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Modelling 3LN redfish with a Schaefer model including one recruitment pulse

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Abstract

Past modelling attempts are revised to consider a Schaefer biomass production extended to include a single exceptional recruitment pulse (in 2003), leading to a pulse of additional biomass moving through the resource over following years. This leads to a good fit to the survey data without the need to include further process error, though estimated catchability coefficients are somewhat low. Plans for further work are set out, including an update of the life-history parameter values used, sensitivities to the assessment, and then progression to the development of CMPs.

Modelling Approach

The underlying (aggregated biomass) population model assumed is a Schaefer production model with a one-year (currently taken to be 2003) pulse in recruitment (whose magnitude is estimated). This leads to a biomass pulse moving through the resource (and fishery) over time, additional to the biomass from other cohorts, which in the absence of the pulse is governed by the Schaefer model.

$$B_y^{tot} = B_y^{prod} + B_y^{pulse} \quad (1)$$

where:

B_y^{tot} is the total biomass of the resource at the start of year y,

B_y^{prod} is the “normal” biomass of the resource from the standard production model, at the start of year y,

B_y^{pulse} is the peak in biomass at the start of year y arising from the recruitment pulse.

Further define:

$$p_y = B_y^{prod}/B_y^{tot} \quad (2)$$

<u>“Normal” biomass dynamics:</u>

$$B_{y+1}^{prod} = B_y^{prod} + P_y - C_y p_y + \mu_y \quad (3)$$

where:

C_y is the total catch taken during year y,

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$P_y = rB_y^{prod}(1-\frac{B_y^{tot}}{K})$ is the net productivity of the resource in year y as a function of parameters K and r, with

K the carrying capacity,

r the intrinsic growth rate, and

μ_y the productivity process error, taken to be distributed as $N\left(0,\left(\sigma_\mu MSY\right)^2\right)$, with the value of σ_μ an input. Note that $MSY = \frac{rK}{4}$.

(The μ_y are all set to zero for this initial application.)

"Pulse" biomass:

For years y_{pulse} (=2003 for this initial application) and thereafter:

Start of year:

$$B_{start\ y}^{pulse} = WAA_{a=y-ypulse} N_y^{pulse} e^{-M/2} \quad (4)$$

where WAA_a is the mass of a redfish of age a at the start of the year

End of year:

$$B_{start\ y}^{pulse} = \left[B_{start\ y}^{pulse} - C_y\left(1-p_y\right)\right] e^{-M/2} \quad (5)$$

i.e. reduce by the proportion of catch taken that year (approximated as taken at the mid-point of the year) from the pulse year-class.

Then:

$$N_{y+1}^{pulse} = B_{end\ y}^{pulse} / WAA_{a=y-ypulse} \quad (6)$$

Model fitting procedure

The likelihood of the model fit is calculated by assuming that the observed abundance indices (CPUE and abundance surveys, though the former is not used for the analysis reported here) are log-normally distributed about their expected values:

$$I_y^i = \hat{I}_y^i e^{\varepsilon_y^i} \quad \text{or} \quad \varepsilon_y^i = \ell n\left(I_y^i\right) - \ell n\left(\hat{I}_y^i\right) \quad (7)$$

where:

I_y^i is the abundance index for year y and series i,

$\hat{I}_y^i = q^i \hat{B}_y$ is the corresponding model estimate,

$\hat{q}^i$ is the multiplicative bias (catchability) for abundance series i, and

ε_y^i from $N(0,(\sigma^i)^2)$.

The contribution of the CPUE (if used) and survey data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell nL = \sum_{i,y}\left[\ell n\sqrt{(\sigma^i)^2+(\sigma_A)^2} + \left(\varepsilon_y^i\right)^2/2((\sigma^i)^2+(\sigma_A)^2)\right] \quad (8)$$

where:

σ^i is the (minimum, when $\sigma_A = 0$) standard deviation of the residuals for the logarithms of index *i*, ,

σ_A is the square root of the additional variance for index series i, which is an input value; this is used as means for specifying an effective lower bound for σ^i.

σ^i is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n_i \sum_y \left(\ell n(I_y^i) - \ell n(q^i \hat{B}_y)\right)^2 - (\sigma_A)^2} \tag{9}$$

where:

n_i is the number of data points for abundance index i.

The catchability coefficient q^i for abundance index *i* is estimated by its maximum likelihood value which is given by:

$$\ell n\hat{q}^i = \frac{1}{n_i}\sum_y \left(\ell n I_y^i - \ell n\hat{B}_y\right) \tag{10}$$

In some cases, a penalty factor has been added to the overall -ln$L$ to avoid the survey q estimates reflecting unrealistic values:

$$\sum_i (q_i - Q)^2/\sigma_q^2 \tag{11}$$

where Q is fixed externally and here $\sigma_q = 0.2$.

The process error on the productivity, μ_y, are estimable parameters, with their estimation being rendered possible by assuming that they follow a normal distribution, so that (in what is an *ad hoc* approach) the following term is added to $-\ell nL$ in equation (8):

$$\sum_y \left[ln\left(\sigma_\mu MSY\right) + \left(\mu_y\right)^2/2\left(\sigma_\mu MSY\right)^2\right] \tag{12}$$

Since the estimation of σ_μ is confounded with that of the observation error variances for the survey series, a value for σ_μ in input with the intent of later investigation the robustness of any MP put forward to alternative values for σ_μ. Initially $\sigma_\mu = 1$, but for the Base Case ("*New*") Operating Model (OM) all process errors are set to zero.

Note for more general extensions:

- The process error term standard deviation is scaled by MSY to allow for a readier interpretation of its value.
- To allow μ_y to be estimable for a specific year requires at least two abundance index series to be available for that year, as otherwise the value of the process error will simply adjust to fit a single index for the year exactly (and naturally is inestimable if there is no index for that year). Hence these errors are estimated from 1985 only; they have to be set to zero for earlier years.
- A similar reason underlies the introduction of an additional variance parameter with a fixed input value for each series; given that the observation error variances for each series are estimated in the fitting process, that process can otherwise become unstable, according inappropriately high relative weight to a single index to fit it exactly while ignoring all the others.
- The choice of the value (1) for σ_μ was dictated by allowing this parameter to be sufficiently large in earlier versions of this model to admit reasonable reflection of the main trends in the abundance indices, but no larger.

Parameter values

For the Base Case ("*New*") OM:

r=0.2

$\sigma_\mu = 1$ (though this has no impact as here all process errors are set to zero)

The r value is a plausible choice simply to get calculations started. The reason for the σ_μ choice is detailed above.

The life-history parameter values used here are given in Table A1 of Appendix A.

Estimable parameters:

- B_{y1}^{prod} First year "normal" biomass
- N_{ypulse}^{pulse} Pulse numbers in year y_{pulse}
- MSY

Data

The catches and survey index values given in Perreault *et al*. (2024) and the CPUE values from Rogers *et al*. (2022) are available for use for this analysis; they are reproduced in Tables A2 and A3 (Appendix A) for the period starting from 1959, as is considered here, though only the survey series are used for this specific analysis.

The choice of the survey series reported in Perreault *et al*. (2024) was made as they were assumed to reflect the SC's choice of which surveys were the most reliable as representative of 3LN redfish abundance. Compared to that selection, which was as made in Butterworth and Rademeyer (2025), the CPUE series has been listed here too so as to allow estimation (especially of process error) to be extended back into the previous century in other implementations.

Further analyses planned

Future dynamics

Future annual productivity process error will take autocorrelation into account (as the model fits suggest that this is not insubstantial):

$$\mu_y = \rho\mu_{y-1} + \sqrt{1-\rho^2}\varphi_y \qquad (13)$$

φ_y from $N(0, \sigma_\mu^{out} MSY)$

$\sigma_\mu^{out} = 0.8$ is the approximate standard deviation of the 1985-2024 estimated μ_y values

ρ is the serial correlation coefficient, which is input ($\rho = 0.3$ was the estimated value in previous versions of the model).

A single (for initial simplicity) future survey index I_y is assumed to be available with a log-normal CV of 0.4. This will later be replaced by a number of the existing survey indices considered likely to continue to be available in the future.

Candidate Management Procedure (CMP)

A simple CMP is planned to be implemented initially to compute TACs in the future which is based on J_y, a three-year average of the survey index I_y; note that the assumption is made that the survey value would be available only up to two years before the year for which the TAC would be being recommended:

$$TAC_y = TAC_{y-1}\left[1 + \alpha(\frac{J_{y-2}}{J_{targ}} - 1)\right] \qquad (14)$$

Furthermore, if J_{y-2} relative to J_{2023} falls below a fixed specified threshold ($J_{threshold}$), the resulting TAC is decreased further:

$$TAC_y \rightarrow TAC_y \frac{J_{y-2}}{J_{2023}} \qquad (15)$$

with

$$J_y = \sum_{y-2}^{y} I_y \Big/ 3$$

$I_y = qB_y e^{\varepsilon_y - \sigma^2/2}$ the survey index in year y

$\varepsilon_y \sim N(0, \sigma^2)$ with $\sigma = 0.4$ and,

α, J_{targ} and $J_{threshold}$ the CMP's tuning parameters.

Note that for $\alpha = 1$, this corresponds to an intended constant fishing mortality strategy, except that if abundance falls below $J_{threshold}$ that mortality is reduced linearly to zero as abundance falls to zero.

Furthermore, TACs are subject to a maximum inter-annual decrease or increase of 20%, except if J_{y-2} relative to J_{2023} falls below $J_{threshold}$, in which case the maximum decrease increases to $(100 - \frac{80}{Jthresh}\frac{J_{y-2}}{J_{2023}})\%$

For this Baseline CMP, $\alpha = 1$, $J_{targ} = 1.2$ and $J_{threshold}$=0.8.

Some assessment model results

Key features of this revised model as at present are as follows:

- A Schaefer model is used as in papers submitted in 2025, but with a single exceptional recruitment pulse in 2003 added; the latter is modelled by its cohort biomass over time less removal of a proportional amount of the catches
- Thus far Schaefer *r* is set to 0.2; the values of the other parameters are all estimated in the model fit
- There is NO further process error, aside from this single exceptional year-class, at this stage

In this initial exploration, a choice for y_{pulse} of 2003 was found to give the best fit of results to the data.

Figure 1 shows results of the fit of this model to the data, giving the historical catch, the annual productivity, the biomass trajectory and the fits to each survey series, while Tables 1 and 2 provide the associated estimates of parameter values. Results are presented for the most recent run ("*New*") and for a run setting *Q*=0.7 (see equation 11), to avoid arguably unrealistic values for survey catchabilities.

The fit to the survey data is good, BUT there is the reverse of the 2025 problem with the values of the q's: they are now generally well below 1, leading to very large biomasses (especially recently).

This can be "solved" this by forcing the q's to be somewhat higher, but there is an associated "negative" which is a marked steady decrease rather than fairly steady biomass increase over the 60s to the 80s.

Immediate plans for future work

1) The values used for life history parameters (M and WAA) (see Table A1 of Appendix A) were taken from a redfish analysis of some years ago. Advice on the best choices for revised choices based on subsequent analyses will be sought from the sub-group for this work appointed by the NAFO SC. Given those, the assessment presented here will be updated.

2) Sensitivity tests to this assessment will be conducted to provide the basis for robustness checks of CMPs. This will include, *inter alia*, consideration of alternative values for M and WAA, and different choices for the year of the recruitment pulse, as well as alternative choices for the value of the intrinsic growth rate parameter r.

3) This will be followed by examination of the performance achieved by the simple form of CMP suggested.

4) CMP testing will need to include a number of robustness tests in addition to the sensitivities suggested above, and probably further values for Q to adjust catchability estimates. Most important here will be specifying different possibilities for further recruitment pulses in the future. Probably this might best start simply by allowing for a single such pulse of the same magnitude as estimated for the current (2003) one, and happening a few years after the projection period starts (say 5 years into a 30 year future management period

References

Butterworth DS and Rademeyer RA. 2025. Outline of a simple approach to develop a MSE for 3LN redfish-extended. NAFO SCR Doc. 25/005, 20pp.

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Table 1. Comparison of -ln*L* and some results for the most recent run (*New*) OM and one run with penalty on the survey *q*'s. Values input are shown **bolded.**

	New	*Q=0.7*
K	585	296
MSY	29.2	14.8
r	**0.20**	**0.20**
N^{pulse}_{2023} (x10^9)	15.09	2.18
-lnL		
Total	17.62	31.39
Process error	-	-
survey *q* penalty	-	3.82
surveys	17.62	27.58
CPUE	-	
B_{1959}	179.4	837.3
B_{1959}/K	0.31	2.83
B_{2024}	1479.3	213.1
B_{2024}/K	2.53	0.72
B_{2024}/MSYL	5.06	1.44

Table 2. Comparison of survey -*lnL, s*'s and *q*'s results for the most recent run (New) OM and one run with penalty on the survey *q*'s.

		Spr_Camp	Au_Camp	EUSp3N	EUSp3L	spr_Tel
New	*q*'s	0.11	0.20	0.11	0.08	0.10
	σ	0.69	0.62	0.85	0.72	0.74
	-*lnL*	4.1	0.6	9.3	2.8	0.8
Qlow=0.7	*q*'s	0.41	1.00	0.64	0.44	0.68
	σ	0.98	0.65	0.83	0.67	0.69
	-*lnL*	14.8	1.9	8.6	1.7	0.5

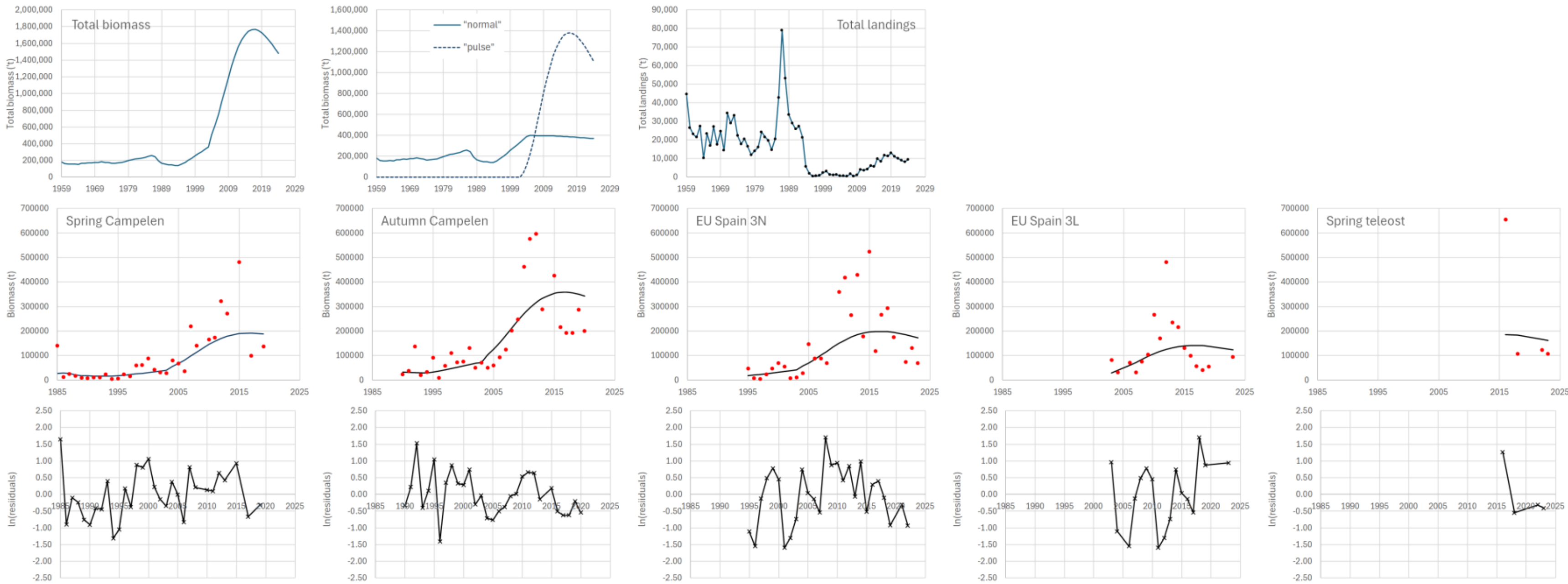

Figure 1a. Estimated annual total biomass, as well as biomass split between "normal" and "pulse", and historical total landings (in kt) **for the most recent run (New) OM.** The fits of the model to the survey data series are also shown.

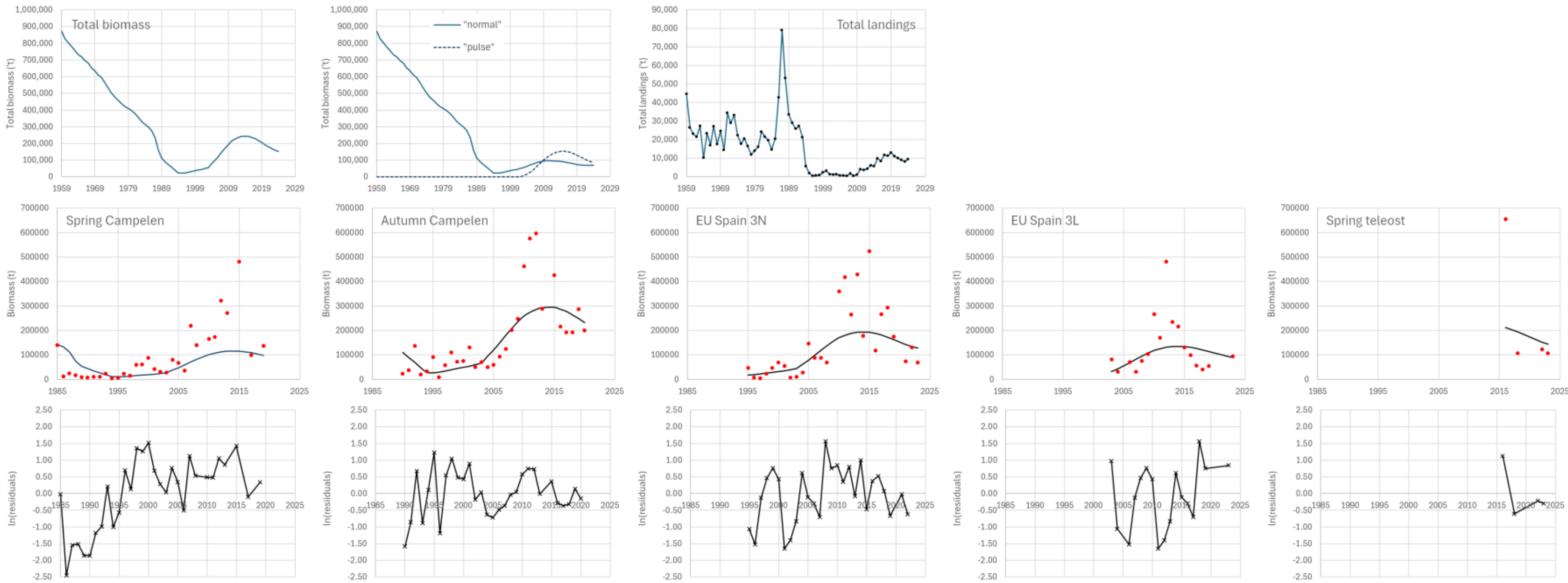

Figure 1b. Estimated annual total biomass, as well as split between "normal" and "pulse", and historical total landings (in kt) **for the most recent run with penalty on the survey *q*'s (*Q*=0.7).** The fits of the model to the CPUE and survey data series are also shown.

Appendix A

Life-history parameters and data used

Table A1. Life-history parameters assumed for 3LN redfish

M	0.1				MacAllister and Duplisea (2011)
	L_{inf}	κ	t_0		
Length-at-age	45.23	0.0698	-1.64	$L_a = L_{inf}(1\text{-}e^{-\kappa(a+0.5\text{-}t0)})$	Don Power, pers. Commn
	α	β			
Weight-at-age	0.0125	3.0935		$W_a = \alpha(L_a)^\beta$	Average of values for *S. fascitius* and *S. mentella* in MacAllister and Duplisea (2011)

Table A2. Total landings (in t) of redfish in Divs. 3LN for 2005 to 2023 (from Perreault *et al.*, 2024). The value for 2024 is as kindly advised by Bell MacCallum (*pers. commn*).

Year	Landings ('000t)	Year	Landings ('000t)	Year	Landings ('000t)
1959	44.585	1981	24.28	2003	1.334
1960	26.562	1982	21.547	2004	0.637
1961	23.175	1983	19.747	2005	0.659
1962	21.439	1984	14.761	2006	0.496
1963	27.362	1985	20.557	2007	1.664
1964	10.261	1986	42.805	2008	0.597
1965	23.466	1987	79.031	2009	1.051
1966	16.974	1988	53.266	2010	4.12
1967	27.188	1989	33.649	2011	3.672
1968	17.66	1990	29.105	2012	4.316
1969	24.75	1991	25.815	2013	6.232
1970	14.419	1992	27.283	2014	5.695
1971	34.37	1993	21.308	2015	9.94
1972	28.933	1994	5.741	2016	8.457
1973	33.297	1995	1.989	2017	11.815
1974	22.286	1996	0.451	2018	11.279
1975	17.871	1997	0.63	2019	13.05
1976	20.513	1998	0.899	2020	11.091
1977	16.516	1999	2.318	2021	10.172
1978	12.043	2000	3.141	2022	8.961
1979	14.067	2001	1.442	2023	8.231
1980	16.03	2002	1.216	2024	9.403

Table A2. Survey biomass estimates ('000 t) from bottom trawl surveys in Div. 3LN considered in the assessment (from Perreault *et al.*, 2024). Standardized CPUE for Div. 3LN, 1959-1994 (from Rogers *et al*., 2022) is also listed, but is not used for the model fit developed in this paper.

	Spring Campelen	Autumn Campelen	EU Spain 3N	EU Spain 3L	Spring Teleost
1985	140	-	-	-	-
1986	11.4	-	-	-	-
1987	24	-	-	-	-
1988	16.5	-	-	-	-
1989	8.4	-	-	-	-
1990	6.8	22.6	-	-	-
1991	10.7	37.9	-	-	-
1992	10.1	136.4	-	-	-
1993	22.7	19.2	-	-	-
1994	4.1	31.8	-	-	-
1995	5.9	90.4	46.1	-	-
1996	22.8	8.8	6.6	-	-
1997	15	57.8	4.8	-	-
1998	59.4	110.6	22.5	-	-
1999	61.5	71.9	46.5	-	-
2000	87.9	75.6	68.9	-	-
2001	41.6	130.3	53.9	-	-
2002	31.4	50.1	7.6	-	-
2003	27.7	70.9	11	81.8	-
2004	79.7	50.1	27	30.8	-
2005	66.5	58.6	146.9	-	-
2006	35.3	91.9	87.8	70.1	-
2007	218.9	124.8	87.6	31.4	-
2008	140.0	201.0	68.1	75.6	-
2009	-	246.8	735.7	103.7	-
2010	165.4	461.5	359.5	266.8	-
2011	173.7	576.2	418.3	170.6	-
2012	322.0	596.7	265.2	481.5	-
2013	271.6	289.0	429.5	235.2	-
2014	-	-	178.1	216.4	-
2015	480.6	425.1	523.5	130.4	-
2016	-	215.3	117.3	98.8	654.3
2017	98.8	192.2	265.9	56.6	-
2018	-	191.4	292.8	40.3	106.0
2019	136.5	286.5	174.6	54.0	-
2020	-	199.3	-	-	-
2021	-	-	73.2	-	-
2022	-	-	131.1	-	121.8
2023	-	-	68.1	94.2	106.7

	Standardized CPUE
1959	1.426
1960	1.602
1961	1.697
1962	1.631
1963	1.632
1964	1.812
1965	2.185
1966	1.781
1967	1.893
1968	0.922
1969	1.338
1970	1.367
1971	1.346
1972	1.387
1973	1.643
1974	1.290
1975	1.669
1976	1.292
1977	1.251
1978	1.106
1979	1.451
1980	1.761
1981	1.594
1982	1.661
1983	1.556
1984	1.049
1985	1.084
1986	1.413
1987	1.374
1988	1.029
1989	1.251