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A note on the mixed species problem

by

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Mixed species can create management problems. This is because where one species is caught as a result of a fishery directed at another species, it may not be possible to achieve the maximum yield for the directed fishery without over-fishing (fishing beyond the MSY level) for the by-catch species. This note is concerned with a simple graphic solution to this problem.

Mixed fisheries have been recognised as a management problem for some time. Ricker, W E, 1958 considered the problem for salmon stocks and these were again considered by Paulik et al in 1967. Suda (1972) considered mixed fisheries as a problem for tuna management. The problem was given more definition by Garrod, 1973 in relation to mixed fisheries in the ICNAF area and these ideas were developed by Brown et al 1973 and Fukuda 1974. Basically Garrod proposed a matrix of directed fisheries and by-catches which assumes that fishing effort directed as species A ( $f_a$ ) produces a fishing mortality on species B such that:

Fishing mortality on B ( $F_{ab}$ ) such that

$$F_{ab} = q_b f_a$$

where the prefix of the catchability coefficient  $q$  refers to the directed fishery species and suffix to the by-catch species. Practically, the species to which the

fishery was directed, was defined as the species which each month gave the highest catch for each nation/gear/vessel class recorded in the ICNAF statistics. The relative size of each  $q_j$  was related to the size of  $q_j$  the catchability of the species in the directed fishery. This enabled Brown et al to consider the problem of maximising the catch from the Southern ICNAF areas subject to the constraints of the catch quotas on each species and subject where necessary to some constraints as to the minimum catch to be permitted for some national catches of specific species. Fukuda studied the problem in more general terms and points out that the solution of any mixed fishery must lie in a cone defined by the by-catch vectors of each directed fishery. The approaches of Brown et al and Fukuda were both concerned with getting the maximum yield in a year within the constraints of certain catch quota, the catch quotas being chosen in most cases to attain the MSY for each stock independently. Thus the solution is not likely to be quite the same as that which would give the MSY for all stocks combined. The purpose of this note is to generalize the arguments of Brown and of Fukuda to the problem of attaining the MSY for all stocks combined. The arguments are mostly graphic but could fairly easily be solved formally.

Let us consider 2 stocks of fish with parabolic yield function. In the interests of topicality these can be named silver hake and haddock but it must be stressed that these yield functions are purely imaginary. The relationship between yield and the total fishing mortality on each species (F) is:

$$\text{Silver hake yield} = A(2.8F_s - F_s^2) \quad 0 \leq F_s \leq 2.8$$

$$\text{Haddock yield} = B(1.2F_h - F_h^2) \quad 0 \leq F_h \leq 1.2$$

In general A and B will be different and the total fishery mortality in each of the species  $F_s$  (silver hake) and  $F_h$  (haddock) will produce an overall yield  $Y_t$  of:

$$Y_t = A(2.8F_s - F_s^2) + B(1.2F_h - F_h^2)$$

Thus lines of equal yield will form ellipses within the ranges given for  $F_s$ ,  $F_h$ . However by a suitable transformation of the  $F_s$  we could convert these into circles and so for simplicity we can consider the case when  $A = B = 100$ . In this case the surfaces of equal yield will form concentric circles within the range specified for  $F_s$ ,  $F_h$ .

Table 1 shows the yield from each stock at various levels of fishing mortality. It is obvious that the greatest yield would occur if  $F_s$  could be made equal to 1.4, and  $F_h$  be made 0.6. At these values  $F_c$  and  $F_h$  the overall yield would then be equal to 232 units.

Table 2 shows the by-catch rates of the species in each directed fishery. It can be seen that there is a considerable mortality induced on haddock as a result of fishing for silver hake and that fishing for haddock produces some mortality on silver hake. The result of these by-catch rates is that if we call the fishing mortality directed at silver hake  $F_1$ , and that directed at haddock  $F_2$ , we find that:

$$F_s = F_1 + 0.400 F_2$$

$$F_h = 0.714 F_1 + F_2$$

To attain  $F_s = 1.4$ ,  $F_h = 0.6$ , we need

$$F_1 = 1.62, F_2 = - .56$$

clearly this violates the practical constraint that  $F_1, F_2 \geq 0$  and therefore the overall maximum cannot be attained in practice unless steps could be taken to change the by-catch rates for example by closed areas.

Since the overall maximum cannot be attained the next question is what is the maximum yield that can be attained by a mixture of fisheries directed at the 2 species. Figure 1, is intended to clarify the solution of this problem. The horizontal axis being the direction of increase of  $F_s$  and the vertical axis indicates the direction of increase of  $F_h$ . The lines  $F_s = 1.4$  and  $F_h = 0.6$  passes through the

overall MSY (which is unattainable). These lines are constraints of Browns et al's treatment of this problem. The contour lines indicate the values of  $F_s$ ,  $F_h$  which give yields of 200, 150, 100 and 50 units. These contour lines are of course concentric circles centred on the overall MSY. The line OA is the locus of possible values of  $F_s$ ,  $F_h$  that would be created by a fishery directed at silver hake. The equation of this line is, from table 2:

$$F_h = 0.714 F_s$$

Thus it can be calculated that the line OA cuts the  $F_s = 1.4$  line at  $F_h = 1.0$ . Similarly the line OB is the locus of possible values of  $F_s$ ,  $F_h$  that would be created by a fishery directed at haddock. The equation of this line is:

$$F_s = 0.4 F_h$$

It can be calculated that the line OB intersects the  $F_h = 0.6$  line at  $F_s = 0.24$ .

Clearly the only values of  $(F_s, F_h)$  that can be attained by positive combinations of the two directed fisheries must lay within the triangle with vertex at O and sides OA, OB. Since the contour lines are continuous the maximum attainable yield must lay on one of the boundaries of this triangle and it can be seen from inspection that in this particular problem it lies on the line OA at the point in figure 1 labelled highest attainable MSY. The contours indicate that this occurs at  $F_s \approx 1.21$ ,  $F_h \approx 0.86$  and the combined yield is about 222 units at this point. The precise parameters and yield for MSY can of course be calculated by finding the equal yield circle to which the line OA is tangential.

Thus a solution for the attainable MSY in this particular mixed fishery situation has been discovered. While the situation is not particularly realistic it does indicate some of the more important points connected with this definition of a mixed fishery. The first point is that possible solutions lay within a triangle whose sides are defined by the loci of species fishing mortality generated by directed fisheries. It can be seen that this might be changed. If for example there were two directed

silver hake fisheries (eg midwater trawl, bottom trawl) having different by-catch rates, the triangle might be extended depending on whether the second silver hake fishery had a locus of  $F_s, F_h$  which was exterior to OAB.

If a constraint were required to ensure some directed fishery on each stock the triangle would have its vertex at the levels of  $F_s, F_h$ , which would be determined by this constraint. Thus if a constraint was made that at least a fishing mortality of 0.3 should be generated in the directed fishery for haddock. The triangle would have its vertex at  $F_s = 0.4 \times 0.3 = 0.12, F_h = 0.3$ . In either case the attainable MSY must always lie on the boundaries of such triangles since the circles of equal yield are continuous. This could however cease to be the case if a constraint on the maximum fishing mortality in any particular species were made since the solution might then lie on the constraint line in the interior of the triangle.

In another example the overall MSY might lay within the triangle OAB in which case of course the overall MSY would be attainable and the solution could be found by solving the simultaneous equations for the necessary levels of directed fishery.

The arguments given here can be extended to three stocks or more and the author hopes to present a more general mathematical argument in due course. It is hoped that this note and the model will have helped to clarify the situation. It is important to appreciate the highest attainable MSY indicated by this method would only be attained by fairly complicated regulations, for example, effort quotas on each species or directed catch quotas on each species. It is interesting to see what the effect of a more simple regulation like an overall quota might be. If for example the catchability of silver hake in the directed fishery was such that a unit effort directed at it produced a fishing mortality  $F_s = 1.0$ . Similarly a unit effort directed at haddock produced a  $F_h = 1.0$  then an effort quota of a 1.21 units of effort would approximately enable the maximum attainable yield to be taken. The line  $A^1 B^1$  indicates the values of ( $F_s$  and  $F_h$ ) which would correspond to such a constraint. It can be seen from the contours that intersect this line that this effort quota could

give the attainable MSY of 222. This would be attained if the effort were directed selectively at only silver hake. Alternatively the yield might be as low as 127 if all the effort were directed at haddock. In this case incidentally the haddock stock would be destroyed. While it is unlikely that effort would concentrate on one species to the exclusion of the others it is possible that this might happen if one particular species were held in particular esteem by a large section of the fishing nations. This is in effect what is happening in world fisheries today except that constraints are now been applied to the distribution of effort, and, indirectly, to its quantity.

This model of a mixed fishery suggest that if two or more stocks of fish independently follow a Schaefer-type yield curve the sum of their steady state yields can itself be a parabola within the constraints that the individual stocks are not destroyed. This is, however, only the case when the directed fisheries are kept in the same proportion. If they are not then the parabolic relationship of total yield to effort would break down. Any line drawn through the origin in the sector AOB would have a relationship between its length of the form  $yield = D(CxL - L^2)$ .

(Where D and C are constants) but only the curve corresponding to line OA would give the maximum yield. Thus a relationship between effort and total catch might in fact show a maximum which was lower than the true attainable maximum. Thus the use of such curve will tend to err on the safe side for choosing second tier MSY's etc. At our present state of knowledge this is a fault in the right direction.

#### References

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Table 1 Hypothetical yield functions for two stocks

|     | <u>Silver hake</u>         | <u>Haddock</u>             |
|-----|----------------------------|----------------------------|
| F   | 100 (2.8F-F <sup>2</sup> ) | 100 (1.2F-F <sup>2</sup> ) |
| 0   | 0                          | 0                          |
| 0.1 | 27                         | 11                         |
| 0.2 | 52                         | 20                         |
| 0.3 | 75                         | 27                         |
| 0.4 | 96                         | 32                         |
| 0.5 | 115                        | 35                         |
| 0.6 | 132                        | 36                         |
| 0.7 | 147                        | 35                         |
| 0.8 | 160                        | 32                         |
| 0.9 | 171                        | 27                         |
| 1.0 | 180                        | 20                         |
| 1.1 | 187                        | 11                         |
| 1.2 | 192                        | 0                          |
| 1.3 | 195                        | 0                          |
| 1.4 | 196                        | 0                          |
| 1.5 | 195                        | 0                          |
| 1.6 | 192                        | 0                          |
| 1.7 | 187                        | 0                          |
| 1.8 | 180                        | 0                          |
| 1.9 | 171                        | 0                          |
| 2.0 | 160                        | 0                          |
| 2.1 | 147                        | 0                          |
| 2.2 | 132                        | 0                          |
| 2.3 | 115                        | 0                          |
| 2.4 | 96                         | 0                          |
| 2.5 | 75                         | 0                          |
| 2.6 | 52                         | 0                          |
| 2.7 | 27                         | 0                          |
| 2.8 | 0                          | 0                          |

Table 2

| Unit fishing mortality directed at | Resulting units of mortalities on |                |
|------------------------------------|-----------------------------------|----------------|
|                                    | <u>Silver hake</u>                | <u>Haddock</u> |
| Silver hake                        | 1.0                               | 0.714          |
| Haddock                            | 0.400                             | 1.0            |

