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Nonequilibrium regulation of fisheries

by

Gilbert G. Walter
Department of Mathematics and Center for Great Lakes Studies
University of Wisconsin-Milwaukee, Milwaukee, WI, USA 53201

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Many of the world's fisheries have until recently been in a virgin state or close to it. The exploitation has been marginal and has not severely affected the stock size. In the last twenty years these fisheries have come under increasing exploitation, which has often reduced the stock size and necessitated some sort of regulation of the catch.

There are many theories available to the fisheries biologist for this regulation. They usually give an estimate for the maximum yield that the fishery can produce on a continuing basis, *i.e.* the maximum sustained yield (MSY). It should be observed that this value is not the maximum possible catch in a given year, but, particularly in a virgin fishery, is considerably less.

The newly exploited stock is usually difficult to manage. Few data years are available and they are often unreliable. Moreover, the stock is not in equilibrium in the presence of fishing. This latter fact is often overlooked and is one of the reasons for the shrinking estimates of MSY that are sometimes encountered.

In this work we shall study the yield of a fishery under nonequilibrium conditions and compare strategies for bringing the stock size to that required for maximum sustained yield. We shall consider reduction to this optimum stock size from above as well as increase from below.

We introduce a procedure based on the equation of Schaefer which assumes the growth rate of the total stock biomass to be a function of the biomass itself and of fishing effort. Although no delayed effects are present in the equation, we shall see that there is considerable delay between the initiation of a regulation and the attainment of the desired equilibrium state.

Mathematical formulation.

The equation of Schaefer (1954) is:

$$\frac{1}{P}\frac{dP}{dt} = r\left(1-\frac{P}{P_\infty}\right)-qf \qquad (1)$$

where P represents the biomass of the stock in question, f the fishing effort, r the maximum instantaneous growth rate of the stock, P_∞ the equilibrium biomass in the absence of fishing, and q the coefficient of catchability. Under conditions of equilibrium the rate growth is zero and the equation becomes:

[1] All ICNAF documents will now be numbered to include the month (in Roman numerals) of the meeting at which they were presented.

$$0 = r(1-\frac{P_e}{P_\infty})-qf \qquad (2)$$

where P_e is the equilibrium biomass. The equilibrium yield is given by:

$$y_e = qfP_e = qf\frac{P_\infty}{r}(r-qf) \qquad (3)$$

The maximum value of the equilibrium yield is attained when $f = \frac{r}{2q}$ and is given by:

$$y_{e,max.} = \frac{r}{4}P_\infty \qquad (4)$$

As Schaefer observed, but didn't exploit in his yield equation, few fisheries are in a state of equilibrium. It could further be added that most are in a state of decline (see Edwards and Hennemuth, 1975). Thus it seems desirable to calculate the yield under nonequilibrium conditions. We must introduce a time variable in order to make this calculation. Accordingly, let y_n be the yield of a fishery during the course of the n^{th} year of fishing at the constant effort f. We assume that the stock biomass satisfies equation (1). Then we see that:

$$y_n = \int_{n-1}^{n} qfPdt = qf\int_{n-1}^{n} Pdt, \qquad (5)$$

from which, by solving (1) for P and substituting the answer in (5), we obtain:

$$y_n = qf\int_{n-1}^{n} P_\infty(1-\frac{dp/dt}{rp}-\frac{qf}{r})dt$$

$$= qfP_\infty-\frac{qfP_\infty}{r}\ln P\Big|_{n-1}^{n} -\frac{q^2f^2P_\infty}{r}$$

$$= \frac{qfP_\infty}{r}(r-qf)+\frac{qfP_\infty}{r}\ln\frac{P_{n-1}}{P_n} \; . \qquad (6)$$

Here P_{n-1} denotes the biomass at the beginning and P_n at the end of the n^{th} year. The first term in the last expression may be observed to be, by comparison to (3), the yield under equilibrium conditions for the year. Thus (6) may be expressed as:

$$y_n = y_e+\frac{qfP_\infty}{r}\ln\frac{P_{n-1}}{P_n} \qquad (7)$$

where the second term on the right represents the yield resulting from changes in the biomass. It will be either positive or negative depending on whether P_{n-1} is greater or less than P_n. Of course P_n cannot be set arbitrarily but depends on P_{n-1} and on f. We may calculate the value of P_n by solving (1) for P with the initial value P_{n-1}. This solution is given by:

$$P_n = (P_{n-1})\frac{(r-qf)P_\infty}{rP_{n-1}-(rP_{n-1}-(r-qf)P_\infty)\exp(-(r-qf))} \qquad (8)$$

This expression may be simplified somewhat. The level of fishing effort f_∞ beyond which the population will be annihilated eventually is given by:

$$f_\infty = \frac{r}{q} \; .$$

We denote by x that fraction of this effort that corresponds to f, *i.e.*:

$$x = f/f_\infty \; .$$

Then (8) becomes, in terms of x:

$$P_n = \frac{P_{n-1}(1-x)P_\infty}{P_{n-1}-(P_{n-1}-(1-x)P_\infty)\exp(-(1-x)r)} \tag{9}$$

In considering the virgin initial state, we take n-1 equal to 0 and set P_o equal to P_∞. In this case (9) reduces to:

$$P_1 = \frac{(1-x)P_\infty}{1-x\exp(-(1-x)r)} \quad . \tag{10}$$

This may be substituted into (6) and f replaced by $\frac{xr}{q}$ to obtain the expression:

$$y_1 = xP_\infty(1-x)r + xP_\infty \ln\left\{\frac{1-xe^{-(1-x)r}}{1-x}\right\} \tag{11}$$

for the yield of a virgin fishery during the first year of exploitation. It has the same general form as the equilibrium yield curve except that the maximum occurs at different values of x (or of f). Figure 1 is a plot of both curves for r = 0.2. If r is small compared to 1, then the exponential function in (11) may be approximated by the first two terms of its power series. The simplified version is then:

$$y_1 \approx rP_\infty x(1-x) + xP_\infty \ln(1+rx). \tag{12}$$

<u>Two possible strategies.</u>

The strategy of choosing that f which maximizes the yield is clearly inadequate in the nonequilibrium case. Indeed, as a reference to figure 1 shows, the value of x which would maximize yield in the first year would be greater than any which could be sustained.

The simplest strategy to use is the constant effort, set at the level that would give maximum sustained yield. That is at x = ½. The yield in this case will decrease asymptotically to the maximum sustainable yield. During the first year the yield will be:

$$y_1 = y_e + \frac{P_\infty}{2}\ln(2-e^{-r/2})$$

during the second,

$$y_2 = y_e + \frac{P_\infty}{2}\ln\left\{\frac{2-e^{-r}}{2-e^{-r/2}}\right\}$$

and during the nth,

$$y_n = y_e + \frac{P_\infty}{2}\ln\left\{\frac{2-e^{-\frac{nr}{2}}}{2-e^{-\frac{(n-1)r}{2}}}\right\} \quad . \tag{13}$$

Thus the yield would be gradually decreasing from a first year which is almost double the maximum sustained yield for small r to one which approaches it as n increases.

An alternate strategy to the constant effort would be one in which the yield is constant in each year at the MSY level. In this case the effort would increase gradually to that necessary for maximum sustained yield. The effort required the first year may be calculated by first setting y_1 in equation (11) to the MSY and then solving it for x. The usual approximations lead to the equation

$$1 + r(\frac{1}{4x} - 1 + x) = 1 + rx \qquad (14)$$

which when solved for x yields:

$$x_1 = \frac{1}{4},$$

the approximate effort needed for a first year yield equal to the MSY. In order to calculate the level of effort for subsequent years, we use equation (9), approximate the exponential function by a linear function and after some simplification find that:

$$P_{n-1}/P_n = r(P_{n-1}/P_\infty - 1) + 1 + rx, \qquad (15)$$

and hence that the yield during the nth year is approximately

$$y_n \doteq xP_\infty(1-x)r + xP_\infty \ln\left\{r(P_{n-1}/P_\infty - 1) + 1 + rx\right\}. \qquad (16)$$

This may be set equal to $rP_\infty/4$ as was done in the previous case and solved for x_n to obtain:

$$x_n = \frac{P_\infty}{4P_{n-1}}. \qquad (17)$$

From equation (15) it follows that P_n/P_{n-1} is always less than 1 for P_{n-1} between $P_\infty/2$ and P_∞, and hence P_n is monotonically decreasing to $P_\infty/2$ as x_n approaches 1/2.

Either of these strategies is successful if the correct values of f_{MSY} and MSY are known. The constant yield strategy would be easiest to regulate through catch quotas and the constant effort through effort regulation, although by using our calculations either regulation is possible in each case. The constant effort strategy reduces the biomass to maximum production more rapidly than the constant yield strategy. Figure 2 shows the successive operating points for both.

Unfortunately, for most fisheries, the values of f_{MSY} and MSY are known only approximately, particularly in the early years of a heavily exploited fishery when the need for regulation becomes apparent. Accordingly we shall investigate the effect of errors in estimation on the strategies.

Effect of errors in the estimates.

Let us first suppose that the maximum sustained yield is underestimated by an amount εP_∞ but that the corresponding effort is known exactly. The effect on the constant effort strategy would be nil if the fishing were regulated through effort. The effect on the constant yield strategy regulated through catch quotas would be to stabilize the fishery at the levels:

$$\hat{y}_e = \frac{rP_\infty}{4} - \varepsilon P_\infty, \qquad (18)$$

$$\hat{x}_e = \tfrac{1}{2} - \sqrt{\varepsilon/r}. \qquad (19)$$

which would lead to underutilization of the fishery through underfishing, but which could be corrected in subsequent years.

If the MSY were overestimated by an amount εP_∞ the constant effort strategy again would be unaffected. For the constant yield case, the biomass during the nth year, by equation (16) would satisfy:

$$\frac{rP_\infty}{4} + \varepsilon P_\infty = x P_\infty(1-x)r + x P_\infty \ln\left\{r(P_{n-1}/P_\infty - 1) + 1 + rx\right\} \quad (20)$$

This may again be solved for the necessary relative effort x which is

$$x_n = \frac{P_\infty}{4P_{n-1}} \left(1 + \frac{4\varepsilon}{r}\right), \quad (21)$$

and which may be substituted into equation (15) to obtain

$$P_{n-1}/P_n = r(P_{n-1}/P_\infty - 1) + rP_\infty/4P_{n-1} + \varepsilon P_\infty/P_{n-1} + 1 . \quad (22)$$

By letting n = 1 and setting $P_0 = P_\infty$, we obtain:

$$P_1 = P_\infty(4/4\varepsilon + 4tr) \leq P_\infty/(1+\varepsilon).$$

To find the limit as $n \to \infty$, we use the fact that for any real number $\alpha > 0$,

$$\alpha + 1/4\alpha - 1 \geq 0.$$

By setting $\alpha = P_{n-1}/P_\infty$, we see that

$$r(P_{n-1}/P_\infty - 1) + rP_\infty/4P_{n-1} \geq 0$$

and hence that

$$P_{n-1}/P_n \geq 1 + \varepsilon P_\infty/P_{n-1} \geq 1 + \varepsilon. \quad (23)$$

Thus the sequence $\left\{P_n\right\}$ satisfies the inequality

$$P_n \leq (1 + \varepsilon)^{-n} P_\infty \quad (24)$$

which in the limit is

$$\lim_{n\to\infty} P_n = 0.$$

Hence this strategy can reduce the biomass to very low levels if not corrected. For example, if the MSY is overestimated by 10%, the biomass will decline to 10% of its equilibrium value in less than 24 years, by equation (24). If the MSY is overestimated by 25%, the decline will take less than 10 years.

We now turn to the question of error in the estimation of f_{MSY}. Suppose it is assumed to occur at:

$$\hat{x} = \tfrac{1}{2}(1\pm\delta),$$

and the fishery regulated accordingly. Under the constant yield strategy, this error will have no effect if the fishery is regulated by catch. Under the constant effort strategy, the yield will stabilize at something less than the optimum level. Indeed, from equation (11) we may calculate that the equilibrium level attained will be:

$$\hat{y}_e = rP_\infty\left(\frac{1}{2} \pm \frac{\delta}{2}\right)\left(1 - \left(\frac{1}{2} \pm \frac{\delta}{2}\right)\right)$$

$$= rP_\infty\left(\frac{1}{4} - \frac{\delta^2}{4}\right). \quad (25)$$

Thus an error of 100δ percent in estimating the optimum effort leads to an error of $(10\delta)^2$ percent in the yield. For example an error of 50 percent in the estimate will lead to a reduction in the ultimate equilibrium yield of 25 percent.

The effect of small errors in the estimates of MSY and corresponding effort generally are not fatal except in the case (see Figure 3) of overestimation of MSY in a fishery regulated by catch in which the constant yield strategy is followed. It is of interest to investigate the nature of appropriate corrective action.

Corrective strategy for initial errors.

Let us suppose that initially the MSY is overestimated by an amount $\varepsilon_1 P_\infty$. After n_1 years the error is discovered and the regulation changed to account for the initial error. What the new allowable catch should be depends on n_1 and the initial error.

The choice of setting the maximum catch at the new estimate for the MSY is a bad strategy whenever the biomass has been reduced by more than half from the virgin level. It would lead to continued reduction of the biomass if the actual catch approaches the allowed maximum. The reason for this is that the effort would be:

$$x_{n_1} = (P_\infty/4P_{n_1}-1)\ (1 + 4\varepsilon/r) > \tfrac{1}{2}(1 + 4\varepsilon/r), \qquad (26)$$

which is greater than the optimum value of $\tfrac{1}{2}$. By repeating the argument which led to equation (24), we can deduce that

$$P_n \leq \left(\frac{1}{rB+1}\right)^{n-n_1} P_{n_1-1}, \qquad (27)$$

where $B = (P_{n_1-1}/P_\infty - 1 + P_\infty/4P_{n_1-1})$. Thus a catch at the MSY level will lead to a continued reduction in the biomass.

Clearly, some other corrective action must be taken. One possibility is to switch to effort regulation which, if the estimate of f_{MSY} is not greatly in error, will lead to recovery of the stock. Another is to set the catch at the level which would correspond to the optimum effort.

This catch would be below the MSY as is clarified by Figure 4. The appropriate catch level, by equation (16) with $x = \tfrac{1}{2}$, is:

$$y = \frac{rP_\infty}{4} + \frac{P_\infty}{2} \ln \{r(P_{n-1}/P_\infty - \tfrac{1}{2}) + 1\} \qquad (28)$$

which, since P_{n-1}/P_∞ is less than $\tfrac{1}{2}$, is less than the MSY $(rP_\infty/4)$. This may be further approximated by replacing the ln function by the first two terms of its power series to obtain:

$$y \approx \frac{rP_\infty}{4} + \frac{P_\infty}{2} \{r\frac{P_{n-1}}{P_\infty} - \frac{r}{2}\} = \frac{rP_{n-1}}{2} \qquad (29)$$

The yield in year n-1 in turn may be obtained from equation (5) by approximating the average biomass (given by the integral) by the biomass at the end of that year:

$$y_{n-1} \approx qf_{n-1}\, P_{n-1}\ . \qquad (30)$$

Here f_{n-1} denotes the effort in year n-1. This in turn may be solved for P_{n-1} and substitutes in equation (29) to obtain

$$y_n \approx \frac{ry_{n-1}}{2qf_{n-1}} = f_{opt.}\ \frac{y_{n-1}}{f_{n-1}} \qquad (31)$$

since $f_{MSY} = 4/(2q)$. This formula gives rise to a simple graphical procedure for determining the total allowable catch (TAC). On a plot of yield versus effort, the most recent point (f_{n-1}, y_{n-1}) is located and a straight line drawn through the origin. The intersection of this line and the vertical line through f_{MSY} locates the TAC point.

The response indicated here is adequate when the reduction in biomass has not been too extreme, *e.g.* when P_{n_1-1}/P_∞ is still larger than 25%. However, in cases where it is greater, the major consideration should be the recovery of the stock rather than the maximization of the yield. Accordingly, we shall calculate the time needed for recovery of the stock to the MSY level when initially it is very low.

<u>Recovery time of overexploited stocks.</u>

Let us suppose the stock has been reduced to a level lower than $P_\infty/4$. Then the maximum speed of recovery is achieved when fishing effort is set at zero. We first calculate the minimum recovery time and then the appropriate levels of effort and yield to achieve recovery in a predetermined number of years.

The recovery time in the presence of fishing at the level x for n years is obtained by first solving equation (1) for P_n when the initial level is some P_0 less than $P_x/4$. The solution is:

$$P_n = \frac{P_\infty(1-x)}{1-(1-(1-x)P_\infty/P_0)\exp(-(1-x)rn)} \qquad (32)$$

which is set equal to $P_\infty/2$, and after inverting both sides, becomes:

$$2(1-x) = 1-(1-(1-x)P_\infty/P_0)\exp\{-(1-x)rn\}. \qquad (33)$$

This may be solved for n to obtain:

$$n_x = \frac{1}{(1-x)r} \ln \left\{\frac{(1-x)(P_\infty/P_0)-1}{1-2x}\right\} , \qquad (34)$$

which will be minimized when x = 0 and will be infinite when x = ½. Thus the fastest possible recovery when, say $P_0/P_\infty = 0.1$ would be:

$$n_0 = \frac{1}{r} \ln \left[P_\infty/P_0-1\right] = \frac{1}{r}\ln q = \frac{2.2}{r} . \qquad (35)$$

This would be 11 years when $r = .2$, the value used in our example.

If a particular recovery time has been specified, equation (33) may be solved for the particular value of x that will bring it about. We shall obtain an upper bound for this value in terms of the maximum recovery time n_x. It may be obtained by taking the ratio of (34) to (35) which is:

$$n_x/n_0 = \frac{1}{1-x} \frac{\ln\left[((1-x)(P_\infty/P_0)-1)/(1-2x)\right]}{\ln\left[(P_\infty/P_0)-1\right]} . \qquad (36)$$

Since the quotient $((1-x)(P_\infty/P_0)-1)/(1-2x)$ is an increasing function of x, it is larger than its value at x = 0. Hence we see that

$$n_x/n_0 \geq \frac{1}{1-x} \frac{\ln\left[P_\infty/P_0-1\right]}{\ln\left[P_\infty/P_0-1\right]} \geq \frac{1}{1-x} ,$$

from which by solving for x we obtain:

$$x \leq 1-n_0/n_x . \qquad (37)$$

For example, if $r = 0.2$, and $P_0/P_\infty = 0.1$, and if we wish the stock to recover in 15 years, we set the fishing effort limitation at

$$x \leq 1-11/15 = 0.27 .$$

This approximation is valid only when n_0/n_x is quite close to 1. For other values we may solve (36) for x numerically, or we may interpolate linearly between x = 0 and x = ¼. Then we obtain a slightly better estimate with the latter procedure. Indeed, since the function $(1-x)n_x/n_0$ is convex for x between 0 and ¼, it is greater than its linear interpolate, *i.e.*

$$(1-x)n_x/n_0 \geq 1-4x\left\{\frac{n_{1/4}(1-1/4)}{n_0}-1\right\} .$$

This may be solved for x to obtain the inequality:

$$x \leq \frac{1-n_0/n_x}{1+(4c-4)n_0/n_x}$$

where

$$c = (1-1/4)n_{1/4}/n_0 = \frac{\ln(3P_\infty/2P_0-2)}{\ln(P_\infty/P_0-1)} .$$

For our previous example, with n = 15, we obtain the value:

$$x \leq \frac{1-11/15}{1+\frac{44}{15}\frac{\ln 13}{\ln 9}} = 0.18$$

which is considerably smaller.

The corresponding values of yield may be obtained from equation (16) through substitution of the values of x obtained by equation (38).

APPENDIX

An example

The Atlantic mackerel fishery seems to fit the description of a fishery not in equilibrium and therefore amenable to our analysis. Anderson (1975) has reviewed the status of this fishery for ICNAF Statistical Areas 5 and 6. Using a modification of the method of Walter (1975), applied to his data (Table 10), we are able to obtain an estimate for MSY and corresponding effort. They are:

$$MSY = 313{,}000 \text{ MT}$$

$$f_{MSY} = 250{,}000 \text{ std. US days.}$$

These estimates correspond to the equilibrium curve shown in Figure 6. On this same graph are plotted the yield and effort for the years 1968 to 1973.

The value of r for this fishery was calculated indirectly by using the catchability coefficient q, which in turn was estimated from the fishing mortality F = 0.6 used in the mackerel assessment (ICNAF Redbook 1974, p. 118) for 1973 and F = 0.55 used in 1972. The effort in those years was 719,000 and 461,000 days respectively. Since qf and F both correspond to the same fishing mortality, the appropriate choice of q would be:

$$q = \frac{0.55 + 0.60}{0.719 + .461} \times 10^{-6} = 0.975 \times 10^{-6} \approx 1 \times 10^{-6} .$$

The equilibrium yield curve of equation (3) for this example would be:

$$y_e = 10^{-6} f \frac{P_\infty}{r}(r - 10^{-6} f) .$$

With $f = f_{MSY}$ this must be equal to the MSY, *i.e.*:

$$.313 \times 10^6 = 10^{-6} \times .25 \times 10^6 \frac{P_\infty}{r}(r - 10^{-6} \times 25 \times 10^6).$$

However, by equation (4) the MSY also equals $rP_\infty/4$. Hence we have:

$$rP_\infty = 4(.25 \frac{P_\infty}{r}(r - .25))$$

which may be solved for r. The solution is:

$$r = 0.50,$$

the intrinsic growth rate for this stock of mackerel. The value of P_∞ is now easily found. It is:

$$P_\infty = .313 \times 10^6 \times 4/r = 2.5 \times 10^6 \text{ M.T.}$$

Using this value, the stock biomass for any year may be estimated by drawing a line through a point (f, y) corresponding to actual effort and catch and the origin. The intersection of this line and the vertical line through f_{MSY}, will give the TAC and the stock biomass as a fraction of $P_\infty/2$. Thus, in Figure 6, it may be observed that the stock biomass in 1971 was at $P_\infty/2$ or 1.25×10^6, in 1972 it was at $P = 0.85 \times 10^6$, and in 1973 at $P = .55 \times 10^6$. The total allowable catch for each year by this method should have been as follows:

Year	1971	1972	1973	1974
TAC	313	313	213	138 .

In order to estimate the TAC for 1975 we must first calculate the stock biomass for 1974. This may be done by using equation (8).

$$P_{74} = P_{73} \frac{(0.5-10^{-6}f)2.5x10^6}{0.5P_{73}-(0.5P_{73}-(0.5-10^{-6}f)2.5x10^6)e^{-(0.5-10^{-6}f)}}$$

$$= \frac{.55x10^6(0.5-0.72)2.5x10^6}{0.5x0.55x10^6-(0.5x0.55x10^6-(-.22)2.5x10^6)e^{+.22}}$$

$$= \frac{-0.303x10^{12}}{(0.275-(0.275+.55)1.246)x10^6} = .40x10^6 .$$

Accordingly, the TAC for 1975 should be 100,000 M.T. if recovery to the MSY level is desired. If the biomass is to be sustained at the 1974 level, a catch of up to 156,000 M.T. could be allowed. A catch in excess of this would cause the stock to decline further according to the model.

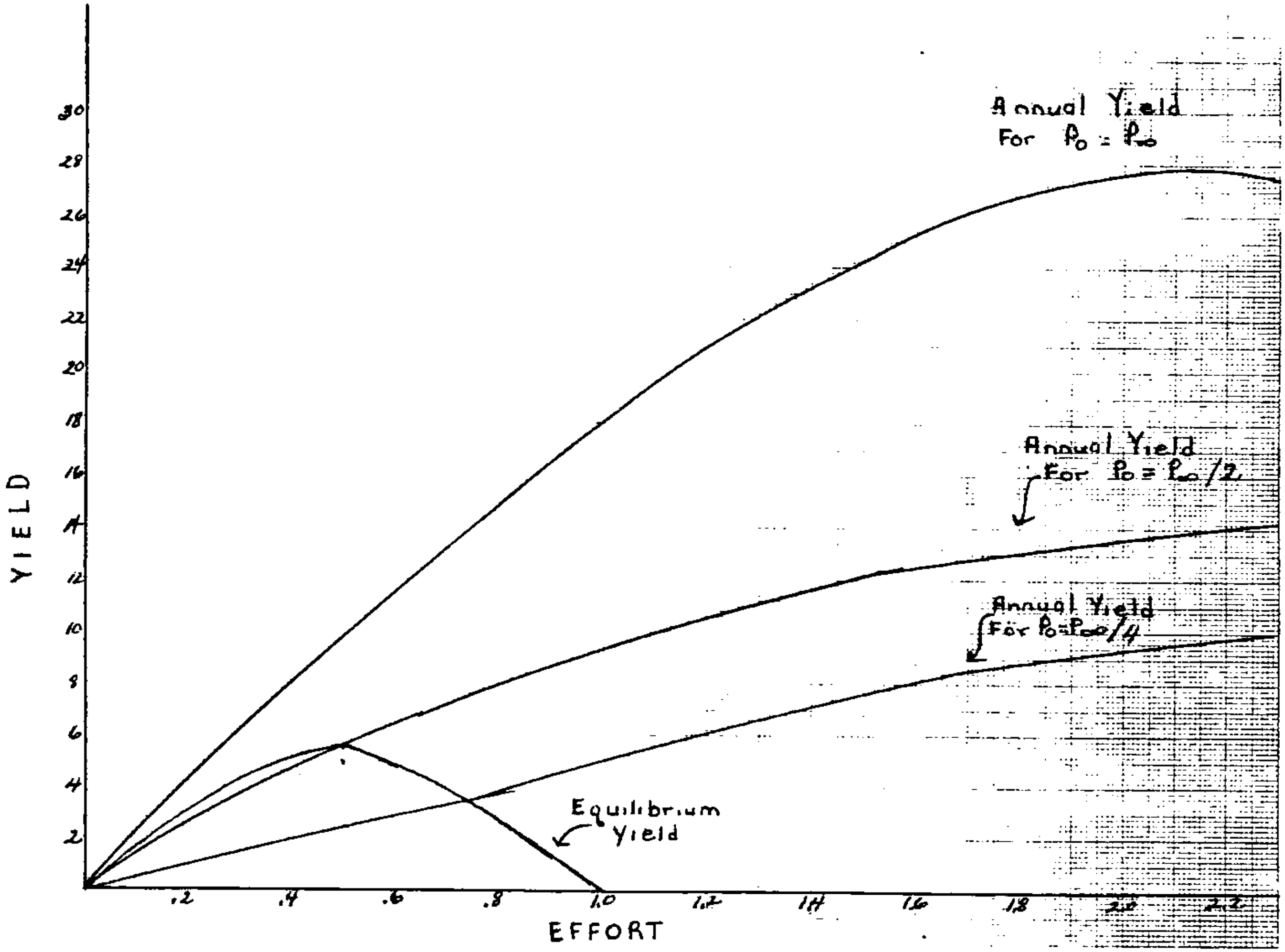

Figure 1. Annual yields and equilibrium yield for a hypothetical fishery with $P_\infty = 100$ and intrinsic growth rate $r = 0.2$. Annual yield is calculated for an initial population biomass to P_∞, $P_\infty/2$, and $P_\infty/4$.

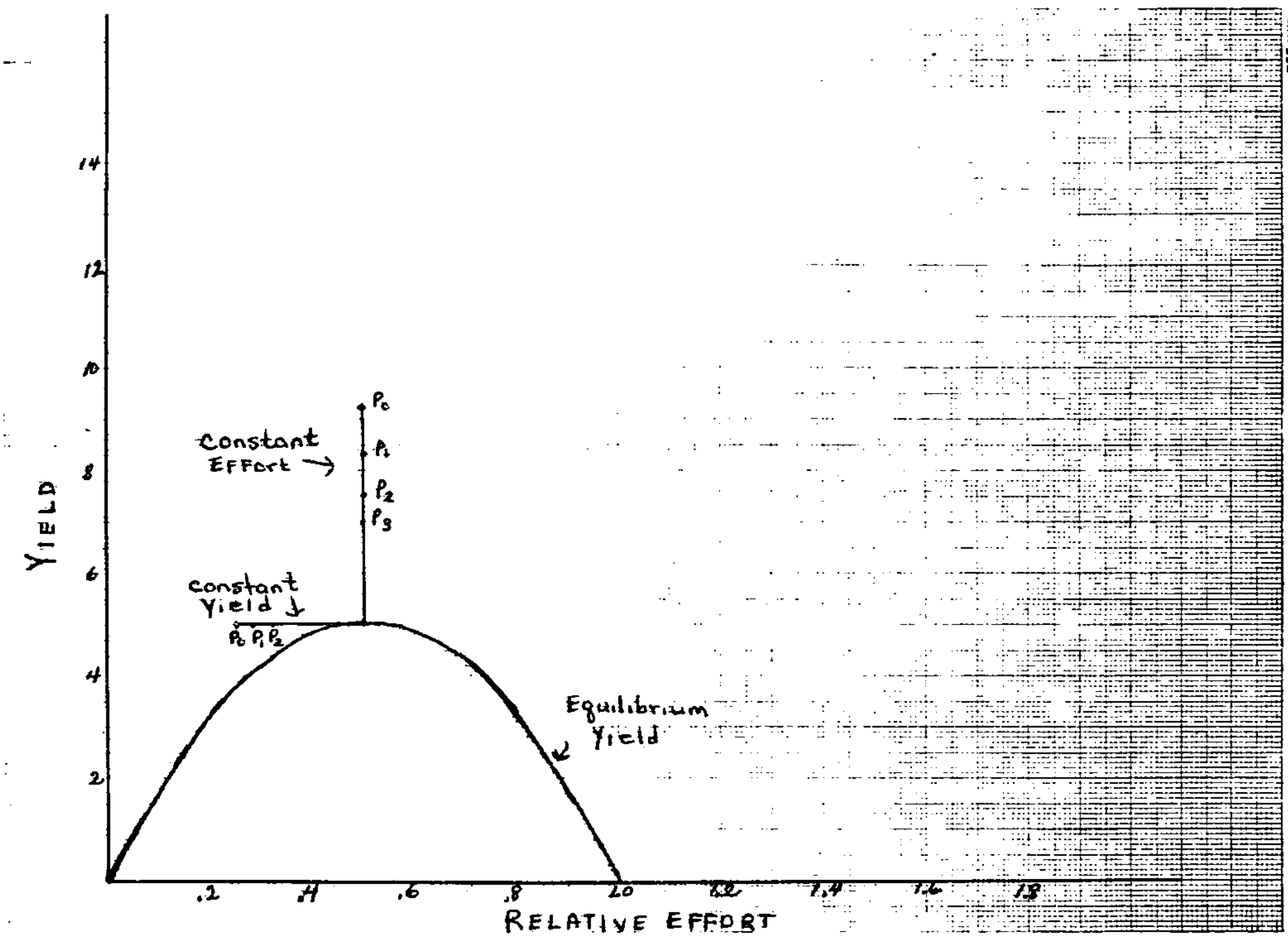

Figure 2. Two strategies for reducing a virgin state fishery to the optimum level of biomass. Parameters are P_∞ = 100 and r = 0.2.

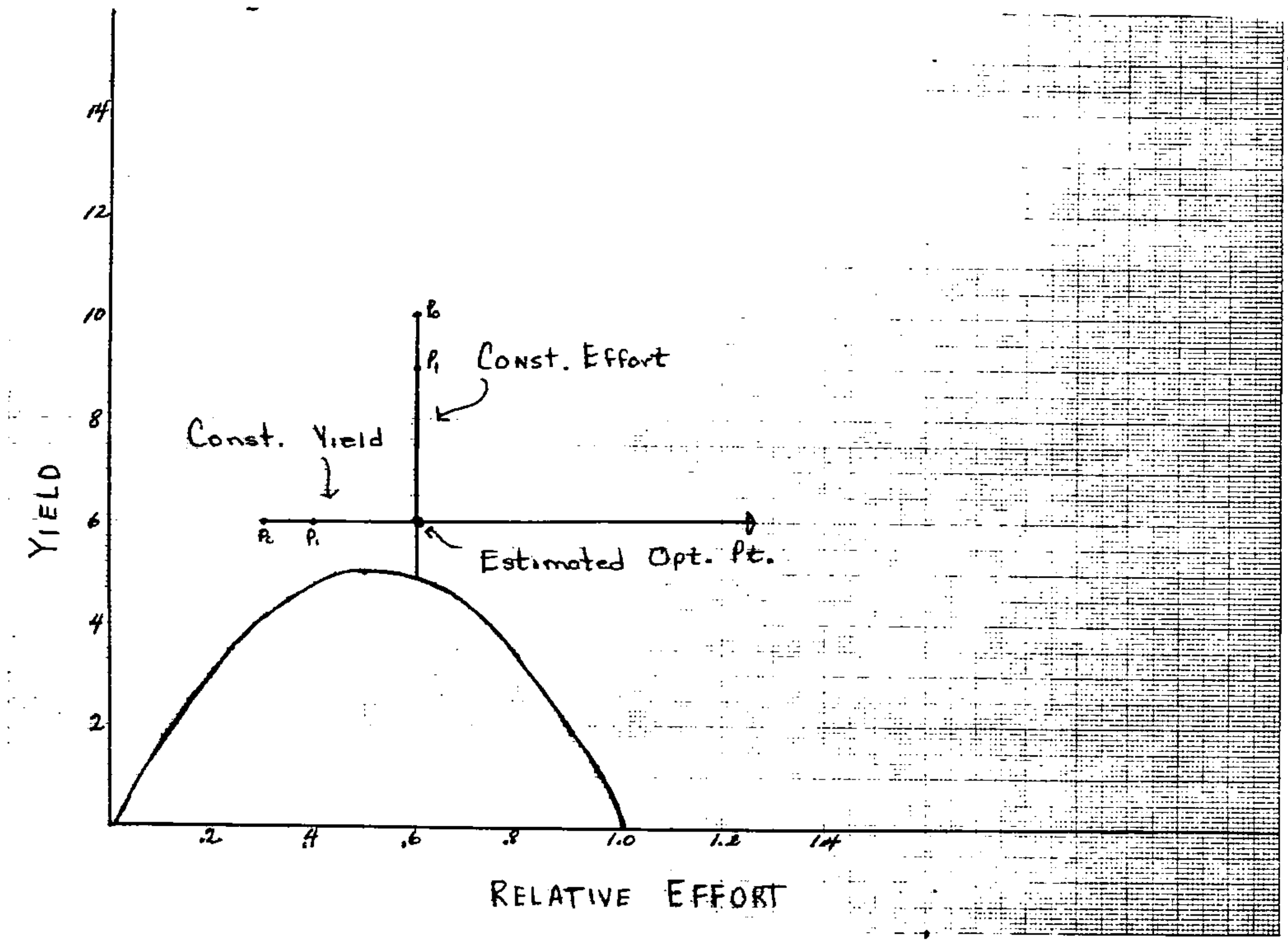

Figure 3. The effect of overestimating MSY and optimum effort on constant yield and constant effort strategy for a hypothetical fishery with P_{∞} = 100 and r = 0.2.

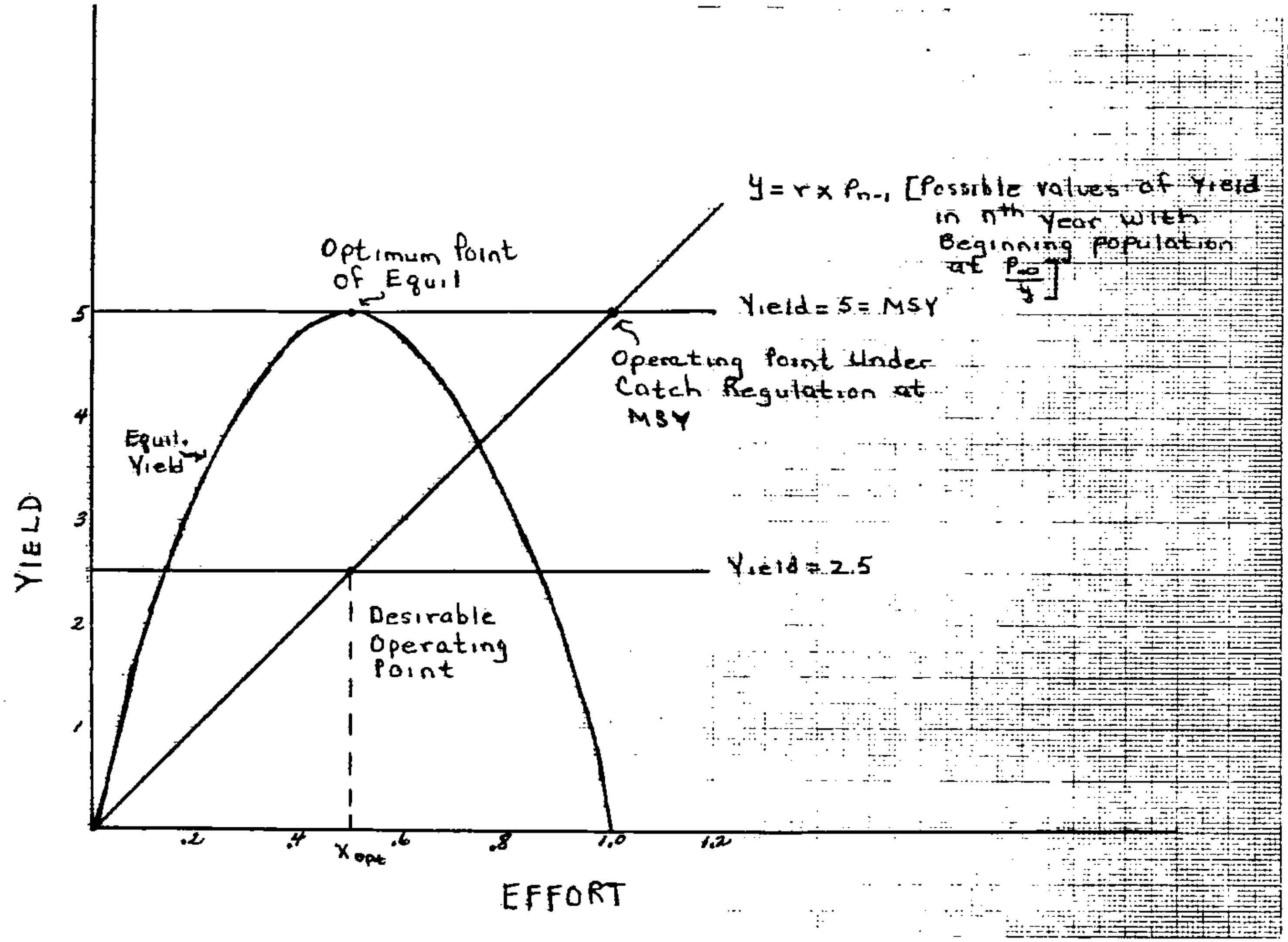

Figure 4. Location of desirable operating points for an overexploited fishery. It is assumed the biomass has been reduced to $P_\infty/4$ in a fishery with $P_\infty = 100$ and $r = 0.2$.

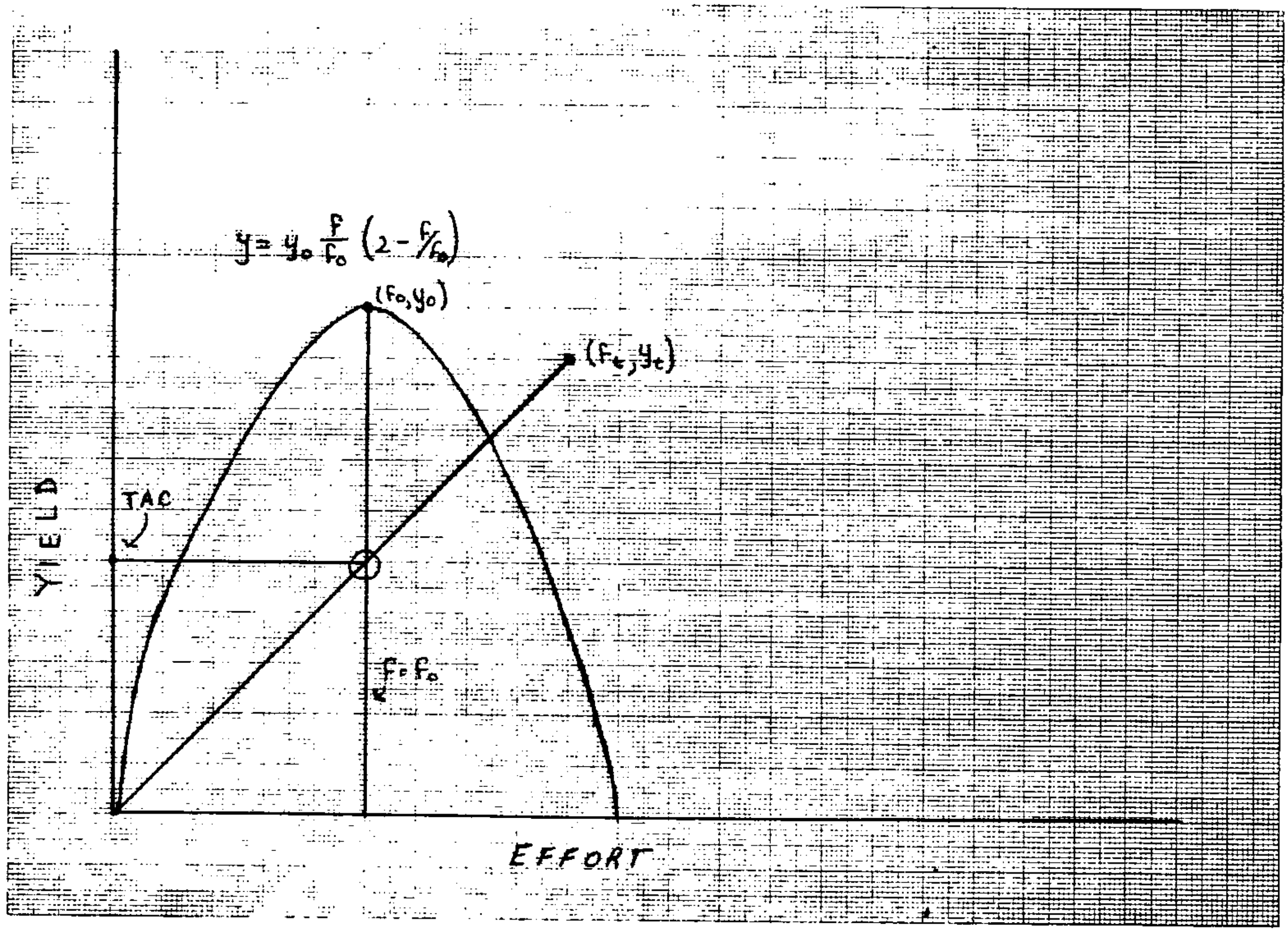

Figure 5. Graphical method for determining TAC.

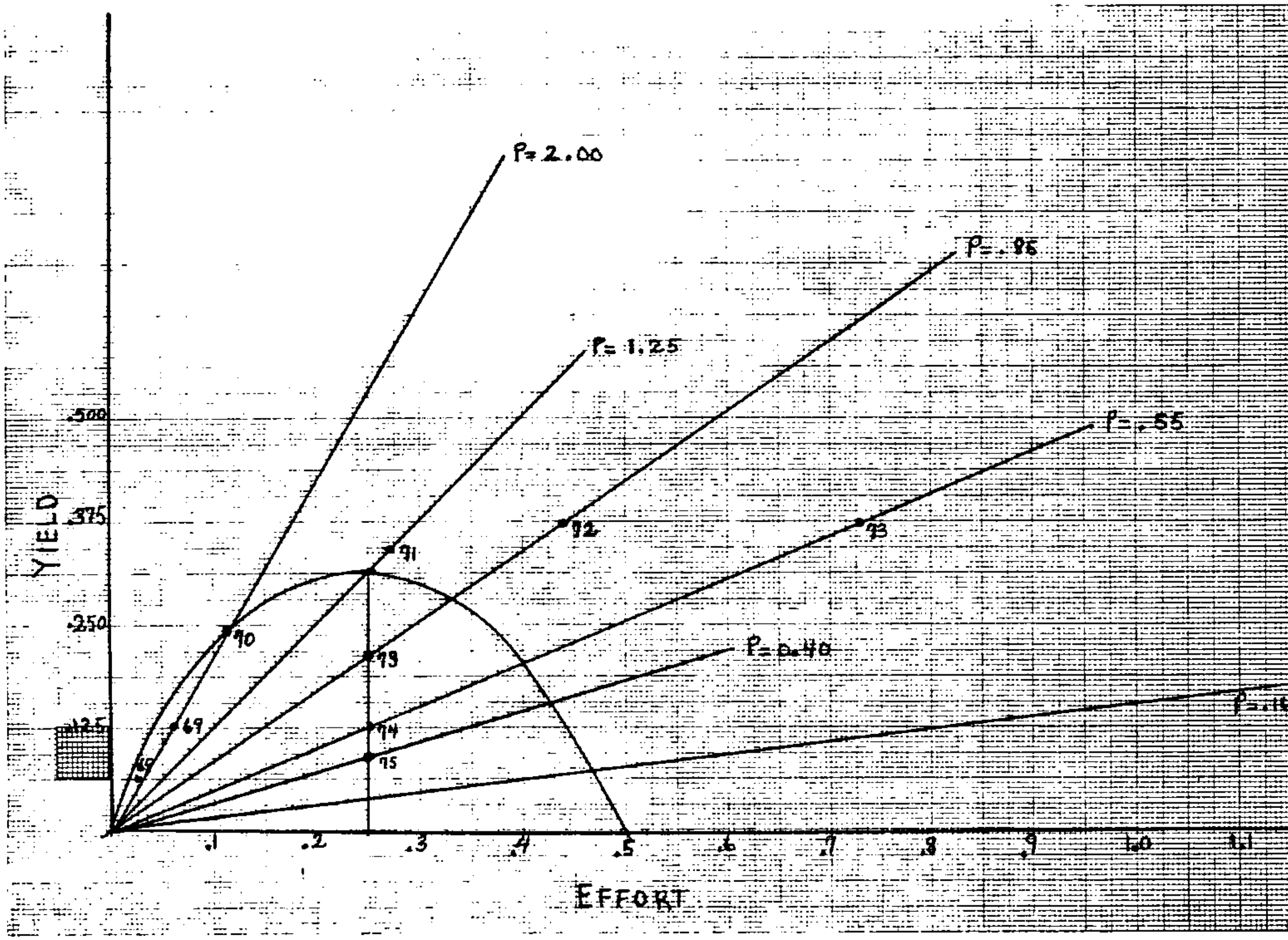

Figure 6. Equilibrium yield curve for mackerel in ICNAF Areas 5 and 6. Straight lines radiating from origin are approximate annual yield curves for various values of biomass. Parameters were calculated to be $P_\infty = 2.5 \times 10^6$. Dots · represent actual values and x's represent calculated TAC's.