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Estimation of commercial stock of Newfoundland capelin within a single mathematical model[1]

by

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Abstract

The paper presents a brief biological characteristic of the Newfoundland capelin and discusses the areas and time of fishing.

Based on biological and statistical data a mathematical model of estimating the commercial stock is worked out to determine the optimal regime of exploiting the commercial stock of the capelin. As a result of computing the commercial stock of the Newfoundland capelin in 1971-1974 can be with a fairly high probability believed not to be below 1.230.000 tons.

According to the results obtained the annual catch of the Newfoundland capelin can be 750.000-800.000 tons.

I.BRIEF BIOLOGICAL CHARACTERISTIC OF NEWFOUNDLAND CAPELIN

The capelin is a small school fish,21cm.long as maximum.The weight of the biggest specimens rarely exceeds 50 gr.Along the Atlantic coast of America capelins are observed from Bay of Hudson to Bays of Fundy and Man. South of the Cabot Strait capelins are hardly observed.

Within the habitat feeding and spawning migrations of capelins are considerable.After spawning the specimens survived leave the spawning grounds (inshore waters of Newfoundland

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and South-eastern slope of the Newfoundland Grand Bank) and migrate to the North via the North-eastern slope of the Grand Bank, the North Newfoundland Bank to the area of fattening (South Labrador). Immature capelins dispersing in the area of the North-eastern slope of the Newfoundland Grand Bank as the water becomes warmer are also bound northward for feeding. By October capelins reach the Northen borders of the habitat and distribute in the area of South Labrador. When the waters become cool and feeding comes to an end they migrate to the area of wintering (The North Newfoundland Bank). Here capelins stay till February. In February individual schools appear on the North-eastern slope of the Newfoundland Grand Bank. In March-April capelins accumulate on the North-eastern slope and migrate to the area of the South-western slope of the Newfoundland Grand Bank and to the Banks of Saint Pierre and Green. In May-June they appear in the area of spawning grounds (the South-eastern slope of the Newfoundland Grand Bank and inshore waters of Newfoundland).

The data obtained during observations in 1971-1974 on the disrtibution abd density of the abundances ,the area of concentrations of different density, size and age structure of the stock and commercial statistics provide grounds to believe that the the abundance of the capelin in the area of the Newfoundland Grand Bank and South Labrador is fairly considerable.

Effective fishing for The capelin in the spring-summer period(March-August) can be carried out in the area of the Newfoundland Grand Bank and in the autumn-winter period (September-December) - in the area of the North Newfoundland Bank and South Labrador.

Increased fishing for the capelin in the Nesth-West Atlantic in recent years has necessitated estimating the permissible removal of this species without detrement of its stock.

For the first time the estimation of the abundance of the capelin spawning stock was made by Norwegian researchers by the hydro-acoustic method and the stock was estimated to be 800.000 tons /5/.

In I973-I974 Canadian researchers also gave considerable attention to determining the optimal regime of exploiting the commercial stock of the Newfoundland capelin /6/. According to them the total catch in I975 can be 750.000 tons and in next years it may be estimated as 500.000 tons. Though this figure may be adjusted.

In I974 an attempt was made to estimate the commercial stock with the help of well-known mathematical methods.

Due to meagre statistic data on hte capelin stock and its ... fishing this attempt has proved a failure. A way out of the situation has been found by the following method.

2. MODEL AND METHODS OF ESTIMATING THE COMMERCIAL STOCK OF THE NEWFOUNDLAND CAPELIN

Assuming the stock of fish caracterized in moments of time t,$t \in n$, N - multitude of integers, by X_t (abundance or biomass). Hence

$$X_t = \sum_{i=1}^{k} X_t^{\male i} + \sum_{i=1}^{k} X_t^{\female i} \qquad (I)$$

where $X_t^{\male i}$ $(X_t^{\female i})$ - male stock(respec.female) of "the age" i in the moment t (the unitary "age" corresponds to the age of recruits).

The state of the stock at the moment t is characterized by the vector $\bar{X}_t = (X_t^{\male 1}, \dots, X_t^{\male k}, X_t^{\female 1}, \dots, X_t^{\female k})$.

The dynamics of the stock is described by the formula

$$\bar{X}_{t+1} = \bar{X}^{1}_{t+1} + \bar{X}_t \cdot S A_{\bar{x}t} \qquad (2)$$

where $\bar{X}_t \cdot S = (0, X_t^{\male 1}, \dots, X_t^{\male k-1}, 0, X_t^{\female 1}, \dots, X_t^{\female k-1})$,

$$A_{\bar{x}t} - I(0, \alpha_{xt}^{\male 1}, \dots, \alpha_{x}^{\male k-1}, 0, \alpha_{\bar{x}t}^{\female 1}, \dots, \alpha_{\bar{x}t}^{\female k-1})',$$

I -unitary matrix, ' - transponing symbol, for all j = I,...,k-I values $\alpha_{\bar{x}t}^{\male j}, \alpha_{\bar{x}}^{\female j}$ depending in a general case on the vector $\bar{X}_t$ and characterizing survivability belong to the interval (0,I), $\overline{X}^{1}_{t+1} = \left(X^{\male_1}_{t+1}, 0, \dots, X^{\female_1}_{t+1}, 0, \dots, 0\right)$ where for all $t \in M$ values $X^{\male_1}_t$ are independant uniformly distributed normal values,

$$X^{\male_1}_t \Big/ X^{\female_1}_t = d = \mathrm{const} \qquad (3)$$

pt.5

In approximation assumption of concerning recruitment can be regarded as performed if the value of recruitment is determined first of all by abiotic factors.Hence it can be taken that the value of recruitment does not depend on the state of the stock, at least for a stock with undisturbed homeostasis /3/.

The available data on capelin year classes' yield depending thermics provide ground to suppose that it is just the case with recruitment for the capelin.

<u>Suggestion I.</u> assuming $M^{\male} = \xi X^{\male_1}_t, \mathcal{D}^{\male} = \mathcal{D}X^{\male_1}_t, M^{\female} = \xi X^{\female_1}_t, \mathcal{D}^{\female} = \mathcal{D}X^{\female_1}_t$

ξ -mathematic expectancy operation, D - dispersion.Under the conditions of pt 5 Xt are normally distributed mean values with mathematic expectancies

$$M_t = M^{\male}\left(1 + \alpha^{\male}_{\bar{x}_{t-1}} + \alpha^{\male}_{\bar{x}_{t-2}} \cdot \alpha^{\male}_{\bar{x}_{t-1}} + \dots + \alpha^{\male_1}_{\bar{x}_{t-k}} \cdot \dots \cdot \alpha^{\male_{k-1}}_{\bar{x}_{t-1}}\right) + M^{\female}\left(1 + \alpha^{\female_1}_{\bar{x}_{t-1}} + \dots + \alpha^{\female_1}_{\bar{x}_{t-k}} \cdot \dots \cdot \alpha^{\female_{k-1}}_{\bar{x}_{t-1}}\right) \qquad (4)$$

and dispersions

$$\mathcal{D}_t = \mathcal{D}^{\male}\left[1 + \left(\alpha^{\male_1}_{\bar{x}_{t-1}}\right)^2 + \dots + \left(\alpha^{\male_1}_{\bar{x}_{t-k}} \cdot \dots \cdot \alpha^{\male_{k-1}}_{\bar{x}_{t-1}}\right)^2\right] + \mathcal{D}^{\female}\left[1 + \left(\alpha^{\female_1}_{\bar{x}_{t-1}}\right)^2 + \dots + \left(\alpha^{\female_1}_{\bar{x}_{t-k}} \cdot \dots \cdot \alpha^{\female_{k-1}}_{\bar{x}_{t-1}}\right)^2\right]$$

The proof is trivial.

It is natural to suppose that X may be of such magnitude that X_t X is equal to the elimination of the stock. In this case thr survival of the stock is characterized by probabilities

$$P_t = P\{x_t \geq x_{cr}\}$$

It is reasonable to believe that I) values P_t are monotonously increasing M_t 2) when M_t are considerable

M_t, $M_t \gg x_{cr}$ $P_t \approx \mathrm{const}$

Following from suggestion I the simpliest condition when properties I) and 2) are satisfied is the condition

$$\frac{M_t}{\sqrt{\mathcal{D}_t}} = f = \text{const} \qquad (6)$$

In further computing (6) is assumed to be satisfied. In this case from (6), (4) and (5) it follows that with $M_t >> X_{cr}$ the matrix $A_{\bar{x}_t}$ in (2) can be regarded constant:

$$\bar{x}_{t+1} = \bar{x}'_{t+1} + \bar{x}_t \cdot SA \ (2').$$

Proceeding further we shall assume that we can observe accidental vectors $\bar{\xi}_t$ distributed as vectors

$$\delta \bar{x}_t \qquad (7)$$

Vectors $\bar{\xi}_t$ are often represented by catches per effort for which the equality (7) in a certain approximation is performed in a number of fishery models /2/.

Suggestion 2. Assume that the dynamics of the stock is described by the formula (2). $\bar{\xi}_t^{\male}$ stands for vector $(\xi_t^{\male_1}, \xi_t^{\male_2} \ldots \ldots \xi_t^{\male_k}$, $\xi_t^{\male}$ for the value $\sum_{i=1}^{k} \xi_t^{\male_i}$, $\rho(\xi, \eta)$ for the correlation coefficient between mean values ξ and η. Hence

$$\alpha^{\male_{k-1}} = \left(\frac{\rho(\xi^{\male}_{t-k+2}, \xi^{\male}_t)}{\rho(\xi^{\male}_{t-k+1}, \xi^{\male}_t)} - 1 \right)^{-1}$$

$$\alpha^{\male_{k-2}} = \frac{1}{\alpha^{\male_{k-1}}} \left[\frac{\rho(\xi^{\male}_{t-k+3}, \xi^{\male}_t)}{\rho(\xi^{\male}_{t-k+1}, \xi^{\male}_t)} - \frac{1 + \alpha^{\male_{k-1}}}{\alpha^{\male_{k-1}}} \right]^{-1} \qquad (8)$$

. . . .

$$\alpha^{\male_{k-2}} = \frac{1}{\alpha^{\male_{k-z+1}} \cdot \alpha^{\male_{k-z+2}} \cdot \ldots \cdot \alpha^{\male_{k-1}}} \left[\frac{\rho(\xi^{\male}_{t-k+z+1}, \xi^{\male}_t)}{\rho(\xi^{\male}_{t-k+1}, \xi^{\male}_t)} - \frac{1 + \alpha^{\male_{k-z+1}} + \alpha^{\male_{k-z+1}} \cdot \alpha^{\male_{k-z-2}} + \ldots + \alpha^{\male_{k-z+1}} \cdot \ldots \cdot \alpha^{\male_{k-1}}}{\alpha^{\male_{k-z+1}} \cdot \ldots \cdot \alpha^{\male_{k-1}}} \right]^{-1}$$

. . . .

$\alpha^{\male_i}$ -recurrently determining factors

Proof : Subsequently computing the ratio of correlation coefficients in the right side of the formulas (8) after elemetary operations (which we omit due to the unwieldiness

of the formulas) we obtain the correlation (8).
Similar formulas must be also true for the factors

$$\alpha^{\female_i}$$

Thus Suggestion 2 provides a method allowing the factors $\alpha^{\male_i}$ and $\alpha^{\female_i}$ to be identified if the state of the stock with accidental recruitment (meaning pt 5) is for from critical.

Unfortunately due to lack of data approbation of this method for the capelin stock is unimplementable.

Suggetion 3. Under the conditions of suggestion 2 are true the formulas:

$$\alpha^{\male_1} = \sqrt{\frac{\operatorname{cov}(\xi_t^{\male_2}, \xi_t)}{\operatorname{cov}(\xi_t^{\male_1}, \xi_t)}},$$

$$(9) \qquad \alpha^{\male_2} = \frac{1}{\alpha^{\male_1}} \sqrt{\frac{\operatorname{cov}(\xi_t^{\male_3}, \xi_t)}{\operatorname{cov}(\xi_t^{\male_1}, \xi_t)}},$$

$$\alpha^{\male_n} = \frac{1}{\alpha^{\male_1} \cdot \ldots \cdot \alpha^{\male_{n-1}}} \sqrt{\frac{\operatorname{cov}(\xi_t^{\male_{n+1}}, \xi_t)}{\operatorname{cov}(\xi_t^{\male_1}, \xi_t)}}$$

where cov (...) -is the covariation symbol.

The proof is performed by direct computing expressions in the right side of the formulas (9).

The result of Suggetion 3 provides another method of identifying the factors $\alpha^{\male_i}$ and $\alpha^{\female_i}$ (see pts II,I2)

Everywhere further the dynamics of the stock is supposed to be described by the formula (2').Besides it is taken that

$$\left.\begin{array}{l} \alpha^{\male_1} = \alpha^{\male_2} = \ldots = \alpha^{\male_{k-1}} = \alpha \\ \alpha^{\female_1} = \alpha^{\female_2} = \ldots = \alpha^{\female_{k-1}} = \beta \end{array}\right\} \qquad (I0)$$

In this case   is true.

Suggestion 4. Assuming $i \leq k-1$

$$\frac{\alpha^i}{\beta^i} = \frac{p_{i+1} \cdot q_1}{q_{i+1} \cdot p_1} \qquad (II)$$

where P_i (respect. q_i) = the share of males (respect.females) among all the fish of the age i and older:

$$P_i = \frac{\sum_{j=i}^{k} X_t^{\male j}}{\sum_{j=i}^{k} X_t^{j}}, \qquad (I2)$$

$$q_i = \frac{\sum_{j=i}^{k} X_t^{\female j}}{\sum_{j=i}^{k} X_t^{j}} = 1 - P_i$$

The abuve results are true for an unfished stock. The suggestion that follows is deals with a stock subject to fishing.

Suggestion 5. Assuming that at the moments t_i, $i = 1, \ldots, z$, $z \leq k-1$ from the stock are withdrown respectively a_i males and b_i females present in the stock at the moment t_i. Hence

$$X_{t_1} = \frac{l_z \sum_{i=1}^{z} \beta^{z-i} b_i - \sum_{i=1}^{z} \alpha^{z-i} a_i}{l_z \beta^z q_{1,t_1} - \alpha^z p_{1,t_1}} \qquad (I3)$$

where $l_z = \frac{p_{k+1,\, t_1+z}}{q_{z+1,\, t_1+z}}$; $q_{i,t} = 1 - p_{i,t}$,

$$p_{i,t} = \frac{\sum_{j=i}^{k} X_t^{\male j}}{\sum_{j=i}^{k} X_t^{j}}.$$

From (I3) it is easy to proceed to similar formulas for $X_{t_1}^{\male}$ and $X_{t_1}^{\female}$.

Following from the results of suggestion 4 and 5 we obtain the estimation from below of the Newfoundland capelin stock. From (I3)

$$X_{1971}^{\male} = \frac{p_{1,1971}\left(l_2 \sum_{i=1}^{2} \beta^{2-i} b_i - \sum_{i=1}^{2} \alpha^{2-i} a_i\right.}{l_2 \beta^2 q_{1,1971} - \alpha^2 p_{1,1971}} \qquad (13')$$

Ignoring the catch of 1971 and computing by Tables I and 2 the values a_2 and b_2 (accurate to 10^6, a higher accuracy standing to no reason) and estimating by Table 4 and 3 the values $\cancel{x}$ l_2, $p_{1,1971}$, $q_{1,1971}$ formula (I3') is reduced to

$$X_{1971}^{\male} = \frac{237 \cdot 10^6}{\beta^2 \left(1,1 - \frac{\alpha^2}{\beta^2}\right)} \qquad (I4)$$

Table I. Age composition of male capelin in catches by on The Newfoundland Grand Bank and South Labrador

year of fishing	age I	2	3	4	5	6
I97I*		87	5258	36II	303	
I972		92775	6I3I33	40I380	I745I	
I973	25350	492088	5563II	I225800	27I607	6887

* for I97I data only covers the spring period on the Newfoundland Grand Bank.

Table 2. Age composition of female capelin in catches on the Newfoundland Grand Bank and South Labrador in thousands of spec.

year of fishing	age I	2	3	4	5	6
I97I*		I287	I8777	6596	I96	52
I972		73096	II337I5	473238	2I884	
I973 666		829636	773537	2665250	5I3337	I3775

*for I97I data covers only the spring period on the Newfoundland Grand Bank.

According to Suggestion 4

$$\frac{\alpha^2}{\beta^2} = \frac{p_{3,1971} \cdot q_{1,1971}}{q_{3,1971} \cdot p_{1,1971}} \qquad (I5)$$

Emperic values $\tilde{p}_{3,1971}$ and $\tilde{q}_{3,1971}$ of $p_{3,1971}$ and $q_{3,1971}$ are equal to $\frac{35}{87}$ and $\frac{52}{87}$ respectively (see Tables 3,4)

Table 3 Age sampling of male capelin on the Newfoundland Grand Bank and South Labrador.

year of fishing	a g e I	2	3	4	5	6	%	male	total
I97I		5,I	20,6	I3,2	I,I		40,0	488	I2I8
I972		I,6	24,3	20,7	0,9		47,5	880	I85I
I973	0,I	5,5	8,9	22,7	5,0	0,I	42,4	I364	3227

Table 4. Age sampling of female capelin in the Newfoundland Grand Bank and South Labrador

year of fishing	age						%	female	total
	I	2	3	4	5	6			
I97I		8,0	37,I	I4,4	0,4	0,I	60,0	730	I2I8
I972		I,2	32,9	I7,5	0,9		52,5	97I	I85I
I973		6,5	8,9	34,9	7,2	0,2	57,7	I863	3227

Checking the hypothesis H: $P_{3,1971} = \frac{37}{87}$, $q_{3,1971} = \frac{50}{87}$.

The value χ^2 for the hypothesis H is equal to

$$2.539 < 2.946 = \chi_{0,6}(3)$$

[I]

Hence χ^2 for the hypothesis H provides the level of magnitude above 0,4. Biological investigations accept hypotheses with the level of magnitude equal to 0.05 and even lower. Accepting the hypothesis H and applying (I5) $1,1 - \frac{\alpha^2}{\beta^2} = 10^{-3}$ hence with allowance for (I4)

$$X^{♂}_{1971} > 237 \cdot 10^9 \qquad (I6)$$

$$X^{♀}_{1971} > 356 \cdot 10^9 \qquad (I7)$$

$$X_{1971} > 593 \cdot 10^9 \qquad (I8)$$

From (I8) using the data given in Table 5 and 6

$X_{1971} >$ I3720270 Tons (I9)

Table 5. Age sampling of capelin on the Newfoundland Grand Bank and South Labrador

year of fishing	age						%	
	I	2	3	4	5	6		
I97I		I3,I	57,7	27,6	I,5	0,I	I00,0	I2I8
I972		2,8	57,2	38,2	I,8		I00,0	I85I
I973	0,I	I2,0	I7,8	57,6	I2,2	0,3	I00,0	3227

Table 6. Mean weight of capelin on the Newfoundland and South Labrador

age	mean weight of one specimen in gr.					
	I97I	n	I972	n	I973	n
I					7,2	I

2	I2,7	I59	I0,2	5I	I7,I	[illegible]86
3	22,4	702	20,2	I060	25,I	573
4	29,8	337	27,I	708	27,8	I864
5	35,9	I9	28.0	32	28,9	394
6	I6,0	I	.		26,5	9

Taking ξ_t, $\xi_t = \sqrt{X_t}$ for relative indices of the biomass of the capelin stock characterizing the productivity of fishing by a large freezing trawler at the Newfoundland Grand Bank from (I9) and Table 7 we can obtain the data cited in Table 8.

Table 7. Fishing productivity by a BMRT on the Newfoundland grand Bank

years	I97I	I972	I973	I974
productivity of 24 hours' fishing in tons	46,5	50,0	64,4	4I,9

Table 8. Estimation "from below" of the biomass of the Newfoundland capelin stock

years	I97I	I972	I973	I974
biomass in tons	I3720270	I4752978	I900I840	I2362997

Table 9. Total catch of the Newfoundland capelin taken by Soviet fisheries in the spring-winter period /Labrador plus Newfoundland Grand Bank/

years of fishing	I97I	I972	I973
total catch in tons	830	60304	I82520

The data of Table 8 and Table 9 allow making up Table IO.

Table IO. Estimation of the commercial removal of capelins by Soviet fisheries

years of fishing	I97I	I972	I973
commercial removal in per cent	[illegible]	0,4I	I

The above method of estimating a stock of fish can be regarded as extension of the method described in paper [4].

Let us obtain another estimation "from below" of the Newfoundland capelin stock by another, a more cautious method.

Assume that I82520 tons (the total catch of capelin taken in the USSR in I973, see Table 9) is close to the critical value of the biomass of the capelin stock X_{cr} (the possibility of which proceeds from the above estimation of the biomass of the capelin stock) and that in natural conditions (unaffected by fishing and other antropogeneous factors) the probability

$$P\{X_t \leq 182520\} \leq 10^{-6}$$

from (20) follows that $\dfrac{\xi X_t}{\sqrt{\mathcal{D} X_t}} = \dfrac{\xi \xi_t}{\sqrt{\mathcal{D} \xi_t}}$

with the help of Table 7 define the values $\xi \xi_t$ and $\mathcal{D} \xi_t$ as equal to 50,7 and 9,7 respectively. Hence according to (2[illegible])

$$\frac{\xi X_t}{\sqrt{\mathcal{D} X_t}} \approx 5.2 \tag{22}$$

By the known formula for normal distribution

$$P\{X_t \leq a\} = \Phi_u\left(\frac{a - \xi X_t}{\sqrt{\mathcal{D} X_t}}\right) \tag{23}$$

where Φ_u is the distribution function of the standardized normal value based on (22) following from (23)

$$\Phi_u\left(\frac{182520 - 5.2\,y}{y}\right) \leq 10^{-6} \tag{24}$$

where y is the estimation of the value $\sqrt{\mathcal{D} X_t}$

Using the table for the function of distributing the standardised normal value (see, for instance /I/) we find that inequality (24) leads to the inequality $\frac{5,2y - 182520}{y} \geqslant 4,5$

Hence $y \geqslant 260714$, $\mathcal{E} X_t = 5,2\sqrt{\mathcal{D} X_t} \geqslant 1303571$, from (20) we obtain the estimation "from below" of the biomass of the capelin stock.

Table 11. Estimation "from below" of the biomass of the Newfoundland capelin stock

years	1971	1972	1973	1974
biomass in tons	1243405	1336995	1722050	1120402

By Table 9 and 11 define the estimation "from above" of the commercial withdrawal of capelins by Soviet fisheries.

Table 12. Estimation "from above" of the commercial removal of capelins by Soviet fisheries

years	1971	1972	1973
commercial removal in per cent	0,06	4,5	10,6

Conclusion

From the results obtained above it follows that the commercial stock of the Newfoundland capelin may be with a fairly high probability understood not to have fallen in 1971-1974 below 1.230.000 tons and that at any rate the estimations "from below" of the commercial stock of the Newfoundland capelin obtained by the second method are trustworthy. Taking these estimations as estimations of the commercial stock of the Newfoundland capelin, i.e. estimating the commercial stock of the Newfoundland capelin with maximum caution it can be agreed upon that the estimation of the catch made for this stock by Canadian researchers equal to 750.000-800.000 tons is possible.

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