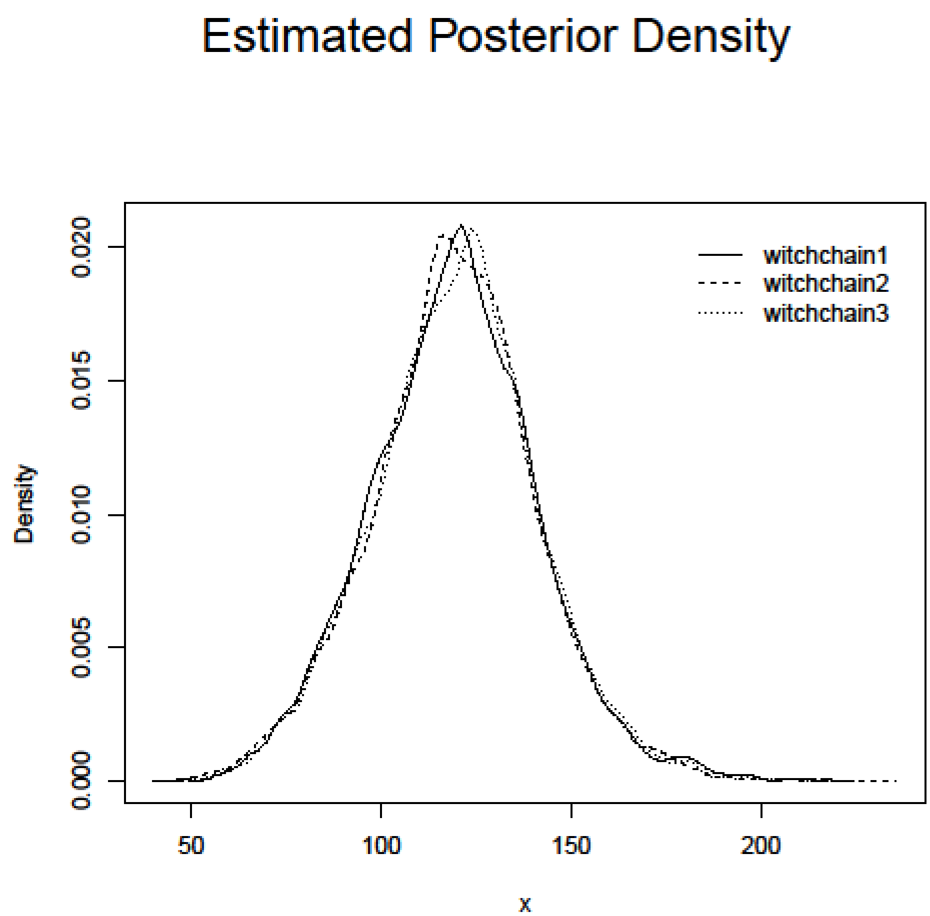
Z-Score -1.95147591

p-value 0.05100046

Chain: witchchain3

Z-Score -0.9032412

p-value 0.3663979


Estimated Posterior Density
witchchain1
witchchain2
witchchain3
Density
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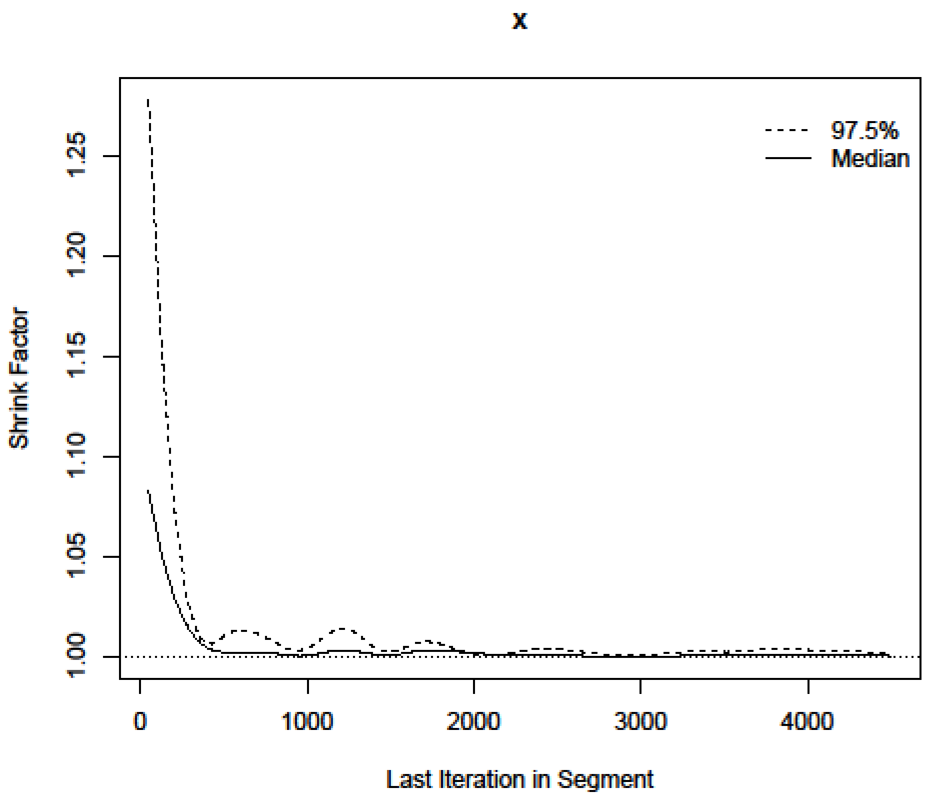
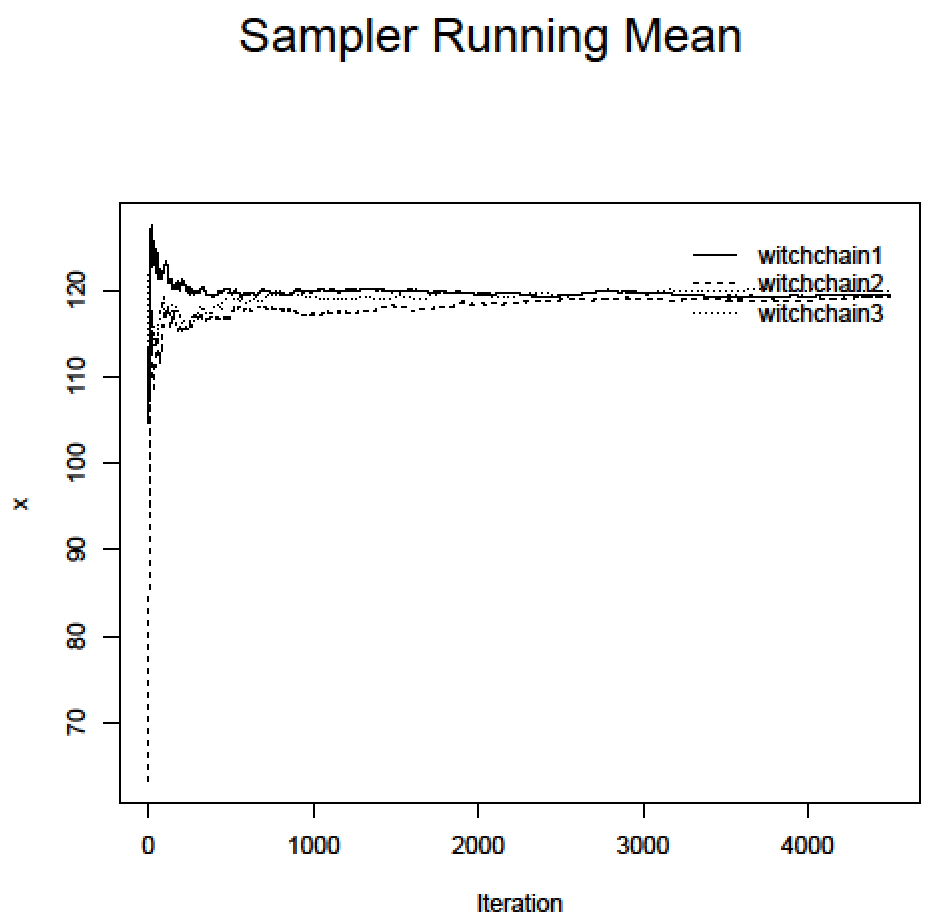
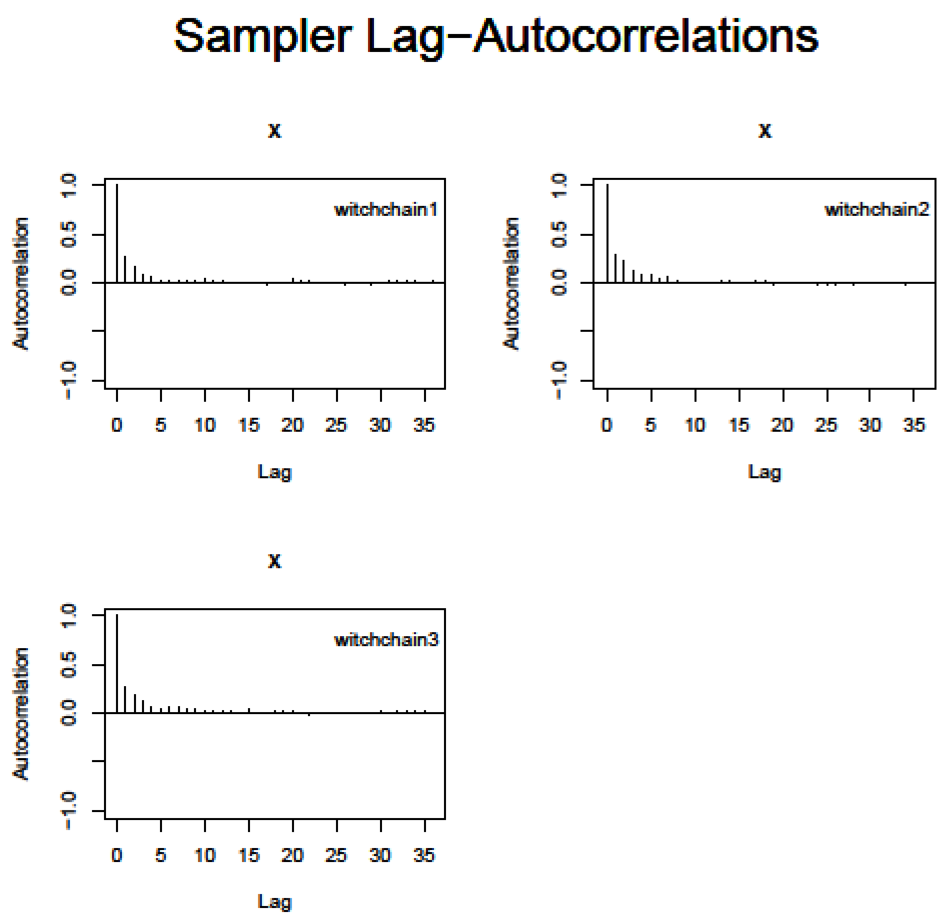
0.005
0.010
0.015
0.020
50
100
150
200
x

Gelman & Rubin Shrink Factors




Sampler Running Mean
witchchain1
witchchain2
witchchain3
x
70
80
90
100
110
120
0
1000
2000
3000
4000
Iteration


Sampler Lag−Autocorrelations
x
witchchain1
Autocorrelation
Lag
x
witchchain2
Autocorrelation
Lag
x
witchchain3
Autocorrelation
Lag

Convergence Diagnostics Sigma

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.0830	0.0674	0.0010	0.0019	0.0019	0.0148	0.0033	0.0676	0.2527

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.0812	0.0675	0.0010	0.0022	0.0020	0.1101	0.0028	0.0657	0.2519

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.0832	0.0656	0.0009	0.0016	0.0015	0.1175	0.0029	0.0690	0.2436

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

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Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000521

Multivariate Potential Scale Reduction Factor = 1.000893

Corrected Scale Reduction Factors

Estimate 0.975

x 1.001081 1.003084

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.7496041

p-value 0.4534932

Chain: witchchain2

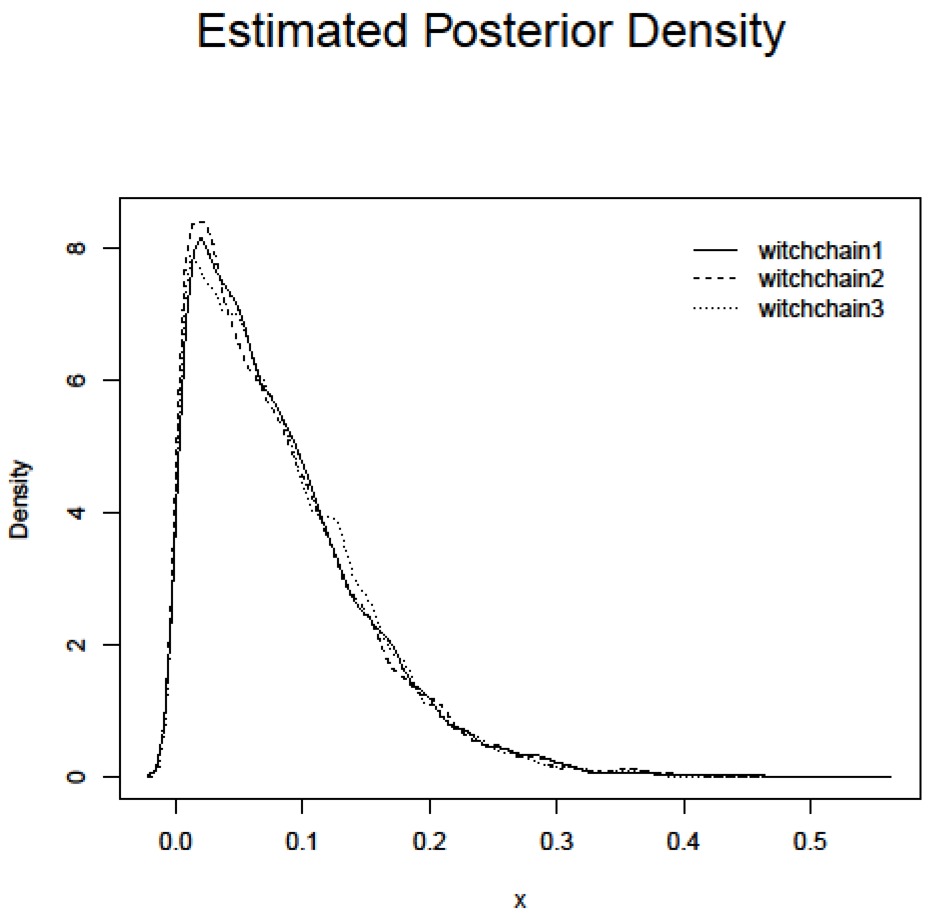
Z-Score 1.1119301

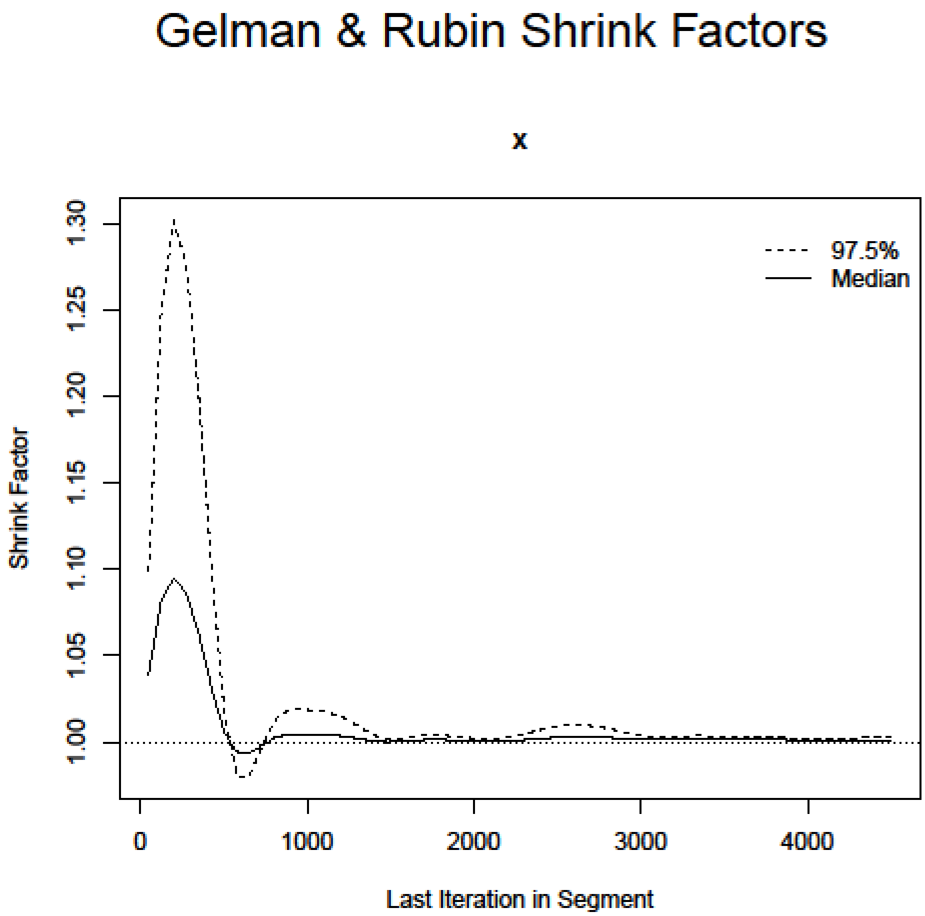
p-value 0.2661682

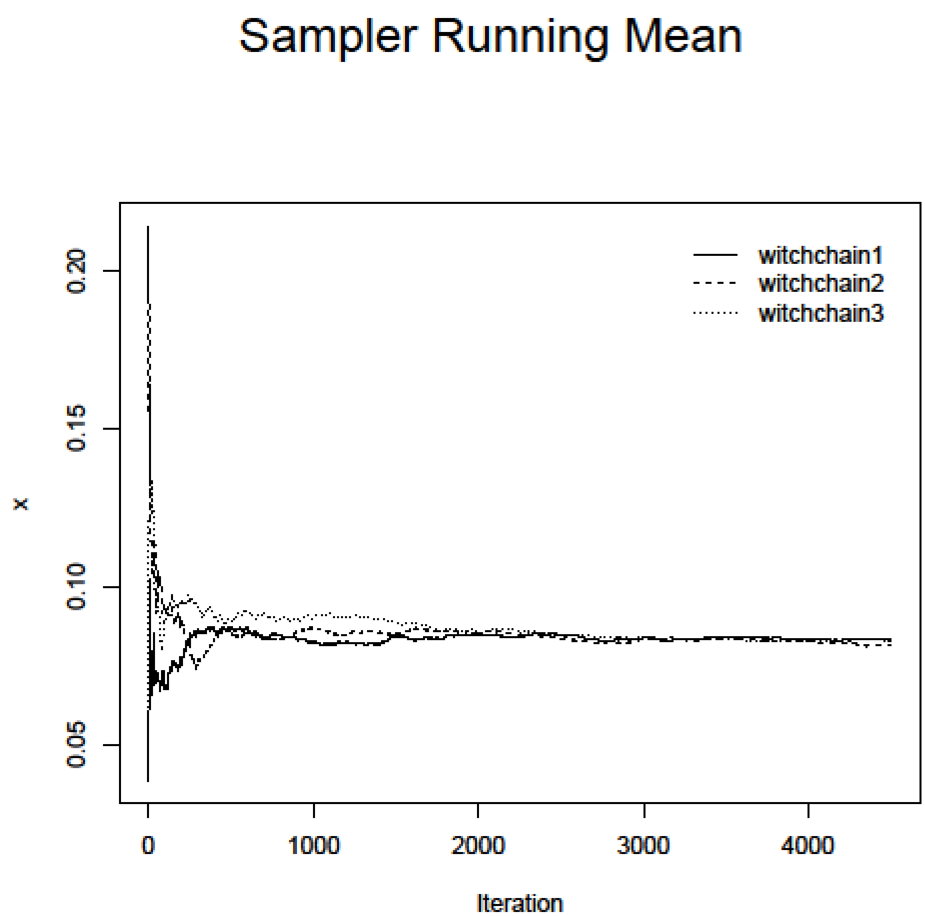
Chain: witchchain3

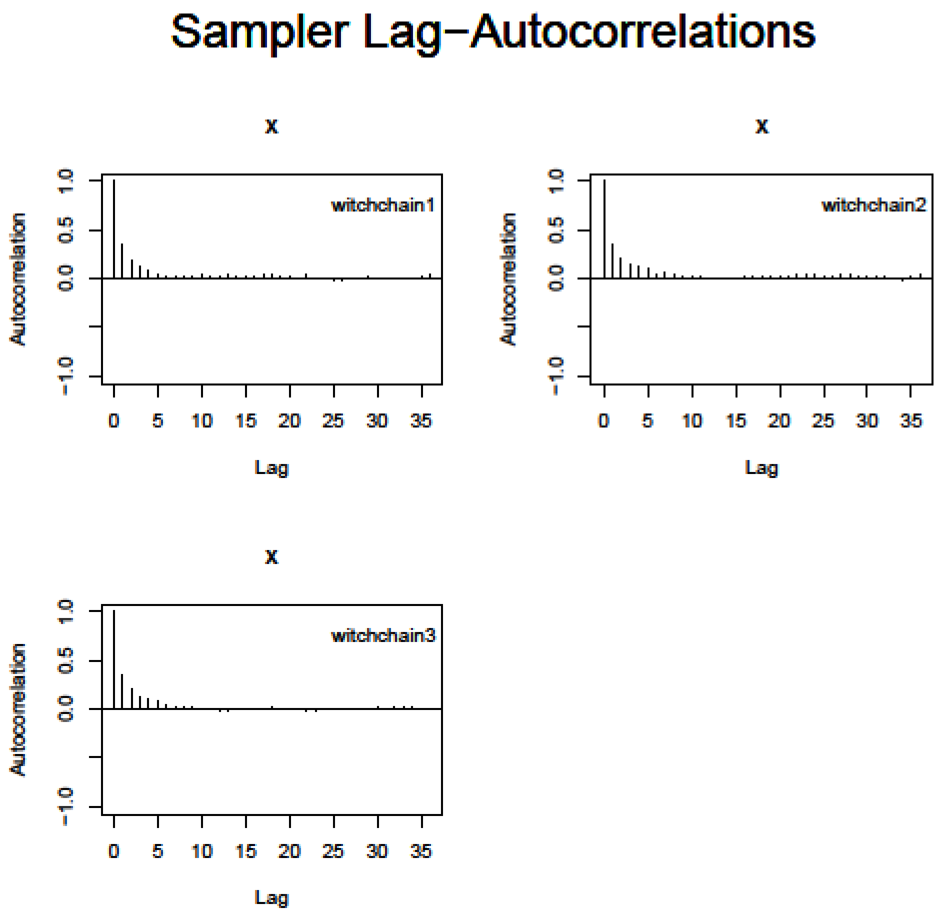
Z-Score 1.3877805

p-value 0.1652039


Estimated Posterior Density
witchchain1
witchchain2
witchchain3
Density
x
0
2
4
6
8
0.0
0.1
0.2
0.3
0.4
0.5


Gelman & Rubin Shrink Factors
x
97.5%
Median
Shrink Factor
1.00
1.05
1.10
1.15
1.20
1.25
1.30
0
1000
2000
3000
4000
Last Iteration in Segment


Sampler Running Mean
witchchain1
witchchain2
witchchain3
x
0.05
0.10
0.15
0.20
0
1000
2000
3000
4000
Iteration


Sampler Lag−Autocorrelations
x
witchchain1
witchchain2
witchchain3
Autocorrelation
Lag
1.0
0.5
0.0
−1.0
0
5
10
15
20
25
30
35

Convergence Diagnostics Q spring late

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.3460	0.1054	0.0015	0.0031	0.0029	0.0657	0.1819	0.3326	0.5931

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.3445	0.1053	0.0015	0.0031	0.0030	0.0090	0.1775	0.3298	0.5925

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.3425	0.1066	0.0015	0.0031	0.0031	-0.1248	0.1803	0.3278	0.5990

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

===

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.00016

Multivariate Potential Scale Reduction Factor = 1.000351

Corrected Scale Reduction Factors

Estimate 0.975

x 1.00042 1.001449

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.1921725

p-value 0.8476071

Chain: witchchain2

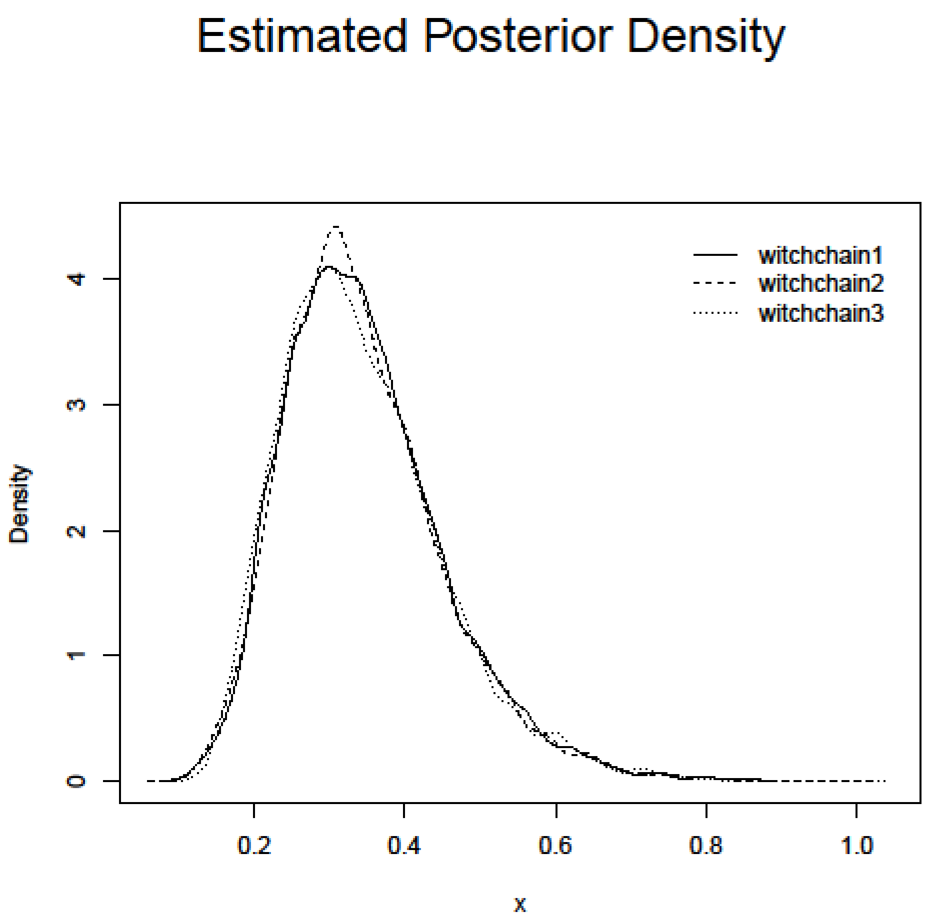
Z-Score 0.9285419

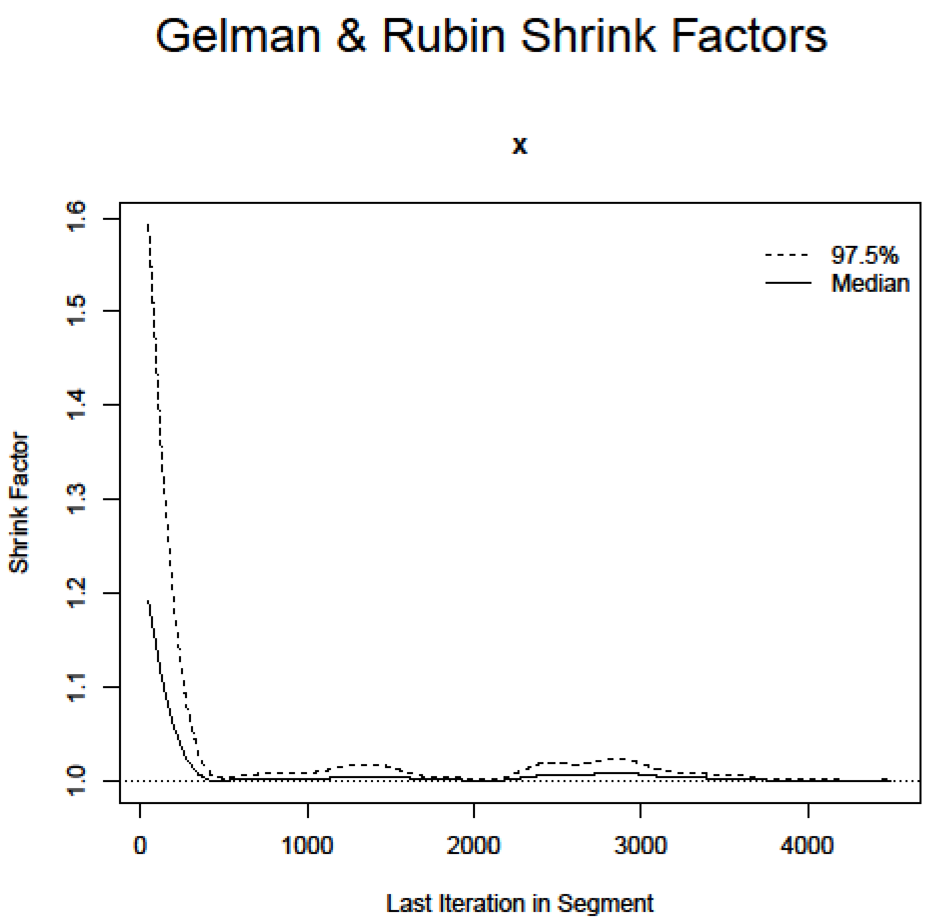
p-value 0.3531265

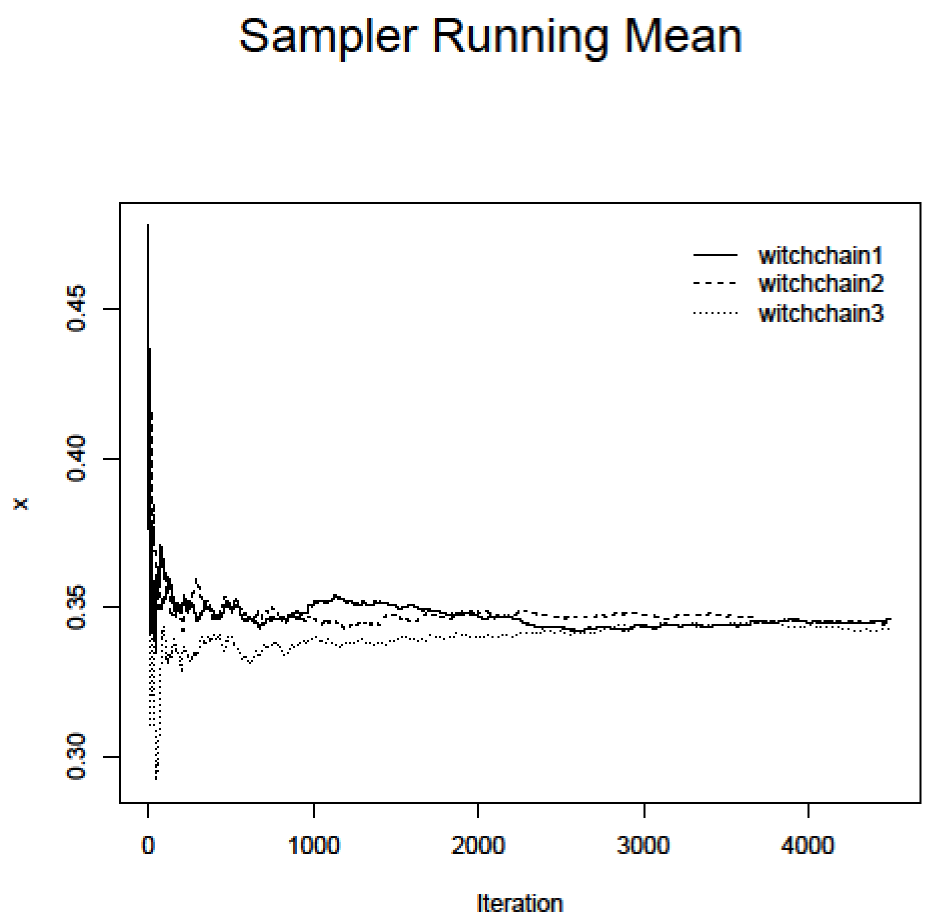
Chain: witchchain3

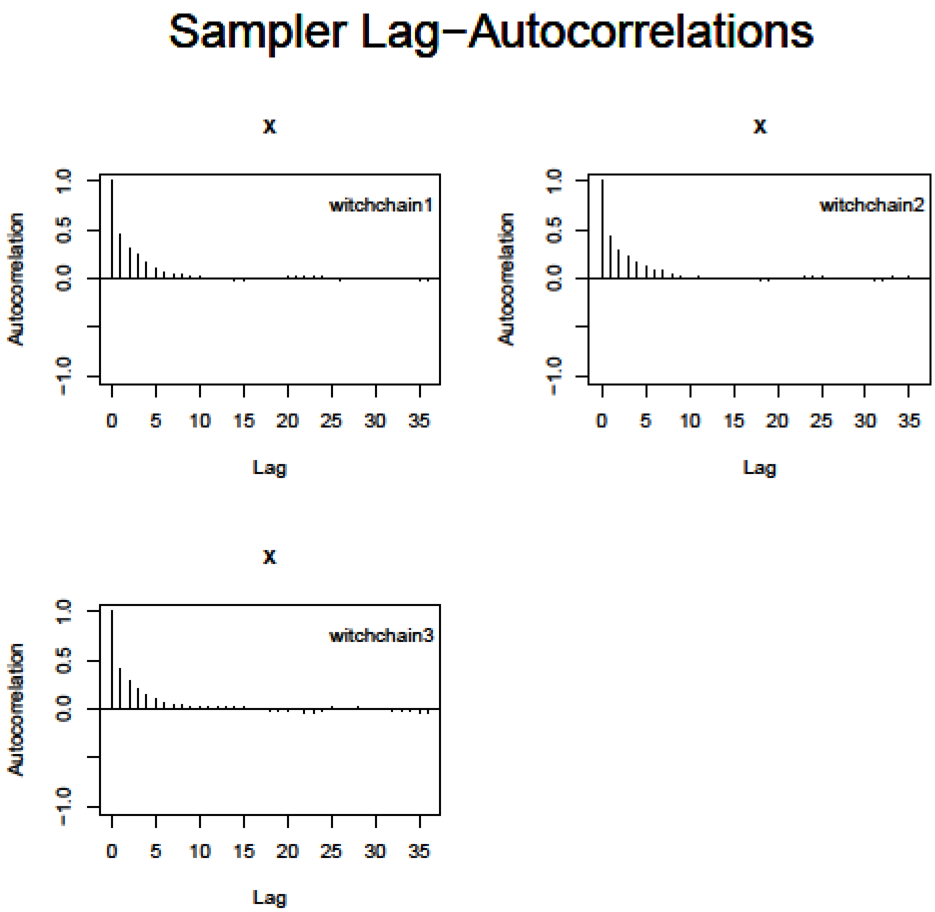
Z-Score -0.6751152

p-value 0.4996026


Estimated Posterior Density
witchchain1
witchchain2
witchchain3
Density
0
1
2
3
4
0.2
0.4
0.6
0.8
1.0
x


Gelman & Rubin Shrink Factors
x
97.5%
Median
Shrink Factor
1.0
1.1
1.2
1.3
1.4
1.5
1.6
0
1000
2000
3000
4000
Last Iteration in Segment


Sampler Running Mean
witchchain1
witchchain2
witchchain3
0.30
0.35
0.40
0.45
x
0
1000
2000
3000
4000
Iteration


Sampler Lag−Autocorrelations
x
witchchain1
witchchain2
witchchain3
Autocorrelation
−1.0 0.0 0.5 1.0
0 5 10 15 20 25 30 35
Lag

Convergence Diagnostics Q fall

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.5159	0.1569	0.0023	0.0044	0.0042	0.0323	0.2732	0.4964	0.8763

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.5130	0.1541	0.0022	0.0047	0.0046	-0.0367	0.2684	0.4965	0.8680

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.5124	0.1570	0.0023	0.0048	0.0048	-0.1279	0.2702	0.4918	0.8905

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

==

Iterations used = 2251:4500

Potential Scale Reduction Factors

x

1.000221

Multivariate Potential Scale Reduction Factor = 1.000443

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000812 1.002008

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score 0.1687223

p-value 0.8660150

Chain: witchchain2

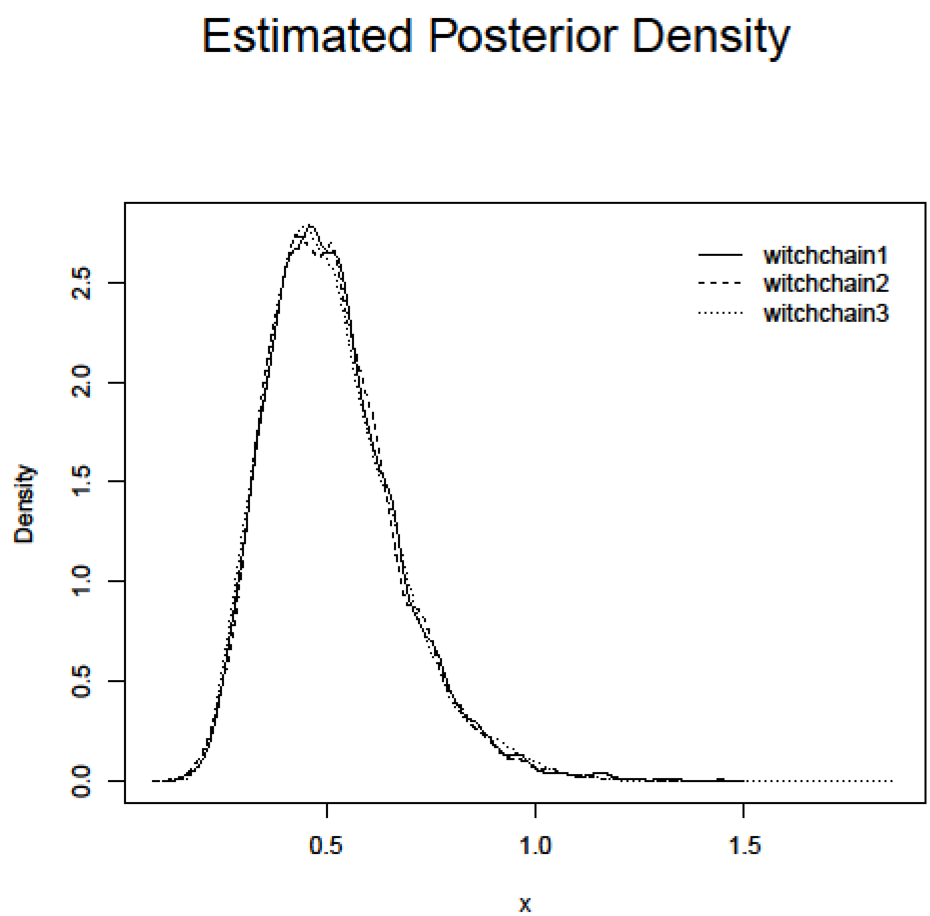
Z-Score 0.8793587

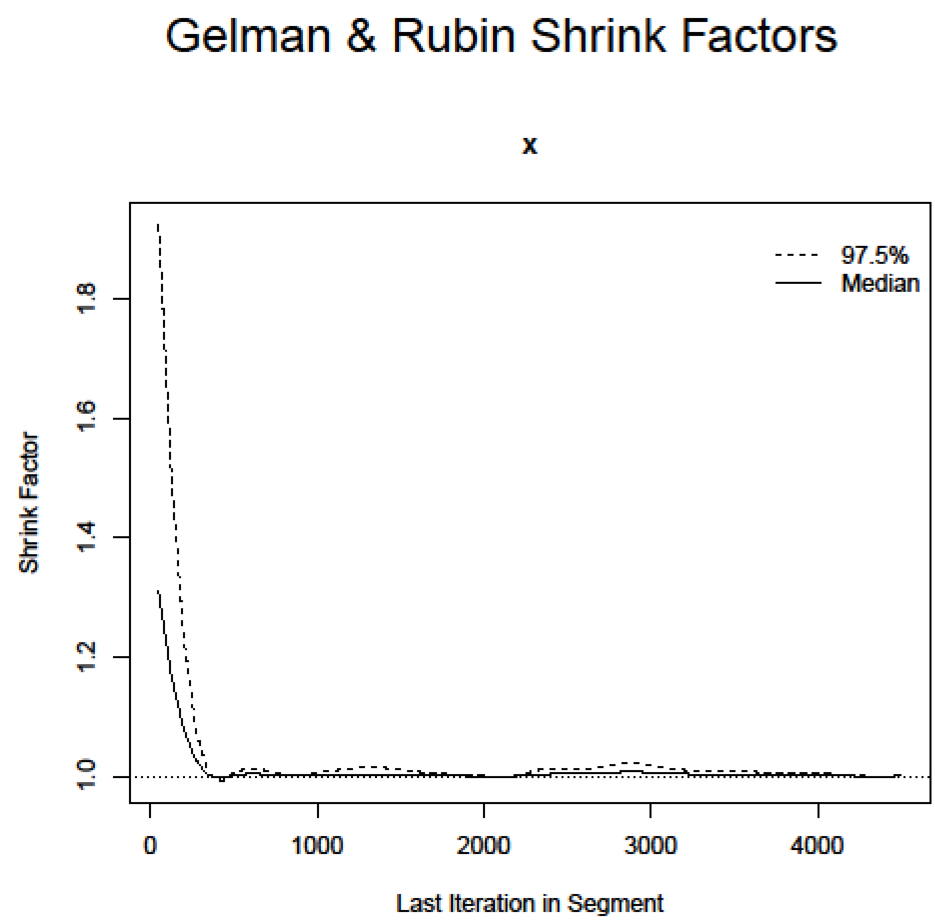
p-value 0.3792068

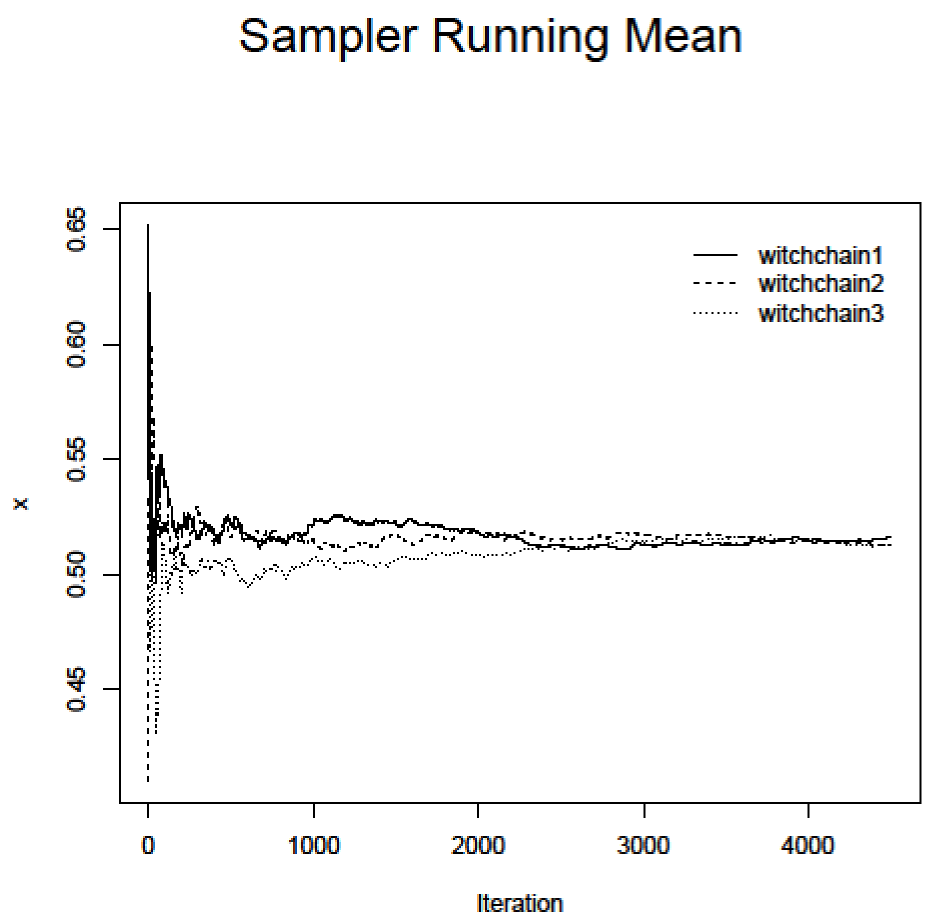
Chain: witchchain3

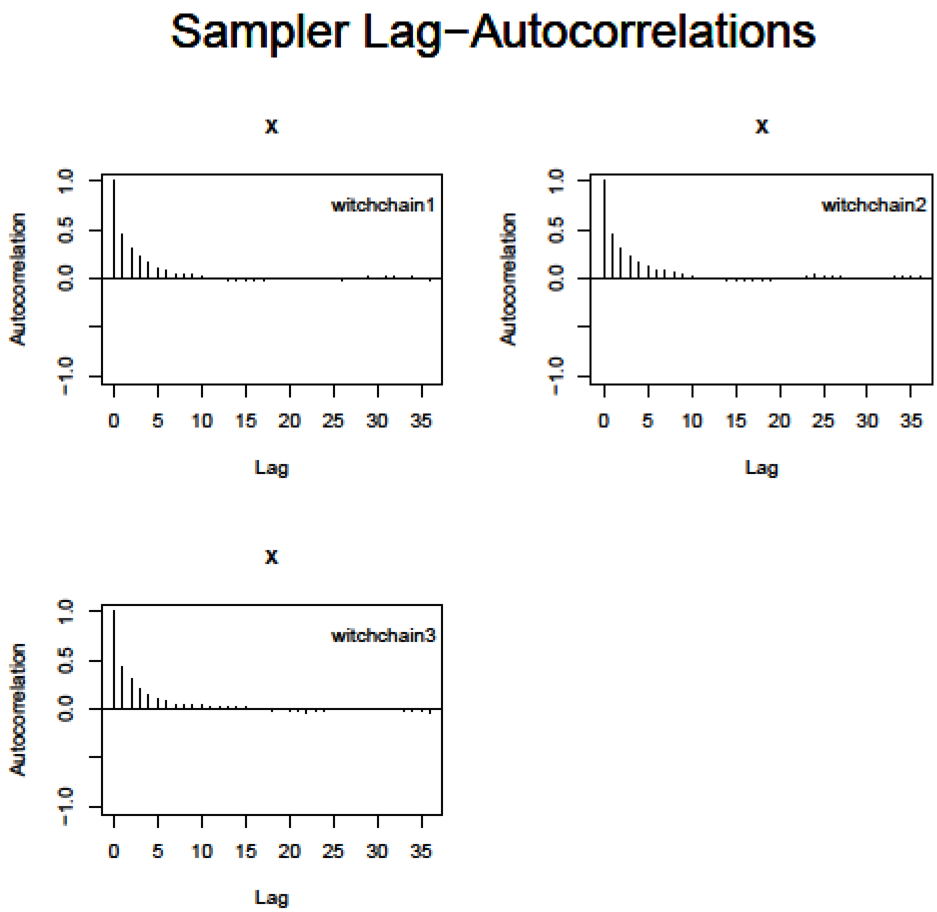
Z-Score -0.8231069

p-value 0.4104472


Estimated Posterior Density
witchchain1
witchchain2
witchchain3
Density
0.0
0.5
1.0
1.5
2.0
2.5
0.5
1.0
1.5
x


Gelman & Rubin Shrink Factors
x
97.5%
Median
Shrink Factor
1.0
1.2
1.4
1.6
1.8
0
1000
2000
3000
4000
Last Iteration in Segment


Sampler Running Mean
witchchain1
witchchain2
witchchain3
0.65
0.60
0.55
0.50
0.45
x
0
1000
2000
3000
4000
Iteration


Sampler Lag−Autocorrelations
x
witchchain1
Autocorrelation
−1.0 0.0 0.5 1.0
0 5 10 15 20 25 30 35
Lag
x
witchchain2
Autocorrelation
−1.0 0.0 0.5 1.0
0 5 10 15 20 25 30 35
Lag
x
witchchain3
Autocorrelation
−1.0 0.0 0.5 1.0
0 5 10 15 20 25 30 35
Lag

Convergence Diagnostics Q spring early

SUMMARY STATISTICS:

===================

Bin size for calculating Batch SE and (Lag 1) ACF = 50

Chain: witchchain1

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.4332	0.1152	0.0017	0.0020	0.0019	0.0976	0.257	0.4165	0.7061

Chain: witchchain2

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.4339	0.1150	0.0017	0.0019	0.0019	0.0040	0.2576	0.4171	0.7099

Chain: witchchain3

Mean	SD	Naive SE	MC Error	Batch SE	Batch ACF	0.025	0.5	0.975
0.4349	0.1310	0.0019	0.0025	0.0022	-0.1126	0.2573	0.4165	0.7134

BROOKS, GELMAN, AND RUBIN CONVERGENCE DIAGNOSTICS:

===

Iterations used = 2251:4500

Potential Scale Reduction Factors

----

x

0.9999342

Multivariate Potential Scale Reduction Factor = 1.000012

Corrected Scale Reduction Factors

Estimate 0.975

x 1.000277 1.000698

GEWEKE CONVERGENCE DIAGNOSTIC:

==============================

Fraction in first window = 0.1

Fraction in last window = 0.5

Chain: witchchain1

Z-Score -0.3218700

p-value 0.7475512

Chain: witchchain2

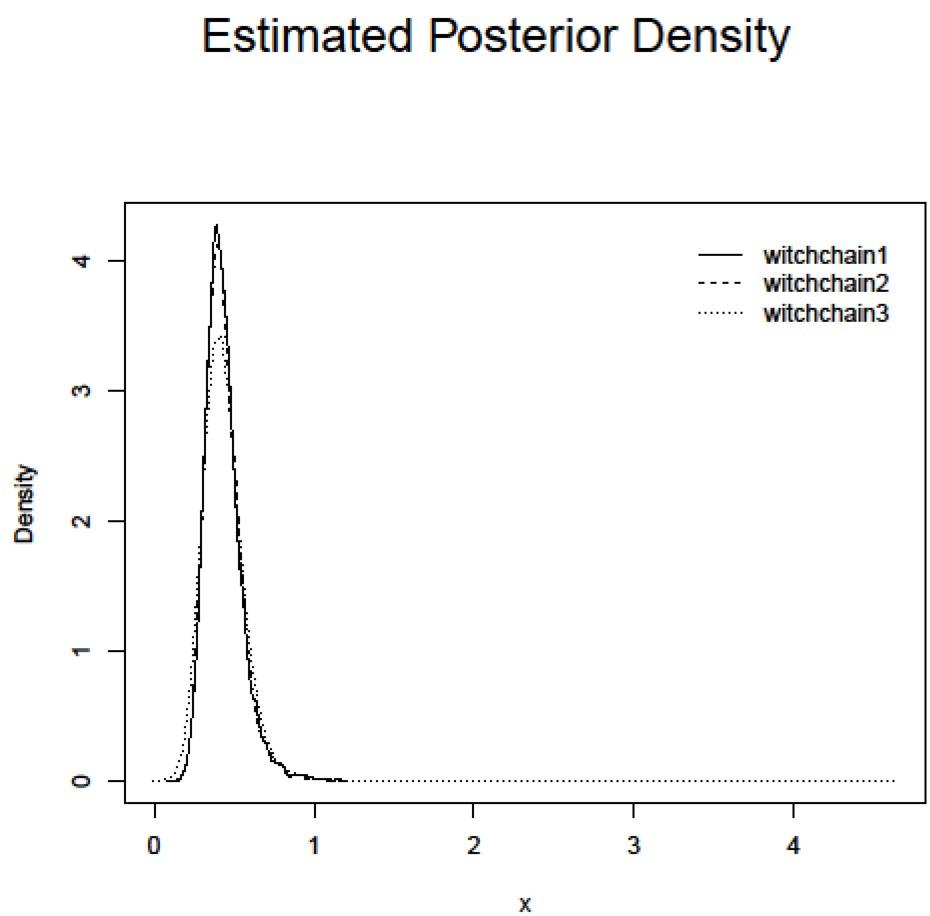
Z-Score 2.23987852

p-value 0.02509881

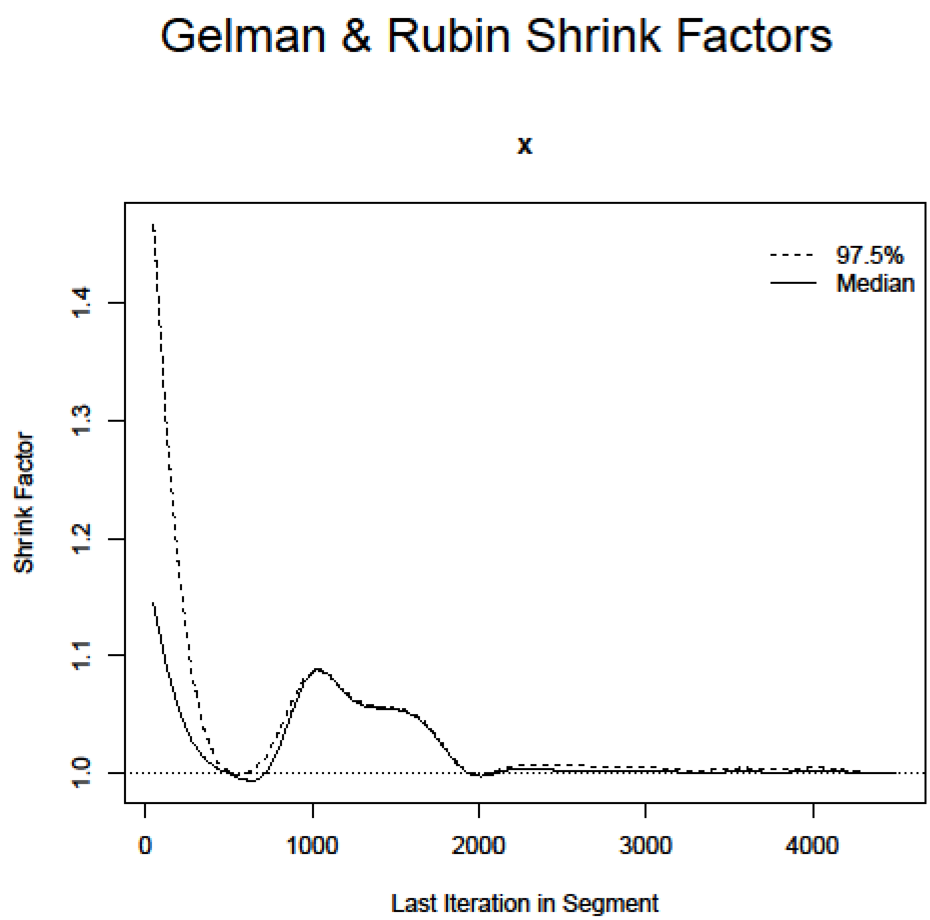
Chain: witchchain3

Z-Score -1.0057220

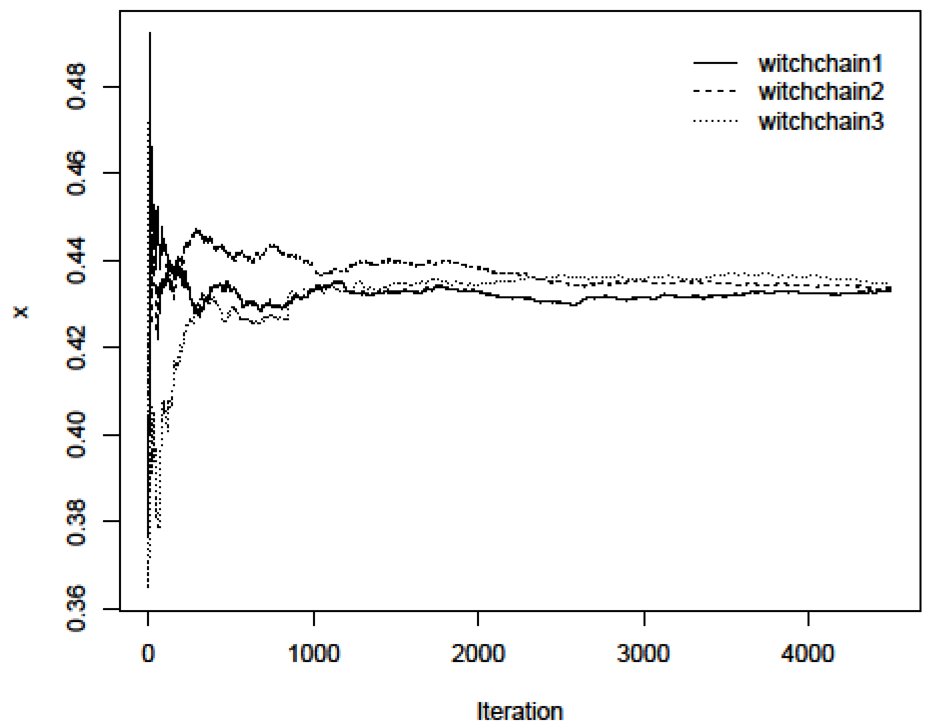
p-value 0.3145493

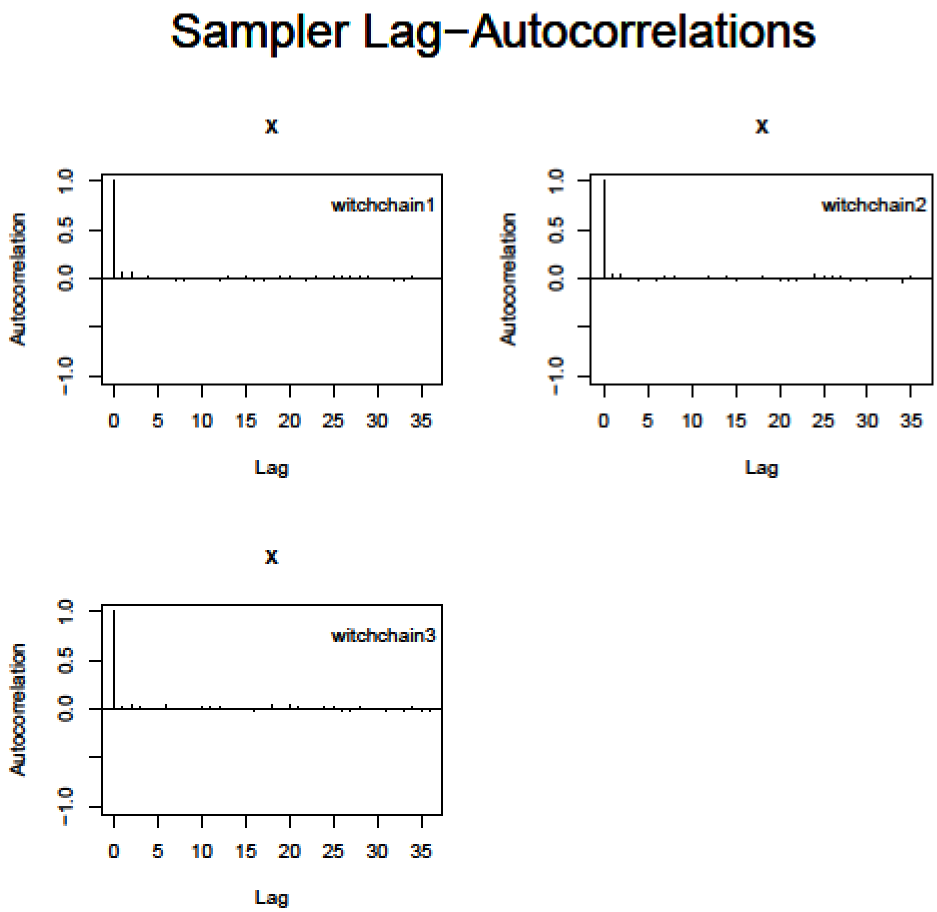


Estimated Posterior Density
Density
x
witchchain1
witchchain2
witchchain3
0
1
2
3
4
0
1
2
3
4


Gelman & Rubin Shrink Factors
x
97.5%
Median
Shrink Factor
1.0
1.1
1.2
1.3
1.4
0
1000
2000
3000
4000
Last Iteration in Segment

Sampler Running Mean




Sampler Lag−Autocorrelations
x
witchchain1
Autocorrelation
−1.0
0.0
0.5
1.0
0 5 10 15 20 25 30 35
Lag
x
witchchain2
Autocorrelation
−1.0
0.0
0.5
1.0
0 5 10 15 20 25 30 35
Lag
x
witchchain3
Autocorrelation
−1.0
0.0
0.5
1.0
0 5 10 15 20 25 30 35
Lag