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International Commission for the Northwest Atlantic Fisher

Serial No. 3676
(D.c.9)

ICNAF Res.Doc. 75/IX/125

SEVENTH SPECIAL COMMISSION MEETING - SEPTEMBER 1975

Interactive fisheries: A two species Schaefer model

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Publ. in ICNAF Sel. Papers No. 1: 151-156

ABSTRACT

A two species fishing model is considered and compared with the results from a grouped Schaefer model. If fishing effort is proportional to the relative number of each species then it is shown that the correspondence between the two models diverges as the ratio of the two species diverges from unity. It is also shown that fishing each species in proportion to their relative numbers does not necessarily take the fishery through its maximum sustainable yield.

INTRODUCTION

Schaefer (1954, 1957) developed a model to evaluate the equilibrium yields from a fishery. This model assumes that the rate of change of stock biomass can be represented as

$$dS/dt = AS - BS^2 - qFS$$ where S is the stock

biomass, A and B parameters of the stock population growth and q and F the familiar parameters of catchability and fishing. Gulland (1974) has pointed out that the Schaefer model is identical with assuming that catch per unit of effort is linearly related to effort. Among others, Pinhorn (1975) and Brander (1975) have fitted Schaefer models to total fish biomass in an area and here the parameters A and B take on different meanings, A being the net rate of increase at low total biomass levels and B representing the inter and intraspecific density dependent regulation.

This study, through consideration of a two species model, helps to elucidate the relationship between the two applications of the Schaefer model.

THE MODEL

Let us assume a two stock fishery where the fishing effort on the individual stocks is proportional to the relative numbers of fish in each stock. To avoid proliferation of the parameters the catchability is put at 1.0 and the preferability of the two species to the fishermen is considered to be the same. This situation may then be expressed as

$$1/p \quad dp/dt = a_1 - b_1 p - F\, p/(p+r) \qquad (1)$$

$$1/r \quad dr/dt = a_2 - b_2 r - F\, r/(p+r) \qquad (2)$$

In the equilibrium state (r^*, p^*) $dp/dt = dr/dt = 0$ and hence

$$r^* = a_2 p^* / (a_1 - p^*(b_1 - b_2)) \qquad (3)$$

and p^* is given by solving the quadratic

$$p^2 b_1(b_1-b_2) + p\left\{F(b_1-b_2) - a_1(b_1-b_2) - a_1 b_1 - a_2 b_1\right\} + (a_1 a_2 + a_1^2 - a_1 F) = 0 \qquad (4)$$

The equilibrium yield is given by (dropping asterisks)

$$F(p^2 + r^2)/(p+r). \qquad (5)$$

As will be appreciated from the form of equation (4), general solutions for r^* and p^* are unwieldy but in all the numerical examples given only one positive pair of r^* and p^* exists, that is, for any value of F two positive equilibria do not exist.

May (1973) and Beddington (1974) have shown how the stability of these valid equilibria may be investigated.

$$\text{If } G_1(p_1 r) = a_1 p - b_1 p^2 - Fp^2/(p+r)$$

$$\text{and } G_2(p_1 r) = a_2 r - b_2 r^2 - Fr^2/(p+r)$$

then the elements of the community matrix $\underline{A}$ are given by

$$a_{11} = \partial G_1/\partial p \qquad a_{12} = \partial G_1/\partial r$$

$$a_{21} = \partial G_2/\partial p \qquad a_{22} = \partial G_2/\partial r$$

all evaluated at $(p_1^* r^*)$; the roots of the matrix are then given by $\det|A - \lambda I| = 0$. For stability max (Real λ) < 0. If max (λ) is complex, the system either decreases by oscillations to the stable part or else becomes unstable in an oscillatory mode. If λ : only imaginary, then the neutrally stable cycles of the basic Lotka-Volterra equations are seen.

Again it will be appreciated that general stability criteria cannot be evaluated due to the large number of parameters, but the stability of any specific case can easily be dealt with, and in general it must be noted that these are a very stable set of equations and unstable equilibria would not be expected.

If equations 1 and 2 are added together the relationship between the two sorts of Schaeffer models can be seen

$$dr/dt+dp/dt = a_1p+a_2r - b_1p^2 - b_2r^2 - F(p^2+r^2)/(p+r)$$

If $p+r = S$, then

$$ds/dt = a_1p+a_2r - b_1p^2 - b_2r^2 - F(p^2+r^2)/S.$$

and if $p \simeq r \simeq \frac{1}{2}S$, then

$$ds/dt = S(a_1+a_2)/2 - S^2(b_1+b_2)/4 - FS/2$$

which is identical with the Schaeffer model fit to the total fish biomass. However as the ratio of $p^*:r^*$ diverges from unity the two models diverge.

The simplest way of comparing the two models is to look at the yield per unit of effort against effort curve. By definition the Schaefer model assumes a linear relationship and the degree of departure from linearity represents the departure from the Schaefer model.

If we arbitarily select $a_1 = 0.35$ and $a_2 = 0.45$, then values of b_1 and b_2 can be obtained if we set $F = 0.2$ and vary the ratio of p:r but keep $p+r = 2000$. This gives

a) If p:r : : 1 : 1 Then $b_1 = 2.5\ 10^{-4}$ and $b_2 = 3.5\ 10^{-4}$

b) If p:r : : 1 : 3 Then $b_1 = 6.0\ 10^{-4}$ and $b_2 = 2.0\ 10^{-4}$

c) If p:r : : 1 : 19 Then $b_1 = 3.4\ 10^{-3}$ and $b_2 = 1.367\ 10^{-4}$

Hence we have three arbitary sets of values which, at F=0.2, have the ratios of p:r as given above. Equilibria values for p* and r* may be obtained from equations 3 and 4 for different values of F, and hence the equilibrium yield from equation 5. These are shown in Figure 1. It will be noted that the characteristic parabolic yield curve is lost as the ratio of p:r diverges from unity. This can be more easily seen in the graph of equilibrium catch per unit of effort against effort (Figure 2) as the departure from linearity.

DISCUSSION

Firstly, is the incorporation of fishing effort in this way valid? Obviously catchability and economic weighting of one species relative to another will complicate the real fishing strategy but this could only increase the non-linear nature of the results and probably the expression of fishing in this model is good for a fishery where total effort is fairly constant.

Secondly, it is noticeable that there is no real difference between the Schaeffer model of total fish biomass and the sum of individual Schaefer models where the two species are in about the same proportions. This can be extended to n species. However as the ratio diverges from 1 the two give quite different results. To fit a yield parabola through the origen one needs two or more points. If these are taken at F = 0.1 and 0.2, for p:r = 1:19, from Figure 1 the resulting parabola would reach zero at about F = 0.5. This can be appreciated better from Figure 2 where a straight line would be fitted through the two points giving an intercept at F = 0.475. Similarly if the values at F = 0.6 to 0.8 are taken, a long low parabola would result predicting a very low maximum sustainable yield.

The maximum sustainable yields for the type of fishing described here can be obtained from Figure 2 but this is not necessarily the maximum sustainable yield that could be achieved if fishing were distributed differently. If the equilibrium populations of the two species are given by

$$a_1 - b_1 p^* - f_1 = 0 \quad \text{and}$$

$$a_2 - b_2 r^* - f_2 = 0,$$

hen the equilibrium yield is then given by

$$f_1(a_1-f_1)/b_1 + f_2(a_2-f_2)/b_2.$$

The yield isopleths are given in Figure 3 along with the locus of equilibrium yields for the original proportional fishing model and it will be noted that, in this instance, the locus does not go through the maximum sustainable yield.

In conclusion two points are worth stressing. First, that the two methods, grouping fish species and taking them as separate species related only by fishing, do not lead to the same results if the species are in differing proportions. This is not to say that on method is better than the other for the underlying assumptions of the Schaeffer model are nebulous. Second, that in this simple, although rational model proportionate fishing does not necessarily take the fishery through the maximum sustainable yield.

REFERENCES

Beddington, J.R. 1974. Age distribution and stability of simple discrete time population models. J. theor. Biol. 47, 65-74.

Brander, K.M. 1975. The population dynamics and biology of cod (Gadus morhua morhua L.) in the Irish Sea. Ph.D. Thesis. University of East Anglia. 103p

Gulland, J.A. 1974. The management of marine fisheries. Bristol Scientechnica (publishers) Ltd. 198pp.

May, R.M. 1973. Stability and complexity in model ecosystems. Princeton University Press. 235pp.

Pinhorn, A.T. 1975. Catch and effort relationships of the groundfish resource in Subareas 2 and 3. ICNAF Res. Doc. 75/55. 20pp.

Schaefer, M.B. 1954. Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. Inter-Amer. Trop. Tuna Comm., Bull. 1(2) 26-56.

1957. A study of the dynamics of the fishery for yellowfin tuna in the eastern tropical Pacific Ocean. Inter-Amer Trop. Tuna Comm., Bull. 11(6) 247-285.

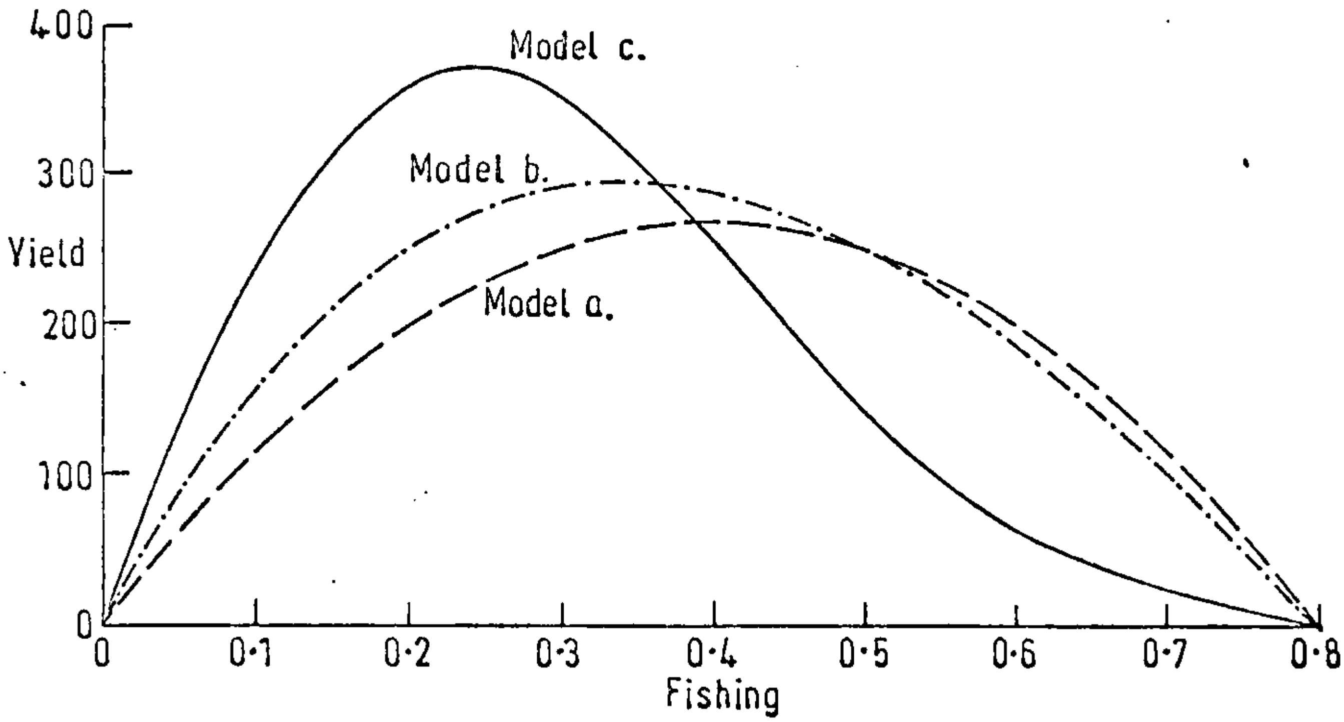

Fig.1. Equlibrium yield and effort for the three sets of model values given in the text.

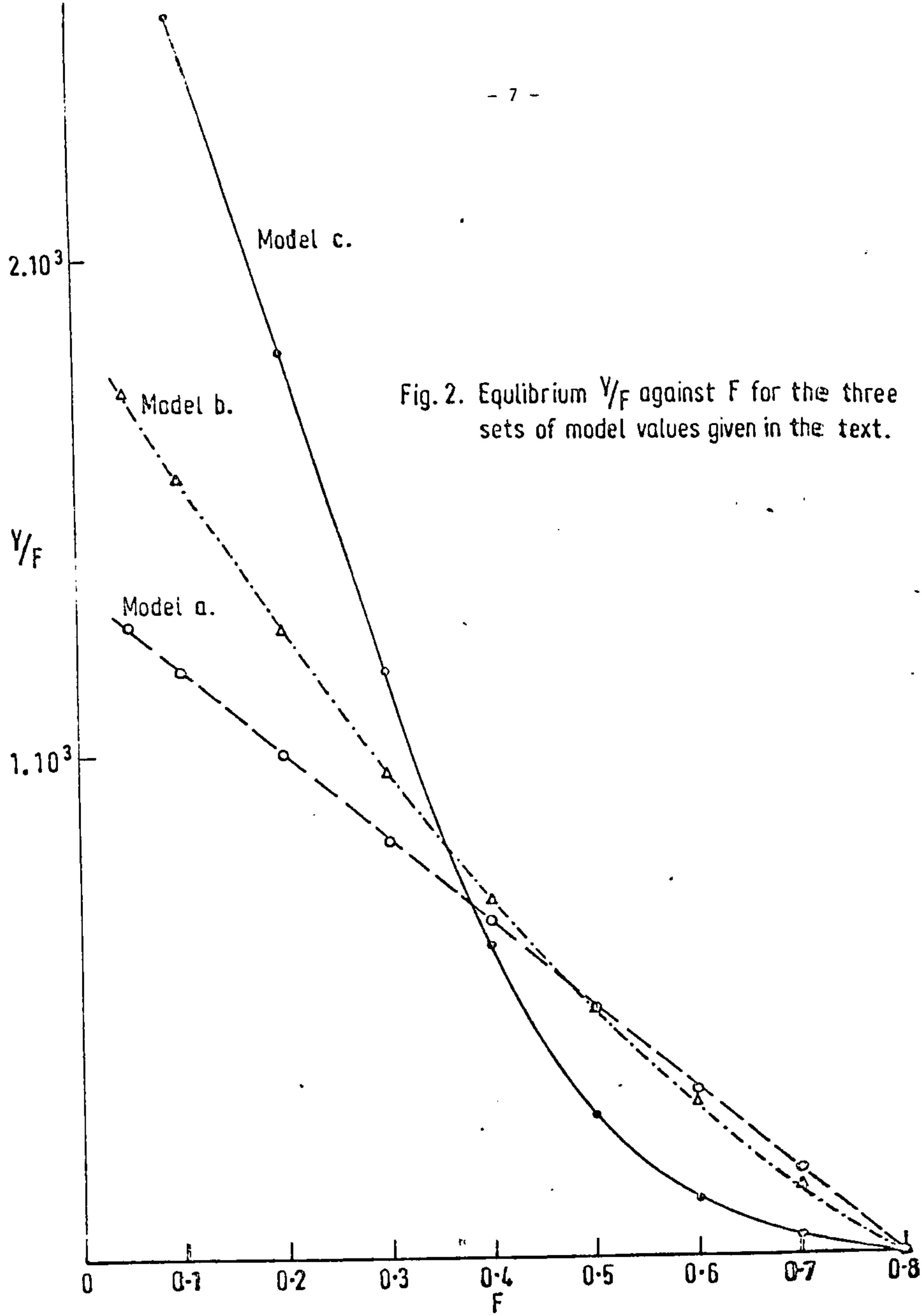

Fig. 2. Equlibrium Y/F against F for the three sets of model values given in the text.

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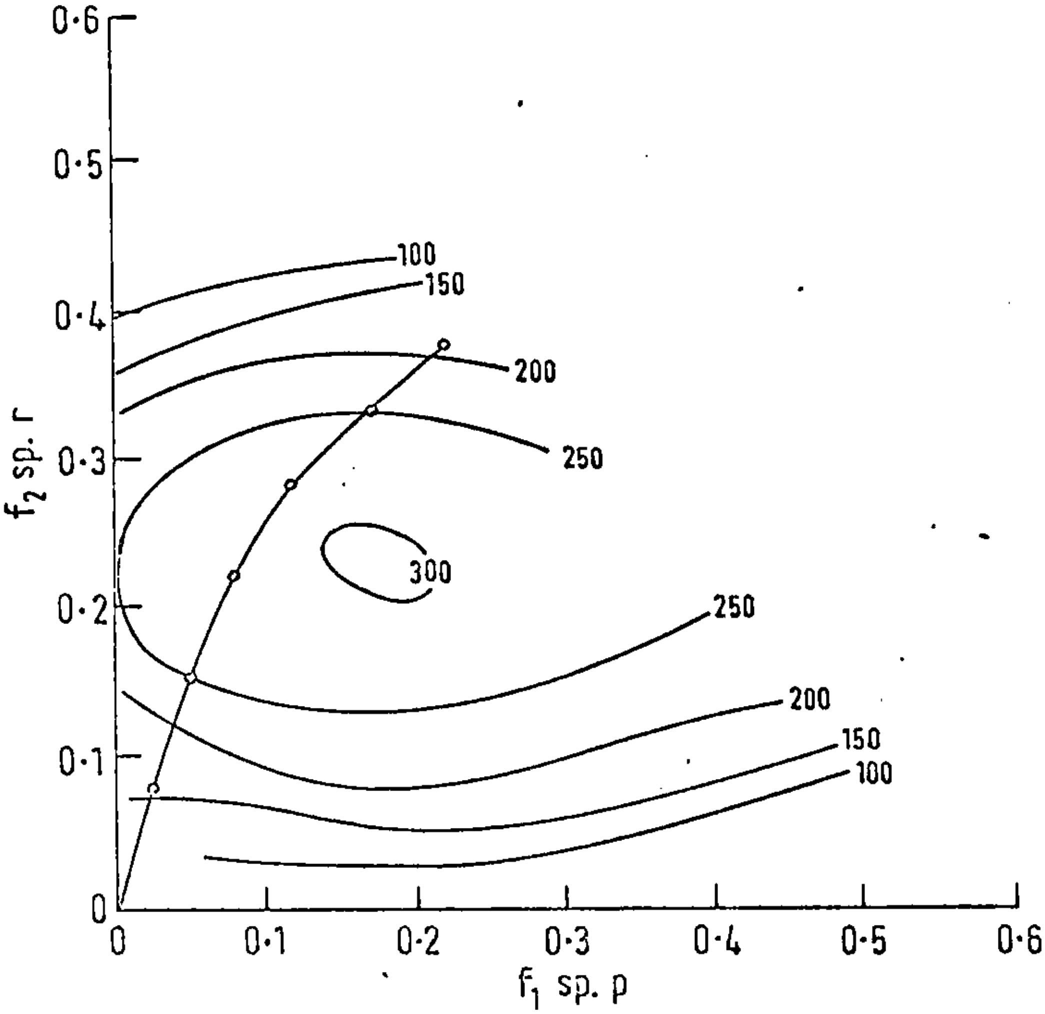

Fig. 3. Sustainable yield isopleths with the locus of yields obtainable with proportionate fishing.