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## Excerpts from "0n statistics of mesh selection"

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#### Abstract

NOTE: In the following we have reproduced the introduction, the third section, and part of the main table from the paper "On statistics of mesh selection" which is currently being prepared for publication. The paper is to appear in the Journal of the Fisheries Research Board. Nultigraphed copies of the complete version may be obtained by the end of July 1964 from either one of the authors.


> On statistics of mesh selection

## Introduction

Lots of experimental fishing has been done by various fisheries research organizations to determine the selectivity of fishing nets. Techniques for such experiments appear to be well established and documented (e.g., 1957 FAO-ICES-ICNAF Lisbon meeting). The most common procedure with nets towed by a vessel is to place a cover over the codend part of the net so as to retain all fish of any length which may have escaped through the meshes of the net. Only the data obtained from such experiments will be studied in this paper.

An examination of the subject matter suggests, at least to the uninitiated, such as the authors of this paper, that there is some uncertainty as to what the solution really depends on. It may well be that the selectivity is influenced by so many factors that can never be really well defined without prohibitive amounts of sampling. At the same time it is fairly obvious that the comparison of experimental results is difficult because no statistical treatment or limits are made available. Indeed it appears that the experimenter does not of ten know himself when two of his fishing experiments have resulted in the same or different gelection.

The purpose of this paper is to set out some statistical techniques applicable to analysis of selection data and to estimation of such things as the $50 \%$ selection point. No emphasis will be placed on the planning of selection experiments, but we hope that it will be fairly obvious from the exposition that the amount of information obtained from a selection experiment will depend on the proper statistical planning of the experiment, and on the successful realization of the plan.

The development of the treatment of the subject matter is almost obvious. Since the selectivity may conceivably depend on the type of catch made, our first concern, Section A, is to consider some suitable criterion for classifying the catches and appropriate statistical techniques for testing and detecting bigger than random variations in size compositions. Tests proposed are the Kolmogorov-Smirnov test and a variant of the contingency table method.

Once one has classified the catches made during the selection experiments to one's satisfaction into more or less homogeneous groups, the next problem is then to find out whether the selection is the same or different within and between the groups. The description of a suitable test forms Section $B$. The suggested test is a slight extension of the contingency table technique used in Section $A_{\text {. }}$

In these first two sections we consider the total size composition and selection from it per se. However, in the literature the results are of ten summarized by giving, not the actual selection, but a summary of the percentages of fish retained in the form of a selection curve and the interpolated $50 \%$ selection point. In Section $C$ we suggest a probit type of analysis and give a method of calculating or fitting a suitable selection curve to the experimental results. The results of our "experimental" fittings are presented in Section $D$, and some comments will also be offered as to when the selection curves apply and when they do not, in our opinion.

Finally, a short section, Section E, is included to make a comment on the variability of the selection of a specified or regulated mesh size. When a mesh size is specified (e.g., by law), the specification pertains to the average size of the meshes. In practice a net is measured with certain precision only, and the selection of a regulated mesh size is more variable than in the selection experiments.

## C. Estimation of the selection curve and $50 \%$ selection point

In the last section we were concerned with detecting difference in the selection of a net between samples or groups of samples. These tests were independent of any assumptions about the selection curves. To summarize the information from the selection experiments one may wish, however, to express the results in the form of a selection curve or curves.

Assume for a moment that we may attach to each length group a fixed probability at which fish in that length group are being retained in the net. A selection experiment or sample may thus be considered as consisting of several independent sub-samples, namely the length groups, each sub-sample having a given total number of fish or sample size and a fixed probability of retention, which, in general, is increasing with the mean length of the group. The situation is in fact analogous to a bioassay, and hence we may apply here a probit type of analysis, the length being equivalent to the dose or dosage and the percentage retained to the percentage affected by the treatment.

In probity analysis one transforms the percentage of fish affected, or, in our case, retained, into probits and plots them against the dose or the length. If the effectiveness of the treatment follows a cumulative normal distribution, this results in a graph to which a straight line may be fitted by weighted least squares; often, however, before a line can be fitted the dose levels must be transformed (to what is called the "dosage") as well. The weights in fitting the probity line depend on the parameters of the line which, of course, are not known but are the object of the estimation; hence to arrive at the final solusdion an iterative procedure is required.

Alternatively to the probity type of transformation one may use what is called legit transformation. The two procedures give pretty much the same results but the logits are computationally much simpler (Berkson, 1944, 1951, 1955). However, if one is contented to use some of the shortcut methods worked out for probity, which are more or less equivalent to fitting the lines by eye, the probits may be preferable since these graphic methods also give the limits for the $50 \%$ selection point.

We have used logits almost exclusively. Since this method is not perhaps as familiar as the probity method, a short resume of it is given here.

The method assumes that the effect of the treatment or, in our case, the fraction retained, $Q$, is graduated by a logistic function

$$
Q=\frac{1}{1+e^{-(a+b x)}}
$$

where $x$ is the dose or dosage or in our case the length. The logit, call it $L$, corresponding to $Q$ is now defined by

$$
L=\ln (1-Q) / Q=a+b x
$$

With this definition the logistic is linearly related to the length.

There is now a simple procedure of estimating the parameters a and b. Let us put, following Berkson (1944)
$q_{i}=R_{i_{0}} / N_{i}=$ observed fraction retained in the net from the eth length group
$p_{i}=1-q_{i}$
$\mathbf{x}_{\mathbf{i}}=$ mean length of the eth length group, and
$\boldsymbol{\ell}_{i}=\ln \left(p_{i} / q_{i}\right)=\operatorname{logit}$ corresponding to observed value of the fraction retained
then $a$ and $b$ and their variances may be estimated from
$b=\sum w_{i} \bar{x}_{i} \bar{l}_{i} / \sum w_{i} \bar{x}_{i}{ }^{2}$
$\operatorname{Var} b=s^{2} / \sum w_{i} \bar{x}_{i}$
$a=\bar{\ell} \quad b \bar{x}$
Var $a=s^{2}\left(\sum w_{i} x_{i}{ }^{2}\right) /\left(\sum w_{i}\right) \quad\left(\sum w_{i} \bar{x}_{i}^{2}\right)$
where $w_{i}$ 's are the weights given by $w_{i}=N_{i}, p_{i} q_{i}$
( $N_{i}$ being the number of fish in the ith group)
$\bar{x}$ and $\mathscr{\ell}$ the mean length and the mean logit calculated from
$\bar{x}=\left\{w_{i} x_{i} / \sum w_{i}\right.$
$\overline{\boldsymbol{\ell}}=\sum \mathrm{w}_{\mathrm{i}} \ell_{\mathrm{i}} / \sum \mathrm{w}_{\mathrm{i}}$
$\bar{x}_{i}$ and $\ell_{i}$ the deviations from the mean, i.e.,
$\bar{x}_{i}=x_{i}-\bar{x}$
$\ell_{i}^{i}=\ell_{i}^{i}-\tilde{\ell}$
and $s^{2}$ what is termed the heterogeneity factor calculated from $s^{2}=\left[\sum w_{i} \bar{\ell}_{i}^{2}-\left\langle w_{i} \bar{x}_{i} \bar{i}_{i} / \sum w_{i} \bar{x}_{i}^{2}\right] / \quad(k-2)\right.$
$k$ being the number of groups and k-2 degrees of freedom
If in an experiment the probability of a fish being retained in a net is strictly the same for all fish in a given length group then $s^{2}$ follows a $X^{2}$ distribution with k-2 degrees of freedom. Significantly large values indicate a non-constant probability or heterogeneity.

The logit transformation has been tabulated and hence the values of $\ell_{i}$ corresponding to any observed $q_{i}$ may be looked up from a table (Berkson).

The $50 \%$ selection point, i.e., the length at which $50 \%$
of fish are retained, has a logit $\ell=0=\ln .5 / .5$ and
hence may be estimated from

$$
x_{50 \%}=\bar{x}-\bar{\ell} / \mathrm{b}
$$

An approximate expression for the variance is given by
$\operatorname{Var}\left(x_{50 \%}\right)=\frac{s^{2}}{b^{2}}\left\{\frac{\bar{x}^{2}}{\left\langle w_{i} x_{i}^{2}\right.}+\frac{1}{\left\langle w_{i}\right.}\right\}$
By using the above variance we get the $5 \%$ limits for $x_{50 \%}$ from $x_{50 \% \pm} t .05 \sqrt{\operatorname{Var}\left(x_{50 \%}\right)}$ where $t .05$ is the corresponding $t-$
value. This approximation is valid only when the value of $g$,

$$
g=t^{2} .05 s^{2} / b^{2}<w_{i} x_{i}^{2}
$$

is small, say less than . 10 . For a larger $g$ the limits become
asymmetrical and should be calculated from
$x_{50}+\frac{g}{1-g}\left(x_{50}-\bar{x}\right) \pm \frac{t}{b(1-05} \sqrt{\frac{1-g}{\sum w_{i}}+\frac{\left(x_{50}-\bar{x}\right)^{2}}{\sum w_{i} \bar{x}_{i}^{2}}}$
The formulae presented in this section have been applied to a number of selection experiments reported in the literature. Some of the results are listed in the accompanying table (Table 5).
Table 5. Summary of Selection curves fitted to data from Report of
Iceland Trawl Mesh Selection Working Group (Doc. 34 ICNAF - 1964)



