RESTRICTED

International Commission for the Northwest Atlantic Fisheries

Serial No. 3513
(E)

ICNAF Res.Doc. 75/34

ANNUAL MEETING - JUNE 1975

A sampling design for combined echo-counting and trawling surveys for groundfish abundance

by

W.G. Doubleday
Environment Canada
Biological Station
St. Andrews, N.B., Canada

Abstract:

Sampling theory is applied to the problem of estimating groundfish abundance by a combined echo-counting and trawling survey. Estimation formulae are presented and compared for precision.

Introduction:

This paper arises from a study of how to incorporate output from an echo-counting device (R. G. Dowd 1967) into the groundfish abundance surveys conducted by the St. Andrews Biological Station. These surveys are currently based on a stratified random sampling design for trawl stations, with population estimates calculated from catch data. During a two-week cruise, with approximately ten days of sampling, about sixty trawl stations may be sampled. No information is collected during the time spent steaming between stations. The echo-counter allows information about total groundfish abundance to be collected while steaming. A length of bottom may be sampled by the echo-counter in a fraction of the time required to trawl over it. The problem considered here is how to combine the expensive but detailed trawl catch information about abundance by age and species with the less expensive but less detailed count information about total abundance.

1. CRUISE TRACK

To construct a cruise track by selecting a simple random sample of trawl stations from the area to be surveyed and joining them by echo-counting over a path of straight line segments causes the central region to have a higher sampling fraction for echo counting than the region on the boundary (Fig. 1). If there is a nonlinear trend in abundance or composition across the area, a bias enters into the resulting estimates of abundance. Nevertheless, it is desirable that trawl stations lie on the path along which counting is to take place and that randomization be applied to the choice of cruise track. If an arbitrary cruise track is chosen by eye, some areas may have higher sampling fractions than others, and it is not possible to construct valid confidence intervals for the abundance estimates. Fortunately, it is possible to choose a cruise track "at random" and to subsample the cruise track to select trawl stations. One solution is described below.

requires assumptions regarding fish distribution to be made which cannot be tested from the survey data since the component of variance due to the path unit may not be estimated. Thus, small sample units which are replicated are preferable to large units which are not.

Trawl stations are to be chosen at random along the counting path. For a given stratum, approximately equal numbers of trawl stations should be chosen from each replicate of the path unit.

If two or more trawl stations are chosen independently from the same path unit the resulting catches are positively correlated. This effect may be reduced by forcing the stations apart. Thus, if two trawl stations are to be selected from a given path unit, the path unit may be divided into two equal parts and the stations chosen independently, one from each part. The probability density of a given point in the stratum being selected remains the same as if the two stations were selected independently from the whole path unit, but the positive correlation of trawl catches on the same path unit is reduced.

The distance of bottom trawled over is assumed to be equal for all trawl stations. It will be referred to as a distance unit so that formulas may be written in terms of the distance unit. If boat speeds are measurable only relative to the water at the surface, the analysis is not invalidated, but there will be an increase in variance of the estimates due to the added uncertainty of the boat's speed relative to the bottom.

The use of small sampling units has been recommended in this section because replication enables the assumptions underlying the survey design to be evaluated in the light of the data collected in the survey. There is another advantage of replicating small units. If storms interrupt a cruise, replication is reduced and variances increased, but all strata are represented in the sample provided one replication for each stratum is carried out before any second replications are begun. If a single complex path unit is interrupted, part of the coverage is lost. Also, if the observations from the first few replications indicate that a larger sample size is required to obtain the desired precision, more replicates may be added without altering the basic path unit provided the extra boat time is available.

2. VARIANCES

Assumptions:

1. All strata are sampled independently.
2. Within a stratum, all trawl catches are uncorrelated.
3. Within a stratum, counts per unit area at trawl stations are uncorrelated.
4. The mean count at trawl stations within a path unit is uncorrelated with the mean count per distance unit on the rest of the path unit.

The use of independent replications of small path units ensures that the first assumption holds and enables remaining assumptions to be tested by analysis of variance of the survey results. Since a path unit will ordinarily have a length of many distance units and trawl stations are forced apart, assumptions 2., 3., and 4. are likely to be nearly correct. Small positive correlations would inflate the variances of the estimates slightly.

The variance of the echo-count over one distance unit is to be estimated by the sample variance of the counts at trawl stations within a stratum. It may be possible to pool estimates from different strata although the distribution of counts, being the sum of counting errors and changes in density is a contagious distribution and therefore skewed so that tests for homogeneity of variance must be applied with caution.

Within a stratum, the variance of the mean count per distance unit of a path unit is to be estimated by its sample equivalent for the replicated path units. If there is no replication, assumptions must be made about the correlation of counts from the same path unit in order to estimate this quantity. The sample variance of count per distance unit of a path unit should be divided by the estimate of variance of count at trawl stations to determine the constant of proportionality q by which the variance of the mean count per distance unit of a path unit is inflated by the correlation of counts on a path unit.

3. A STATISTICAL MODEL FOR THE CATCH COUNT RELATIONSHIP

The following considerations serve as a guide in the choice of estimates and variance formulae. Correctness of the following model is not essential in the analysis of estimates in section 5, but it is important in the choice of an index using transformed observations discussed in section 4.

Consider the total catch and the count per unit area at a trawl station. Even if counting and trawling are conducted simultaneously, different areas of bottom are covered by the trawl and counter so that different fish are caught and counted. Let the fish density per square unit trawled over be X. Then the catch may be written as $\alpha X + \varepsilon_1\sqrt{X}$ where α is a multiple of the area swept by the trawl and ε_1 is a random variable representing the sampling error of the trawl. Thus, the catch differs from the expected catch at density X by $\varepsilon_1\sqrt{X}$. Taylor (1953) found that the observed distribution of trawl catches on Georges Bank could be accounted for by Poisson sampling error of fish densities which were themselves distributed in space as gamma random variables. Thus, ε_1 is independent of X and has the same distribution for all trawl catches, provided the area trawled over is constant. If the total fish density per unit area over the bottom covered by counting is Y, then a similar argument suggests that the number of fish counted per unit area may be written as $\beta Y + \varepsilon_2\sqrt{Y}$ where β is a constant and ε_2 is a random variable independent of Y. However, in this case, the distribution of ε_2 depends on depth since the area sampled and amount of overlap of sonified volumes from successive sonar pulses increase with depth.

If a trawling station is chosen by randomization, then the densities X and Y are random variables. From the above considerations, the means and variances of catch and count may be derived.

Let $E[X]=M_x$, $E[Y]=M_y$, $Var[X]=\sigma_x^2$, $Var[Y]=\sigma_y^2$, $E[\varepsilon_1]=0$,

$E[\varepsilon_2]=0$, $Var[\varepsilon_1]=\sigma_1{}^2$, $Var[\varepsilon_2]=\overline{\sigma_2^2(d)}$ where the variance in the sampling error for counts is averaged over the depths of all possible trawl stations. Then:

$$E[\text{Catch}]=E[CA]=\alpha M_x$$

$$E[\text{Count per unit area}]=E[CO]=\beta M_y$$

$$Var[CA]=\alpha^2\sigma_x^2 + \sigma_1{}^2 M_x$$

$$Var[CO]=\beta^2\sigma_y^2 + \underset{d}{E}[\sigma_2^2(d)M_y(d)]$$

$$\text{Correlation}[CA\ CO] = \frac{COV(X,Y)}{\sqrt{\sigma_x^2 + \frac{\sigma_1{}^2 M_x}{\alpha^2}}\ \sqrt{\sigma_y^2 + \frac{E[\sigma_2^2(d)M_y(d)]}{\beta^2}}}$$

This formula shows that the correlation between catch and count is due to the correlation between fish densities sampled by the trawl and counter. Simultaneous counting and trawling would be expected to yield a higher correlation than separate counting and trawling at the same station since navigational errors would be eliminated. The contribution of counting sampling error, $E[\sigma_2^2(d)M_y(d)/\beta^2]$ should be smaller in strata of deep water. If fish densities X and Y are low, then counting and catching sampling errors reduce the correlation.

A similar model could be constructed to examine the correlation of the catch of one species or one age group of one species with count per unit area. In this case the density X would represent the density of that species or species-age group. Unless percentage catch composition were very stable, the resulting correlation would be low since the correlation between X and the total fish density Y would be low. The reason for choosing strata with homogeneous catch composition is now clear.

If fish densities are high enough that counting errors are small relation to the count per unit area, then catch and count per unit area are related by:

$$E[CA\ CO] \sim \text{const}_1 CO$$

$$Var[CA\ CO] \quad \text{const}_2 CO$$

These conditions are ideal for the use of a ratio estimator (Cochran 1963). A ratio estimator is considered in section 5.

The above model suggests that sampling errors in catch and count at constant fish density have variances proportional to the density being measured. If the catches and counts per unit area are transformed by taking the square root of each observation, the sampling error variances become independent of fish density. Thus $\sqrt{CA} = \sqrt{\alpha}\sqrt{x} + \varepsilon_3$, $\sqrt{CO} = \sqrt{\beta}\sqrt{y} + \varepsilon_4$ when ε_3 and ε_4 are the sampling errors.

$E[\varepsilon_3]=0$, $Var[\varepsilon_4]=\sigma_3^2$, $E[\varepsilon_4]=0$, $Var[\varepsilon_4]=\sigma_4^2$. $Var[\varepsilon_4]$ depends on the distribution of depths on a stratum. If the square root transformation achieves a bivariate normal distribution for catch and count per unit area, then the regression of catch on count per unit area will be linear and will have constant residual variance. However, the slope of this regression will be $\rho\sqrt{\alpha}/\sqrt{\beta}$ where ρ is the correlation coefficient between $\sqrt{CA}$ and $\sqrt{CO}$ so that $E[\sqrt{CA}] > \sqrt{\alpha}\sqrt{CO}/\sqrt{\beta}$ for low counts per unit area and $E[\sqrt{CA}] < \sqrt{\alpha}\sqrt{CO}/\sqrt{\beta}$ for high counts. If ρ is greater than 0.8, then a ratio estimator is a reasonable choice.

Sometimes (Taylor 1953, Gulland 1956, Grosslein 1971, Jones and Pope 1973) a logarithmic transformation is used to stabilize the variances of catches. In the model, this transformation retains an approximately linear relationship in the expectation of catches given counts, but the variance about the line decreases with higher fish density. In the above references, the log transformation stabilizes the total variance pf catch, not the sampling variance for constant fish density. Thus, the log transform, while appropriate for pure trawl surveys does not seem to be appropriate for combined surveys. Despite this, estimators based on log transformed observations are considered in section 5. The transformations with the highest correlation between catch and count may be chosen.

4. INDICES OF ABUNDANCE

The ideal of an abundance survey is to estimate the numbers of fish by age and species within a given area. However, the number of fish in a trawl catch is a random variable with a highly skewed distribution (Taylor 1953). The result of this skew is that an abundance estimate based on ten tows may change by a factor of 1/2 if it is recalculated omitting the largest catch. This instability necessitates the use of larger sample sizes than are required for more symmetrical distributions to obtain the same precision of estimation.

One approach to this problem is to transform the catches by taking (for example) logarithms or square roots and basing an index of abundance on the means of the transformed quantities. The logarithm transformation stabilizes the measurement variance if the variance in catch in numbers is proportional to the square of fish density. However, the mean of the log catches measures both overall abundance and variability of catch size. Thus, if log transformed catches have a normal distribution with mean m and variance σ^2, then the untransformed catches have mean $e^m e^{(\sigma^2/2)}$ and variance $e^{\sigma^2} e^{m^2} (e^{\sigma^2/2} - 1)$, so that mean abundance may remain constant with the variance of catch changing from survey to survey to produce the same effect as mean abundance changing and variance of catch size remaining fixed. In the log-normal case $\widehat{\exp}\ \overline{(\log_e \text{catch})}$ underestimates mean abundance by a factor of $\exp(\sigma^2/2)$ where σ^2 is the variance of log catch. Reference to the log-normal distribution is intended to indicate qualitatively the effect of the log transformation but not to suggest correction factors since such a factor is sensitive to the form of the distribution of catch. However, the variance of log catch should be constant from cruise to cruise for the resulting indices to be comparable.

The above remarks about log transformations apply to square root transformations as well, although the bias involved is smaller.

If the limitations imposed by transforming the observations are acceptable to the designer, the resulting index may be very attractive due to the increased precision over direct estimators of abundance.

In the next section, two types of estimators are considered. The first type is based on the correlation of catch and count and assumes only that this correlation is positive. A larger correlation results in a more precise estimator and a small correlation makes this type of estimator less effecient than the corresponding estimator for a pure trawl survey. The second type assumes that counts are proportional to fish abundance and that the constant of porportionality is independent of the age and species composition of the fish counted. The last assumption is not strictly valid since there is a tendency of larger fish to be counted more often than smaller fish as the boat passes over them because stronger echos are received from a larger area of bottom than weaker ones so that, although an index with a percentage composition component from catches and a total abundance component due to counts may be calibrated with a trawl survey to estimate the constant of proportionality, there remains a variable and, at this time, unknown bias of such an index compared to a similar index from a pure trawl survey. In spite of this drawback, the added precision of this type of index may make it preferable to the former type if comparisons are made only between combined surveys.

5. ESTIMATORS

Six estimators are considered in this section. They are divided into two groups according to whether simultaneous catch-count information is used or whether the composition of catches is combined with an index of counts. Untransformed, square root transformed, and log transformed observations are used. In each case, separate within stratum estimates are made and these are averaged with weights proportional to stratum areas.

Notation

The subscirpt i, i=1, ..., I refers to stratum, j, j=1, ..., J refers to trawl station j in stratum I (J may vary from stratum to stratum).

X_{ij} number of fish caught in a given species or species-age class at trawl station j in stratum i

X'_{ij} $\sqrt{X_{ij}}$

X''_{ij} $\ln(X_{ij} + 1)$

Y_{ij} Count per unit area at trawl station j in stratum i

Y'_{ij} $\sqrt{Y_{ij}}$

Y''_{ij} $\ln(Y_{ij} + .01)$

A_i area of stratum i

f_i area of stratum if area trawled over in stratum i

P_{ij} X_{ij} as a fraction of the catch of all species, similarily P'_{ij}, P''_{ij}

$\overline{Y}_i$ mean count per unit area for stratum i for the whole cruise track, similarily $\overline{Y}'$, $\overline{Y}''$

h_i number of path units in stratum i

l_i average number of distance units per path unit in stratum i

C total time allowed for the survey

T_i time allowed for the survey in stratum i

C_1 extra time required to count over one distance unit as opposed to counting

C_2 time required to count over one distance unit

C_3 time including steaming required per trawl station in a pure trawl survey

q Var of count per unit area per path unit/Var of count per unit area per distance unit

ρ, ρ', ρ'' correlation between (X,Y), (X',Y'), (X'',Y'') respectively.

The estimators using simultaneous catch-count information are:

$$A \quad \frac{\sum_{i=1}^{I} \frac{A_i}{A} \left(\sum_{j=1}^{J} f_i X_{ij} \right) \quad \bar{Y}_i}{\sum_{j=1}^{J} Y_{ij}}$$

$$A' \quad \sum_{i=1}^{I} \frac{A_i}{A} \left(\frac{\sum_{j=1}^{J} f_i X'_{ij}}{\sum_{j=1}^{J} Y'_{ij}} \right) \quad \bar{Y}'_i$$

$$A'' \quad \sum_{i=1}^{I} \frac{A_i}{A} \left\{ \frac{\sum_{j=1}^{J} f_i X''_{ij}}{J} - b f_i \left(\frac{\sum_{j=1}^{J} Y''_{ij}}{J} - \bar{Y}''_i \right) \right\}$$

where b is the regression coefficient of X'' on Y'', i.e. $S_{X''Y''}/S^2_{Y''}$)

Since the ratio estimator in this case is equivalent to a regression through the origin, to a first order approximation in sample size, the variance formulae for all three estimators have the same form:

$$\text{Var} = \sum_{i=1}^{I} \left(\frac{A_i}{A} \right)^2 f_i^{\,2} \frac{\sigma_i^2}{J} \left\{ 1 - \rho^2 \quad (1 - \frac{Jq}{h_i}) \right\}$$

(Hansen et al.), where σ_i^2 represents σ_X^2 for A, $\sigma_{X'}^2$ for A', and $\sigma_{X''}^2$ for A'' and ρ, ρ', ρ'' as appropriate.

If time C_i is allocated to stratum i and $C_1 J + C_2 h_i l_i = T_i$ then the allocation of time between counting and catching which minimizes the contribution of stratum i to the total variance is:

$$h_{i\ opt} = \left(l_i q \ C_2 + C_1 \sqrt{\frac{1-\rho^2}{\rho^2} \ \frac{C_2}{C_1} \ l_i q} \right)^{-1} q \ T_i$$

$$J_{opt} = h_{i\ opt} \sqrt{\frac{1-\rho^2}{\rho^2} \frac{C_2}{C_1} 1_i q} \Big/ q$$

$$Var_{opt} = \sum_{i=1}^{I} \frac{A_i}{A}^2 f_i^2 \frac{\sigma_i^2}{T_i} (\rho\sqrt{C_2 1_i q} + \sqrt{(1-\rho^2)C_1})^2$$

Optimal allocation of time to strata is given by

$$T_i \propto A_i \sigma_i (\rho\sqrt{C_2 1_i q} + \sqrt{(1-\rho^2)C_1})$$

A combined survey has smaller variance than a pure trawl survey in stratum i if

$$\sigma_i^2 (\rho\sqrt{C_2 1_i q} + \sqrt{(1-\rho^2)C_1})^2 < \sigma_i^2 C_3$$

If $C_3 = C_1$ then

$$\rho^2 > \frac{4 C_1 C_2 1_i q}{(C_1 + C_2 1_i q)^2}$$ is required for an improvement over a pure trawl survey estimate based on the same transformation. In practice, the marginal cost of adding a trawl station to a pure trawl survey is greater than for a combined survey due to the extra steaming time required.

From the above analysis, the precision of estimation (at a first order approximation) depends on the variance of (possibly transformed) catches and the correlation between (possibly transformed) catches and counts. The estimated variances may be compared after the cruise to select the estimator with the smallest variance or coefficient of variation.

The estimators using counts as an independent measurement of abundance are:

$$B \quad \sum_{i=1}^{I} \frac{A_i^2}{A} \frac{\sum_{j=i}^{J} P_{ij}}{J} \quad \bar{Y}_i$$

$$B' \quad \sum_{i=1}^{I} \frac{A_i^2}{A} \frac{\sum_{j=1}^{J} P'_{ij}}{J} \quad \bar{Y}'_I$$

$$B'' \quad \sum_{i=1}^{I} \frac{A_i^2}{A} \frac{\sum_{j=1}^{J} P''_{ij}}{J} + \bar{Y}''_{ij}$$

B and B' have the same formula for variance

$$\mathrm{Var} = \sum_{i=1}^{I} \left(\frac{A_i}{A}\right)^2 A_i^2 \left\{ \frac{\sigma_p^2}{J} E[Y_i]^2 + \frac{E[P_{ij}]^2 q \, \sigma_y^2}{h_i} \right\}$$

where P and Y are replaced by P' and Y' for B'.

If $C_1 J + C_2 h_i l_i = T_i$, then the optimal choices of J and h_i for stratum i are:

$$J = T_i \left(\frac{\sigma_p \; E[Y_i] C_1^{-1/2}}{C_1^{+1/2} \sigma_p \; E[Y] + (C_2 \; l_i q)^{+1/2} \sigma_y E[P_{ij}]} \right)$$

$$h_i = T_i \left(\frac{\sigma_y \; E[P_{ij}] \; \sigma_2^{-1/2}}{C_1^{1/2} \sigma_p \; E[Y] + (C_2 \; l_i q)^{1/2} \sigma_y E[P_{ij}]} \right) q$$

$$\mathrm{Var}_{opt} = \frac{A_i^2}{T_i} \left(C_1^{1/2} \sigma_p E[Y_i] + (C_2 \; l_i q)^{1/2} \sigma_y E[P_{ij}] \right)^2$$

This represents an improvement over the corresponding pure trawl survey estimate if

$$\frac{C_1^{1/2} \sigma_p E[Y_i] + (C_2 \; l_i q)^{1/2} \sigma_y E[P_{ij}]}{K} < C_3 \; \sigma_x$$

where K is the conversion factor from counts to catches derived from previous calibration.

B'' has the variance formula:

$$\mathrm{Var} = \sum_{i=1}^{I} \left(\frac{A_i}{A}\right)^2 A_i^2 \left\{ \frac{\sigma_{p''}^2}{J} + \frac{q \sigma_{y''}^2}{h_i} \right\}$$

with optimal allocation in stratum i

$$J = T_i \left(\frac{\sigma_{p''} \; C_1^{-1/2}}{C_1^{1/2} \sigma_{p''} + (C_2 l_i q)^{1/2} \sigma_{y_i''}} \right)$$

$$h_i = T_i \left(\frac{\sigma_{y''} \; \sigma_2^{-1/2}}{C_1^{1/2} \sigma_{p''} + (C_2 \; l_i q)^{1/2} \sigma_{y_i''}} \right)$$

$$Var_{opt} = \frac{A_i^2}{Ti} \left(C_1^{1/2} \sigma_{p''} + (C_2 \, l_i q)^{1/2} \sigma_{y''} \right)^2$$

which has a smaller variance than the mean log catch from a pure trawl survey if

$$C_1^{1/2} \sigma_{p''} + (C_2 \, l_i q)^{1/2} \sigma_{y''} < C_3^{1/2} \sigma_{x''}$$

6. DISCUSSION

The requirement of direct comparability of abundance of estimates with those of pure trawl surveys means that the correlation between catch and count must be high. This condition may not be met for all strata and/or all species or species-age classes. Only correlations between total catch and count density have been examined to date. In this case a randomized trawl survey may be applied to those strata where the correlation falls below that required for the auxiliary information to produce a gain in precision. It is valid to treat the catches from the combined catch and count survey as trawl stations from a pure trawl survey for species or species/age classes which are not well estimated by combining information from catches with counts.

The use of transformations such as square roots or logarithms may improve the precision of estimators at the cost of introducing sensitivity to changes in the form of the distribution of catches and counts.

If the requirement of direct comparability with pure trawl surveys is dropped, the counts may be used directly to measure total fish abundance. Indices based on the percentage composition of catches and the average of (transformed) counts are likely to have smaller variances or coefficients of variation than the corresponding quantities from pure trawl surveys for most species, since percentage composition is more stable than numbers caught.

The choice of estimator between pure trawl estimators and those which combine catches and counts can be made after the survey on the basis of their estimated variances. Biases introduced by selecting the method of estimation a posteori on the basis of variance estimates (not agreement with the designer's expectations) are unlikely to be appreciable (Hansen, Hurwitz, and Madow (1953)).

When designing the survey, estimates of variances and correlations are required for the allocation of sampling effort. These need not be more accurate than plus or minus twenty per cent since variance optima are stable under moderate errors of estimation (Cochran 1953). If the within stratum variances of the estimators do not differ by a factor of two or more, the allocation of effort to strata should be proportional to stratum area since the gain in precision in this case is likely to be slight and poor estimates of the variances may result in a loss in precision in choosing the estimated optimum over proportional allocation (Hansen et al.). After the survey is complete, more precise estimates of the design parameters are available based on the sample variances of catches and counts.

The linear cost formula of section 5 is only valid if the path unit is fixed. Different costs of time are involved in adding a trawl station to a pure trawl survey and to a combined survey since steaming time must be considered in the former case but not the latter.

The allocation formulas are intended **as a guide** to the division of time between counting and catching. The actual allocation used should be chosen near the indicated optimum by the judgement of the designer.

The designer may wish to extraploate the observed abundance estimates to areas of bottom unsuitable for counting. This is a matter for judgement and experience and has no theoretical basis on the survey findings outside these areas.

7. CONCLUSION

The untransformed ratio estimator is recommended when an estimate of abundance comparable to a pure trawl survey is required. The square root transformation with a ratio estimator is likely to result in a better correlation between catch and count and hence a more efficient estimator than the regression estimator with a log transformation.

If the use of counts as an absolute index of total abundance is acceptable, any of the three estimators is satisfactory and the choice may be made on the basis of estimated variances. The square root transformation is to be preferred to the logarithm transformation if the model of section 3 is valid.

The counter is most useful when catches vary widely in size but not in percentage composition. The counter is more effective in complimenting catch information when abundance by species as opposed to abundance by species and age is required.

References

1. COCHRAN, W. G. 1963. Sampling Techniques. Second Edition. Wiley

2. DOWD, R. G. 1967. An Echo Counting System for Demersed Fishes. FAO Conf. on Fish Behaviour in Relation to Fishing Techniques and Tactics. FB/67/E/7: 1-6

3. GROSSLEIN, M. D. 1971. Some observations on Accuracy of Abundance Indices Derived from Research Vessel Surveys. ICNAF Res. Doc. 71/59.

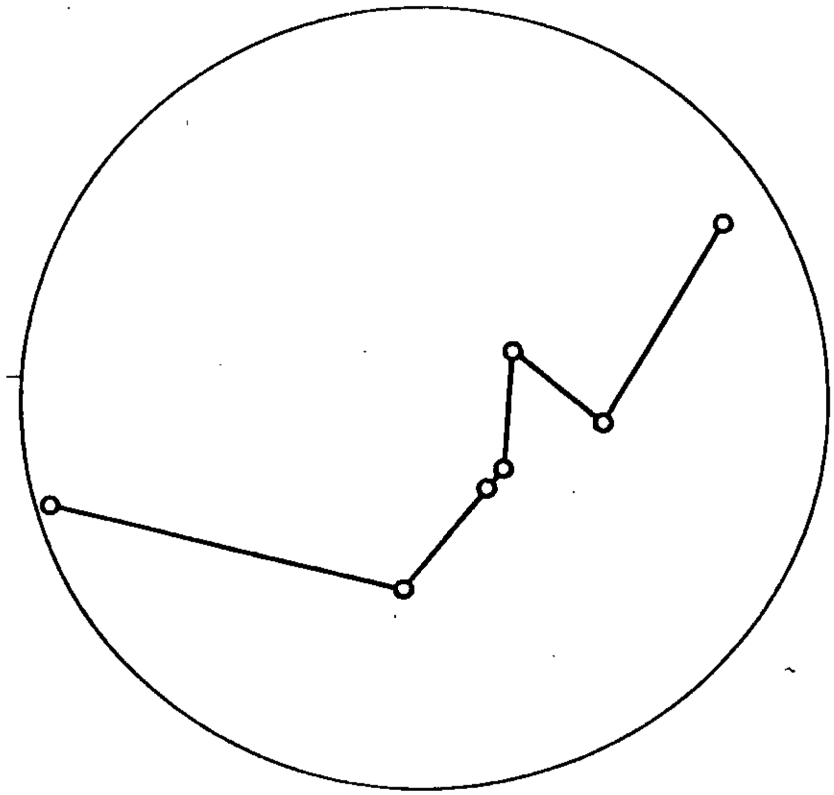
4. GULLAND, J. A. 1956. A Note on the Statistical Distribution of Trawl Catches. I.C.E.S., Papp. et. Proc. Verb. 140 Part 1: 28-29.

5. HANSEN, M. H., HURWITZ, W. N., and W. G. MADOW. 1953. Sample Survey Mehtods and Theory. Wiley

6. JONES, B. W. and J. G. POPE. 1973. A Groundfish Survey of Faroe Bank. ICNAF Res. Bull. #10: 53-62.

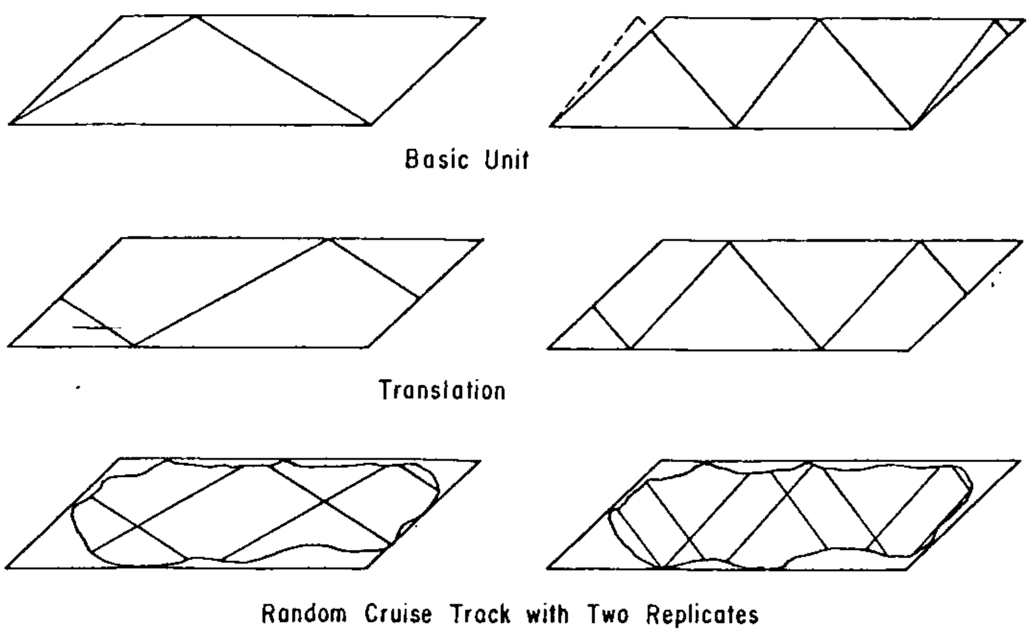
7. TAYLOR, C. C. 1953. The Nature of Variability in Trawl Catches. Fish. Res. Bull. 83, Fish. Bull. of the Fish and Wildlife Service (U.S.), Vol. 54.

Fig. 1



Random Stations Joined by a Cruise Track

Fig. 2



Choice of Cruise Track by Randomized Zig-zags