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A LEAST SQUARES APPROACH TO ANALYSING CATCH AT AGE DATA

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Abstract

Starting from the assumptions of a known, time and age independent, natural mortality rate and unknown, time independent, selection (partial recruitment) at age factors, a model is constructed to estimate recruitment, fishing mortalities and an effective effort multiplier from catch at age data. A computer program applying standard least squares technique to a series of linear approximations to catch equations is presented, together with a second program which may be used to provide initial estimates of the fishing mortalities. The new method is compared with the cohort analysis of J. Pope and applied to data on Greenland cod previously analysed by A. Schumacher.

Introduction

Virtual population methods (Fry 1949; Murphy 1964; Jones 1964; Gulland 1965; Pope 1972) have been developed to estimate fish population (stock) sizes and fishing mortalities from catch at age data. These methods analyse the catches of a single year-class as a unit so that there are more parameters (population size and fishing mortalities) to be estimated than there are observations. Thus, each mortality estimate is supported by somewhat less than one observation. Mortalities for the same age and different year-classes are sometimes averaged to overcome this inherent instability, but this solution is not completely satisfactory since there are often considerable changes in fishing effort over a 2- or 3-year period.

Recently, Pope (1974) suggested a model where the instantaneous fishing mortality rate ${}_aF_n$ at age a in year n is the product of a year effect (effort multiplier) $\mathcal{F}_n$ and an age effect (gear selection or partial recruitment) ${}_a\mathcal{S}$. Thus ${}_aF_n = {}_a\mathcal{S}\mathcal{F}_n$. He estimated these parameters by a modified steepest descent method applied to minimize the sum of squared differences of observed and predicted logarithms of catch ratios of the same year-class in successive years. Since the same ${}_a\mathcal{S}$ and $\mathcal{F}_n$ are applied to several year-classes in this approach, the parameter estimates are supported by more than one observation each.

The model presented here arose out of an attempt to solve Pope's model by applying least squares methods to a series of linear approximations. This technique is called linearization and is discussed by Draper and Smith (1966). The reasons for linearizing are twofold. It was felt that linearization would contribute to a better understanding of the structure of the model and that information concerning the reliability of parameter estimates obtainable from regression analysis would be of value. Problems of instability with this approach were traced to the excessive nonlinearity of predicted catch ratios when fishing mortality rates are small (Fig. 2). This nonlinearity was removed by estimating the age and year effects on a logarithmic scale (Fig. 3):

$$f_n = \ln \mathcal{F}_n \quad , \quad {}_as = \ln {}_a\mathcal{S}$$

Unfortunately, the resulting model was found to be extremely insensitive to proportional changes in all fishing mortalities together. Higher population size together with lower mortalities could produce identical catches. Therefore, the model was restricted so that $_{a}s$ for the oldest age and f_n for the last year were specified once and held fixed. At this stage, the model was able to recognize trends in mortalities.

Dissatisfaction with the necessity of fixing some parameters and with the correlated sampling errors in the catch ratio logarithms (the same catch occurs in the numerator of one ratio and the denominator of another so that a negative correlation results) led to the direct use of the logarithms of the individual catches at age. The use of logarithms stabilizes sampling variances and emphasizes percentage errors in prediction. This approach retains the information about population size which is lost in taking catch ratios.

Methods and Procedures:

First, the cohort analysis of J. Pope (1972) is examined for bias and variance in estimation, then the new model is developed, and finally there is an application to the catches of West Greenland cod from 1956 to 1966.

Throughout this paper, natural mortality will be assumed to occur at a constant rate, independent of age and year and that all other removals from the population are contained in the catch data. More detailed knowledge about natural mortality could easily be incorporated into the following treatment, but catch-at-age data contains very little information about the rate of natural mortality so that such information must come from another source. Failure of these assumptions is not investigated. The standard catch equation of Beverton and Holt (1957) is used as a starting point and the adequacy of such an equation to describe a fishery is not investigated.

Notation

In what follows, subscripted prefixes refer to ages and subscripted suffixes to years. Hats ^ are placed over parameter estimates. A catch-at-age matrix consisting of N rows (years) and A columns (ages) is the raw data.

M instantaneous coefficient of natural mortality for age a and year n

${}_aC_n$	catch in numbers of age a fish in year n
${}_as$	logarithm to base e of availability at age a
f_n	logarithm of effective effort multiplier (fishing intensity) in year n
${}_aF_n$	$\exp({}_as + f_n)$
${}_aP_n$	stock size in numbers of age a fish in year n
exp	exponential function
ln	logarithm to base e

Cohort analysis

We begin with a brief description of cohort analysis as developed by Pope (1972) in our terminology since his paper is not universally available. Cohort analysis was chosen for comparison because the consequences of sampling errors and choice of initial parameters are particularly clear and the method itself has acceptance, especially among the scientists who provide management advice to the International Commission for the Northwest Atlantic Fisheries (ICNAF).

The catch equation

$$ {}_aC_n = \frac{{}_aF_n \quad (1 - \exp(- {}_aF_n - M))\, {}_aP_n}{{}_aF_n + M} $$

may be written as

$$ {}_aC_n = \frac{{}_aF_n \; ({}_aP_n - {}_{a+1}P_{n+1})}{{}_aF_n + M} $$

so that

$$ {}_aP_n = {}_aC_n \; \frac{{}_aF_n + M}{{}_aF_n} + {}_{a+1}P_{n+1} \tag{1} $$

Pope approximated this formula by

$$ {}_aP_n \simeq {}_aC_n \; \exp(M/2) + {}_{a+1}P_{n+1} \; \exp(M) \tag{2} $$

which is correct to the first power of ${}_aF_n$ and M. Pope claims that the approximation (2) is usable at least up to values of $M = 0.3$ and $F = 1.2$.

From (2)

$$ {}_aP_n = {}_aC_n \; \exp(M/2) + {}_{a+1}C_{n+1} \; \exp(3M/2) + \ldots $$

If the series is terminated before fishing is complete, the last term (for the final year N) is

$$_{a+N-n}P_N = \frac{_{a+N-n}C_N \; (_{a+N-n}F_N + M)}{_{a+N-n}F_N \; (1-\exp(-_{a+N-n}F_N - M))} \qquad (3)$$

while, if fishing is complete at year N, and no fish from the year-class under consideration survive,

$$_{a+N-n}P_N = \frac{_{a+N-n}C_N \; (_{a+N-n}F_N + M)}{_{a+N-n}F_N} \qquad (4)$$

$_aF_n$ is estimated by

$$_aF_n = \ln\,(_aP_n \,/\, _{a+1}P_{n+1}) - M \qquad (5)$$

$$= \ln\left(\frac{_aC_n \exp(M/2) + {_{a+1}P_{n+1}} \exp(M)}{_{a+1}P_{n+1}}\right) - M$$

Ordinarily, the initial mortality estimate, $_{a+N-N}F_n$ is unknown and must be specified (usually arbitrarily) in order to use (3) or (4). Fig. 1 illustrates the effect of the choice of $_{a+N-n}F_N$ on the estimate $_{a+N-n}\hat{P}_N$ for various values of M. The logarithm of the population size estimate is very nearly linearly related to the logarithm of $_{a+N-n}F_N$ and is nearly independent of M. As $_{a+N-n}F_N$ changes from 0.05 to 1, $_{a+N-n}\hat{P}_n$ changes by a factor of thirteen. This strong influence is steadily reduced in its effect on population estimates for younger ages. The size of the effect depends on M and on the catches as far back as the age of interst. If the contributions of all catches, discounted for natural mortality, were roughly equal, a choice of 0.01 instead of 1 for $_{a+N-n}F_N$ would change the population estimate 9 years earlier by a factor of two. Of course, small changes in $\ln {_{a+N-n}F_N}$ have very little effect a few years back. One could describe the influence of errors in $_{a+N-n}F_N$ as higly damped by the earlier catches. Also, the effect of changes in $_{a+N-n}F_N$ is diminished if the discounted earlier catches are much larger than the last as is often the case in practice. Usually, the simplest expedient is to repeat the analysis with different values for $_{a+N-n}F_N$

in order to ascertain the size of possible biases.

Overestimation of ${}_{a+N-n}F_N$ results in overestimation of all fishing mortalities. This effect also diminishes in back calculation similarily to the effect on population size estimates. The relative error in one is approximately the inverse of the relative error in the other.

Variance in ${}_aC_n$ due to sampling errors produces variance in ${}_a\hat{F}_n$. Equation (5) may be written as

$${}_a\hat{F}_n = \ln \left(1 + \frac{{}_aC_n \exp(-m/2)}{{}_{a+1}\hat{P}_{n+1}} \right) \qquad (6)$$

Thus, to a first order approximation, ${}_a\hat{F}_n$ has about the same coefficient of variation as a single observed catch.

The contribution of variance in ${}_aC_n$ to the rel-var of the estimate ${}_a\hat{P}_n$ depends on the sizes of the contributions of all component catches of ${}_a\hat{P}_n$. If all K components contribute equally, the contribution to rel-var due to ${}_aC_n$ is $\frac{1}{K^2}$ rel-var $\left({}_aC_n\right)$, while if ${}_aC_n$ dominates, it contributes rel-var $\left({}_aC_n\right)$. In practice, the effect lies somewhere between these extremes.

In conclusion, with a good choice of the final F value, for estimating mortalities, Pope's cohort analysis is about as good as the sampling error in a single observation, and for estimating population sizes, it is somewhat better. With a poor choice of the final F, serious errors can arise, especially with older fish.

Example: West Greenland Cod (from Shumacher (1971)).

Table 1 contains catch at age data and cohort analyses for one cohort of West Greenland cod. These data were analysed by Schumacher by a virtual population method other than cohort analysis, but Pope's cohort analysis is applied to them as an illustration for comparison with results from the new model. The value of 0.8 for ${}_{14}F$ was used by Schumacher. The extremes of the population estimates at age five differ by a factor of 1.43.

In the cohort analysis, catches at age 6 and 7 each account for about 1/6 of the population estimate at age 5.

Rel-variance in these two catches is twice as important as in the remaining catches so far as estimation at age 5 is concerned. A ten percent change in the catch at age 6 produces a two percent change in the estimated population at age five. The population estimate at age seven is dominated by the catch at age seven (1/4 of total).

If the sampling variance associated with each catch were known, then confidence intervals could be constructed for the population size estimates given a value for F at age fourteen.

The New Model

The following equation is derived from the catch equations

$$ {}_aC_n = {}_aP_n \frac{{}_aF_n}{{}_aF_n + M} (1-\exp(-{}_aF_n - M)) $$

and $\quad {}_{a+1}P_{N+1} = {}_aP_n \exp(-{}_aF_n - M)$

of Beverton and Holt by taking logarithms and expanding $\ln {}_aF_n$ as ${}_as + f_n$:

$$ \ln {}_aC_n = \ln {}_rP_{n-a+r} - (a-r)M - \sum_{i=r}^{a-1} \exp\left({}_is+f_{n-a+i}\right) + {}_as+f_n $$

$$ -\ln\left(\exp\left({}_as+f_n\right) + M\right) + \ln\left(1-\exp\left(-\exp\left({}_as+f_n\right) - M\right)\right) + {}_a\varepsilon_n \qquad (7) $$

where ${}_a\varepsilon_n$ represents the sampling error in observing $\ln {}_aC_n$ and is assumed to have zero mean and constant variance for all a and n and to be statistically independent for different catches. r is the age at which the year class of ${}_aC_n$ enters the table of catches.

In the main program, POPO, the nonlinear terms in (7) are expanded in linear approximations by Taylor series at an initial set of estimates of ${}_as$ and f_n for all ages and years. Standard least squares procedures are applied to the linear approximations and the resulting estimates are either, at the option of the operator, accepted or averaged with the previous estimates of ${}_as$ and f_n and ${}_rP_{n-a+r}$ to produce a revised set of estimates. This whole process of linear approximation and estimation is repeated until two successive estimates of all parameters differ by less than 10^{-2}. Since the contribution of ${}_rP_{n-a+r}$ is already linear, it does not enter into the Taylor series approximations.

Since ${}_as$ and f_n only affect the predicted catches through the sum $({}_as+f_n)$, there is an indeterminancy in the model. A constant could be added to all ${}_as$'s and subtracted from all f_n's without changing the predictions. In POPO, one of the ${}_as$ is held fixed, the choice of which is left to the user.

This model for ${}_aC_n$ is rather nonlinear in the terms expressing fishing mortality up to age a-1, i.e. mortality before the year under consideration, and this often leads to instability if the initial parameter estimates make the mortalities badly out of proportion. The option of averaging old and new estimates stabilizes the process to some extent, but not enough to allow arbitrary starting estimates.

Therefore, a second computer program, POPI, is used to provide starting values. Here the logarithms of catch ratios are used.

$$\begin{aligned}\ln\left({}_aC_n/{}_{a+1}C_{n+1}\right) = {}& {}_as+f_n - \ln\left(+\exp\ ({}_as+f_n) + M\right) \\ & + \ln\left(1-\exp\ (-\exp({}_as+f_n) - M)\right) + \exp\ ({}_as+f_n) + M \\ & - {}_{a+1}s - f_{n+1} + \ln(+\exp({}_{a+1}s+f_{n+1}) + M) \\ & - \ln(1-\exp(-\exp({}_{a+1}s+f_{n+1}) - M)) + {}_a\varepsilon_n\end{aligned}$$

This model is very nearly linear for fishing mortalities less than one and, if the values of f_n for the last year and ${}_as$ are specified and held fixed, convergence is rapid from arbitrary starting values.

The advantages of the least squares approach over cohort analysis (or other virtual population methods) are as follows:

(1) There are fewer paramters than observations so that the extra "fudge factor" is removed and sampling errors have the opportunity to neutralize one another.

(2) The residual mean square indicates how well the catches are explained.

(3) Indication is given (variance estimates) of the reliability of parameter estimates.

(4) There are residuals which may be examined to detect anomalies and indicate the appropriateness of the model to the data.

(5) The amount of information contained in the catch data about population sizes is indicated by the variance estimates associated with the population size estimates.

The proposed model has, however, some serious imperfections.

(1) There is no guarantee of a unique solution. Different starting values may produce different solutions, although solutions with comparable error mean squares usually differ by less than one standard deviation.

(2) Long series of well sampled catches are required, but few fisheries retain the same age selection characteristics for more than a few years. With A ages and N years there are 2(A+N)-1 parameters to be estimated so that A and N must be large for good results. With twenty ages and one hundred years there would be eight observations per parameter, while with nine ages and eleven years there are only two and one half. It follows that, the analysis of variance table of the regression may mean little. The residual mean square often has a smaller component due to sampling (pure) error than due to systematic errors (lack of fit).

(3) Even when the catches are well explained, confidence intervals for the fishing mortalities can be wide. This is largely due to the lack of orthogonality between popultion size and fishing mortalities as evinced in the negative correlations between their estimates. One may

increase and the other decrease with little effect on catch.

Application: West Greenland Cod 1956-1966 (Schumacher (1971))

Table 2 consists of catch at age data for West Greenland cod from 1956 to 1966. These data were thoroughly analysed by Schumacher (1971) and serve to illustrate the new method. Table 3 contains Schumacher's estimates of fishing mortalities. His choice of M = 0.2 was retained in the least squares analyses. The program POPI was run for ages five to thirteen with the initial values of -1.8 for $_{13}$s and 0 for all remaining parameters. The restriction to nine ages was due to the restrictions for subsequent analysis by POPO. POPI produced the estimates of table 4 with a residual mean square of 0.1432. The values in table 4 were used as input to POPO with $_{2}$s held fixed. After fourteen iterations, the estimates had converged to the values of table 5.

Numbers are assigned to year classes by the following system. Thirteen year olds in 1956 are year class one, twelve year olds in 1956 are year class two, ..., five year olds in 1956 are year class nine, five year olds in 1957 are year class ten, ..., five year olds in 1966 are year class nineteen.

The statistics which best describe the success of the solution are the residual mean square, called "Estimated SIGMA SQUARED" by POPO, which should be less than 0.1, and the variance of the ln catches. The usual analysis of variance is misleading since there is no grand mean to be removed and hence all sums of squares due to parameter estimation are inflated resulting in high F values even if the model fits badly.

Table 7 shows the estimated variances of the logarithms of the fishing mortalities. These are derived from the estimated dispersion matrix which is printed out by POPO together with the corresponding correlation matrix. The variances shown are slight underestimates of the rel-variances of the mortalities.

Table 7 shows decreasing confidence with the first and last two years and with the last few ages. This behaviour is similar to the increase in size of confidence limits about a regression line towards the ends of the range of observations. The estimates for 1966 have very large variances associated with them. The area of greatest confidence consists of ages five to ten and years 1957 to 1963.

The estimated fishing mortalities are given (on a linear scale) in table 8. With the exception of 1966, a very consistent picture is presented. Most of the mortality estimates are lower than those of Schumacher. This discrepancy is largely due to his choice of 0.8 for the fishing mortality of very old fish. 95% confidence intervals for the mortalities would range from ±20% of the best table entry to ±1000% of the worst entries. The population size estimates were positively correlated among themselves and negatively correlated with estimates of fishing intensity. This is the practical consequence of the non-ortho-gonality mentioned in the last section. Accurate estimates of population size and fishing mortalities from the same catch data cannot be fully determined. Horsted (1968) analysed recaptures of tagged cod and fixed a total mortality rate of 0.61 from 1955 to 1958 and 0.86 from 1960 to 1963. These estimates are fairly close to the least squares estimates.

The analysis of residuals proceeded as follows: The residuals were plotted on probability paper (Fig. 4). Most of the residuals resembled those from a normal distribution with an estimated variance of 0.02, but there were two large negative and two large positive residuals. The probability paper plot suggests that as much as 60% of the residual mean square could be due to lack of fit. A projection of a three dimensional graph of residuals against age and year (Fig. 5) was drawn. It showed clearly that there was a serious lack of fit in 1957 and 1958 due to a shift in age selection. 48% of the residual mean square was due to these two years. In an assessment of the fishery, it is recommended either to remove these data or to examine more closely the offending catches with a view to modifying the model equations for these two years. A plot of residuals against predicted values showed no trends. A plot of residuals against age showed greater scatter for ages five and six than for other ages, although there was a slight tendency for variance to increase with age after age seven. Evidently, the selection of ages five and six is not so stable or else these ages are poorly sampled. A plot of residuals against year showed wide scatter for 1956, 1957 and 1958, tending to a minimum at 1960 and increasing slightly towards 1966. This behaviour corresponds to the variance estimates for fishing mortalities.

Horsted (1965) calculated an index of fishing intensity shown in table 9. His definition of fishing intensity is the relative probability of capture of a marked cod. A straight line, Fig. 6, was fitted by least squares to predict f_n using Horsted's intensity. The equation was $f_n = -1.58 + 0.082i_n$ where i_n represents Horsted's fishing intensity in year n. r^2 was 0.92. The standard error of the slope was 0.0097 and of the intercept was 0.129. The value of students t for the slope was 8.2 with 6 d.f. as compared with a value of student's t of 4.9 for the slope of the regression of Schumacher's average fishing mortalities on Horsted's fishing intensity.

To test for sensitivity to the initial choice of parameters, POPO was run again with the logarithms of Schumacher's average fishing mortalities for 1956-64 for initial $_a$s's and -0.22 for f_n up to 1960 and 0.22 for f_n for 1961-1966. Differences of about 10% were found in fishing mortality estimates for ages ten to thirteen but were less than 5% elesewhere.

Conclusion:

A method of analysing catch at age by linearized least squares has been presented. The new method provides not only estimates of population size and fishing mortalities, but also an indication of the reliability of these estimates. In particular, the inverse relationship between population size estimates and fishing mortality estimates is underlined and the ability of a given matrix of catch data to resolve this antipathy is indicated.

The method requires a long history of well-sampled catches during which selection at age has not changed greatly. The new model satisfactorily explained most of the West Greenland Cod data, but not the catches of 1957 and 1958.

Even if the model is found lacking for a given fishery, examination of the residuals and of the trends in selection at age and fishing mortality may provide insight into changes in a fishery.

Appendix:

Listings of POPI and POPO and documentation of the programs is available on request from:

Fisheries and Marine Biological Station,
Brandy Cove,
St. Andrews, N. B.

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TABLE 1.

Cohort Analysis of a Year-class of West Greenland Cod

Age	5	6	7	8	9	10	11	12	13	14
Catch	4996	9362	7501	3881	2743	2333	1709	1156	321	34
F	.09	.24	.30	.25	.28	.42	.62	1.22	1.67	.8
aPn	65712	49280	31876	19311	12299	7587	4101	1811	437	67
F	.08	.22	.28	.23	.26	.36	.49	.74	.47	.08
aPn	68253	51360	33579	20705	13440	8522	4866	2438	950	487
F	.06	.15	.18	.13	.16	.16	.16	.16	.06	.008
aPn	93783	72262	50692	34716	24912	17914	12556	8733	6104	4707

Catch and population sizes in 1000's

TABLE 2.

West Greenland Cod
Number of fish landed per year and age group (in 1000's)

Age (Years)	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
2		544	488			24		296	8	2752	88
3	209	1177	348	578	435	2946	869	7612	8655	14718	1294
4	1758	19353	1772	2866	6186	22958	11423	6589	27181	58619	7738
5	4996	12493	15136	5464	5168	19756	70311	19301	11407	53331	59987
6	17901	9362	6751	27411	4652	8055	29344	48418	18264	8994	40726
7	6622	17367	7501	6622	20250	6980	7816	22517	30864	9152	5791
8	6400	3967	17177	3881	4492	23126	5050	3973	11355	15125	4403
9	24418	4061	3181	5996	2743	4359	13772	1708	2543	2595	6667
10	2345	8893	3652	1124	5363	2333	2433	6768	1027	539	1166
11	4106	1271	12981	1477	805	4724	1709	1104	4138	472	276
12	1014	1899	1691	4327	1438	528	2599	1156	591	1864	122
13	1363	485	2168	999	5195	1138	720	2325	321	73	981
14	2893	436	725	836	741	5052	1219	189	933	34	137
14+	1194	1383	3271	960	1859	2383	2897	3718	747	265	234

TABLE 3.

West Greenland Cod Fishing Mortality (Schumacher (1971))

Age (Years)	YEARS 1956	1957	1958	1959	1960	1961	1962	1963	1964	$\bar{F}$ 1956-61	1961-64	1956-64
3	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.02	0.01
4	0.02	0.04	0.04	0.05	0.04	0.08	0.11	0.11	0.09	0.05	0.10	0.06
5	0.09	0.23	0.08	0.15	0.13	0.18	0.35	0.28	0.27	0.14	0.30	0.20
6	0.16	0.23	0.19	0.21	0.18	0.30	0.43	0.44	0.47	0.21	0.45	0.29
7	0.30	0.22	0.29	0.28	0.23	0.45	0.55	0.70	0.55	0.30	0.60	0.40
8	0.22	0.30	0.36	0.24	0.31	0.44	0.70	0.61	0.96	0.31	0.76	0.46
9	0.29	0.21	0.42	0.28	0.27	0.55	0.52	0.55	1.03	0.32	0.70	0.45
10	0.24	0.16	0.29	0.26	0.27	0.39	0.69	0.52	0.76	0.27	0.66	0.40
11	0.40	0.20	0.37	0.18	0.30	0.40	0.56	0.79	0.70	0.31	0.68	0.43
12	0.34	0.33	0.43	0.20	0.27	0.33	0.40	0.97	1.04	0.32	0.80	0.48
13		0.27	0.78	0.48	0.38	0.35	1.00	0.75		0.45	0.88	0.50

TABLE 4.

Output from POPI

Age	5	6	7	8	9	1C	11	12	13		
a^*	-2.146	-1.501	-1.261	-1.075	-0.991	-1.062	-1.245	-1.387	-1.800		
Year	1	2	3	4	5	6	7	8	9	10	11
f_n	0.587	0.609	0.708	0.510	0.445	0.718	0.915	0.863	0.936	.581	0.0

TABLE 5.

Parameter Estimates from POPO

Age	$_aS$	Year	f_n	Year Class	ln $_rP_{n-a+r}$	Year Class	ln $_rP_{n-a+r}$
5	-2.466	1956	-1.004	1	9.130	12	11.195
6	-1.501	1957	-0.847	2	10.167	13	12.205
7	-0.867	1958	-0.708	3	9.768	14	13.022
8	-0.586	1959	-0.874	4	11.745	15	11.899
9	-0.533	1960	-0.786	5	10.621	16	11.389
10	-0.642	1961	-0.363	6	10.303	17	12.880
11	-0.856	1962	-0.021	7	11.891	18	13.422
12	-0.876	1963	0.006	8	11.307	19	11.813
13	-1.044	1964	0.126	9	11.478		
		1965	-0.032	10	12.795		
		1966	-0.263	11	11.033		

TABLE 6.

Analysis of variance of ln catches

Source	Sum of Squares	D.F.	F.
Parameters	12459.211	39	5092.941
Residual	3.763	60	
Estimated Sigma squared	0.062727		
Var of ln catch	1.591		

TABLE 7.

Estimated Variances of logarithms of fishing mortalities

Year	5	6	7	8	9	10	11	12	13
					Age				
1956	0.04	0.10	0.05	0.06	0.10	0.18	0.31	0.51	0.83
1957	0.03	0.09	0.03	0.05	0.08	0.15	0.27	0.46	0.77
1958	0.03	0.08	0.02	0.04	0.07	0.13	0.24	0.42	0.71
1959	0.03	0.08	0.02	0.03	0.05	0.11	0.21	0.38	0.66
1960	0.03	0.08	0.01	0.02	0.04	0.09	0.18	0.34	0.60
1961	0.03	0.08	0.01	0.02	0.04	0.08	0.16	0.31	0.57
1962	0.05	0.09	0.02	0.03	0.05	0.09	0.18	0.33	0.58
1963	0.09	0.12	0.05	0.06	0.08	0.14	0.23	0.38	0.63
1964	0.21	0.22	0.15	0.15	0.19	0.25	0.35	0.50	0.76
1965	0.49	0.48	0.39	0.40	0.44	0.52	0.62	0.78	1.03
1966	0.99	0.94	0.84	0.85	0.91	1.00	1.10	1.25	1.49

TABLE 8.

ESTIMATED FISHING MORTALITIES

Year	5	6	7	8	9	10	11	12	13
				Age					
1956	0.03	0.08	0.15	0.20	0.21	0.19	0.15	0.15	0.12
1957	0.03	0.09	0.18	0.23	0.25	0.22	0.18	0.17	0.15
1958	0.04	0.10	0.20	0.27	0.28	0.25	0.20	0.20	0.17
1959	0.03	0.09	0.17	0.23	0.24	0.21	0.17	0.17	0.14
1960	0.03	0.10	0.19	0.25	0.26	0.23	0.19	0.18	0.16
1961	0.05	0.15	0.29	0.38	0.40	0.36	0.29	0.28	0.24
1962	0.08	0.21	0.41	0.54	0.57	0.51	0.41	0.40	0.34
1963	0.08	0.22	0.42	0.55	0.59	0.52	0.42	0.41	0.35
1964	0.09	0.25	0.47	0.63	0.66	0.59	0.48	0.47	0.39
1965	0.08	0.21	0.40	0.53	0.56	0.50	0.41	0.40	0.34
1966	0.06	0.17	0.32	0.42	0.45	0.40	0.32	0.31	0.27

TABLE 9.

Estimated Fishing Intensity

Year	1956	1957	1958	1959	1960	1961	1962	1963
Intensity	5.94	7.99	11.01	9.79	10.50	16.95	17.31	18.04

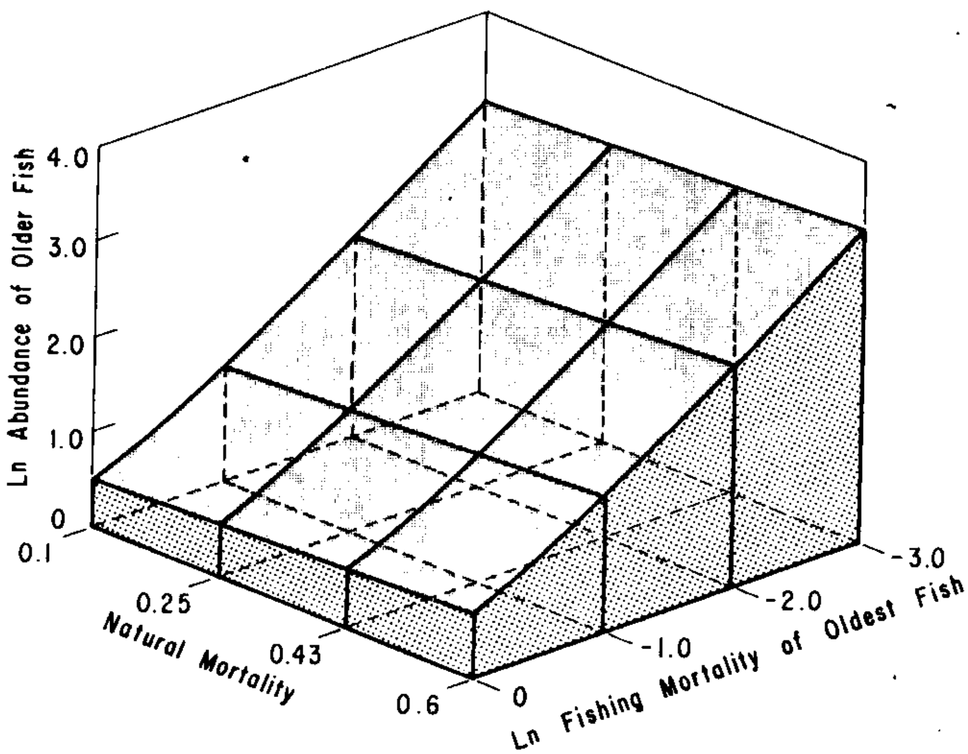

Fig. 1. Effect of final F and M on estimated population size of oldest fish.

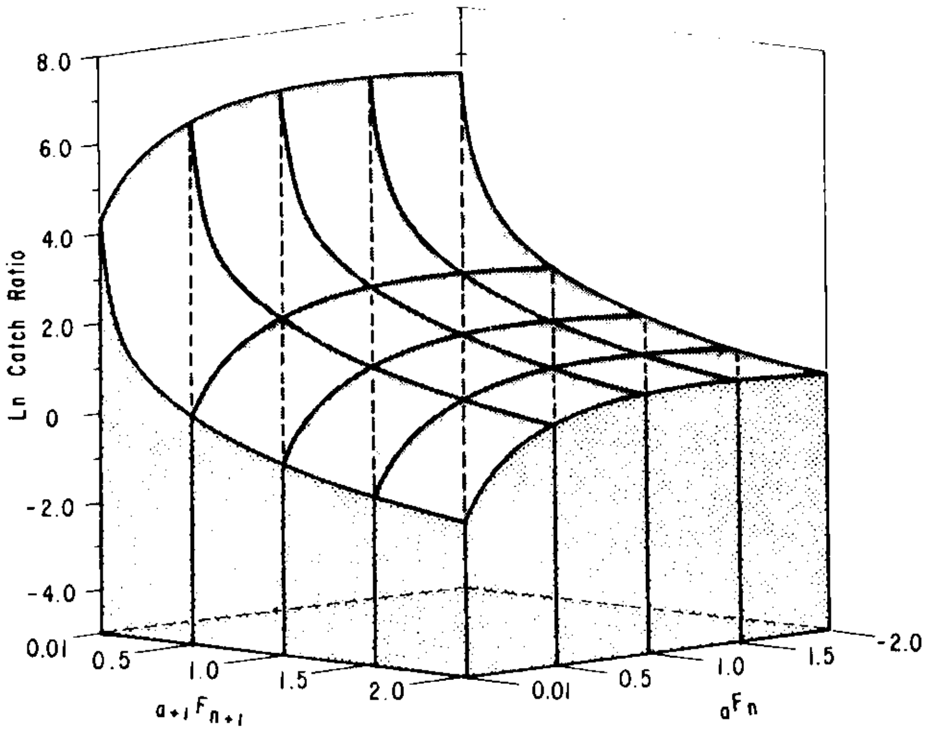

Fig. 2. Response of ln catch ratio to fishing mortalities at M = 0.2.

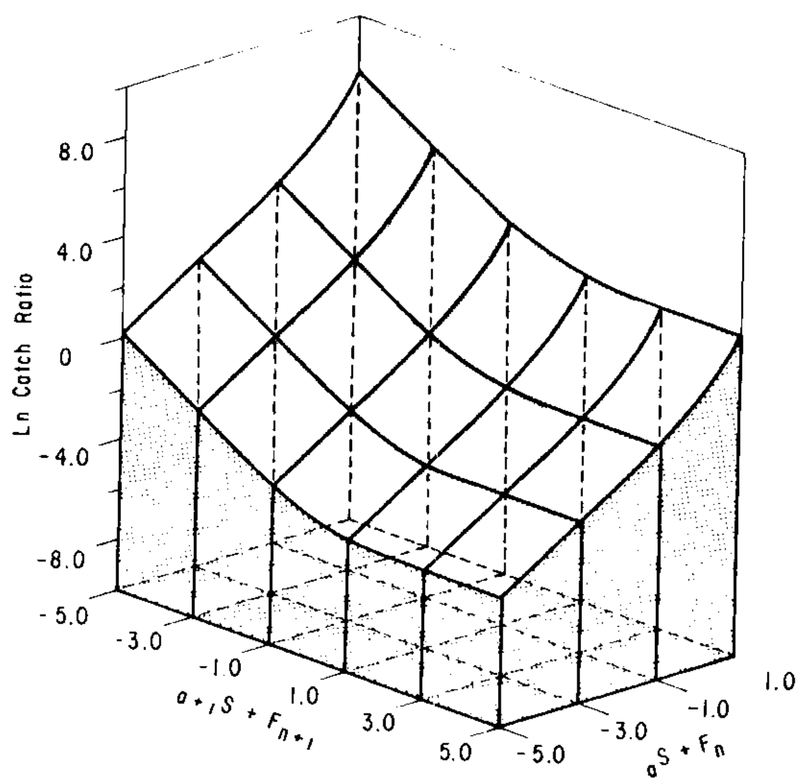

Fig. 3. Response of ln catch ratio to fishing mortalities at M = 0.2 (log scale).

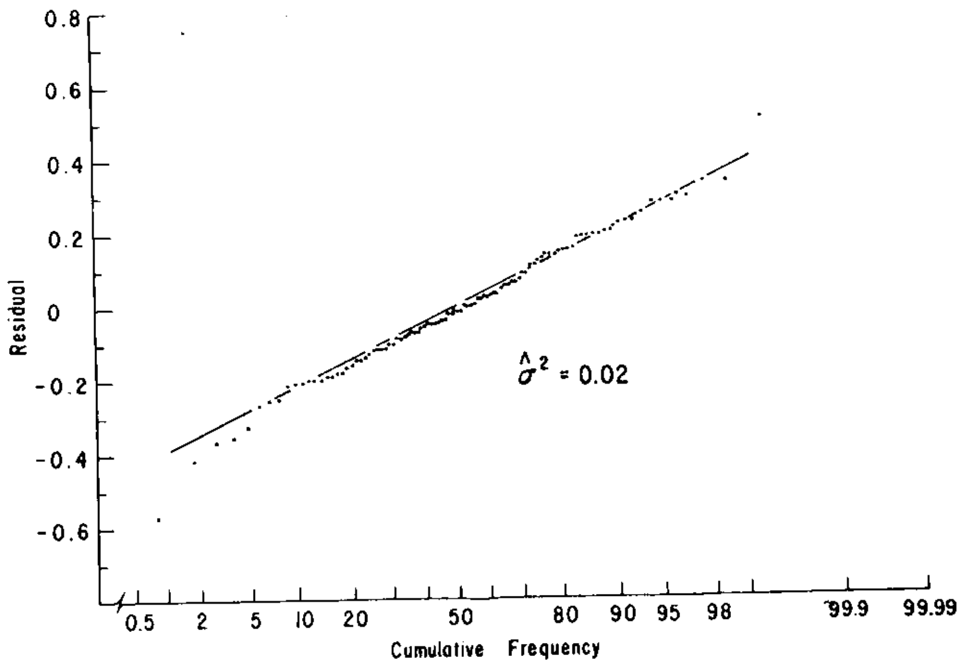

Fig. 4. Probability paper plot of residual $\log_e$ catch for Greenland cod.

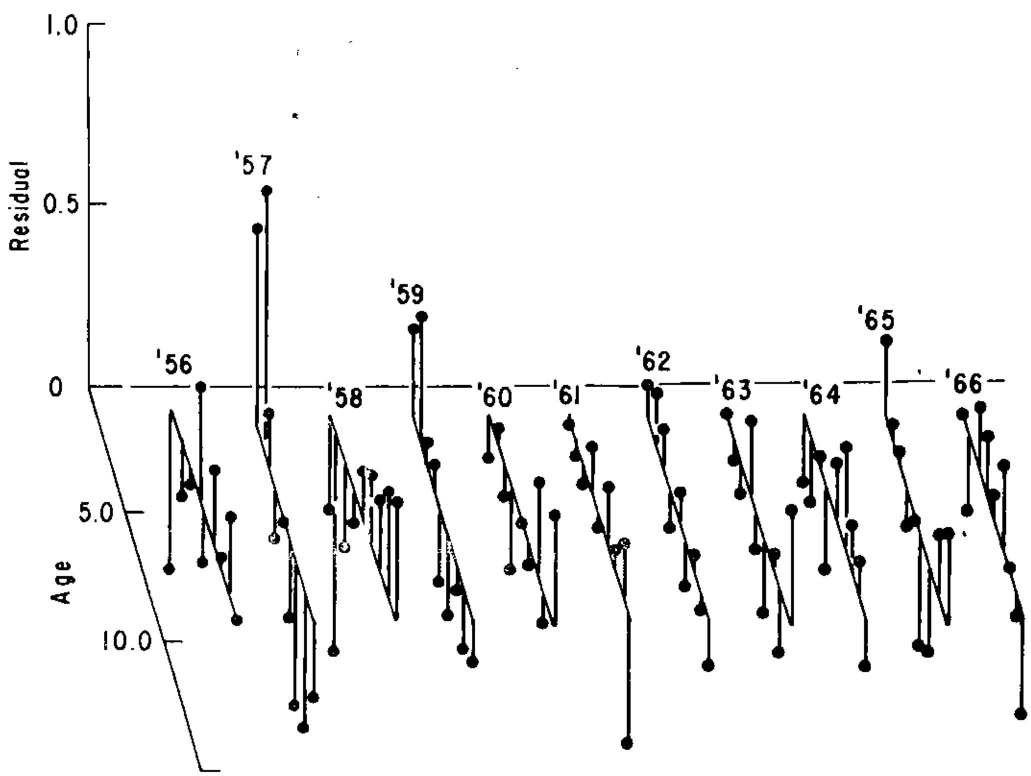

Fig. 5. Plot of residual $\log_e$ catch of Greenland cod vs age and year.

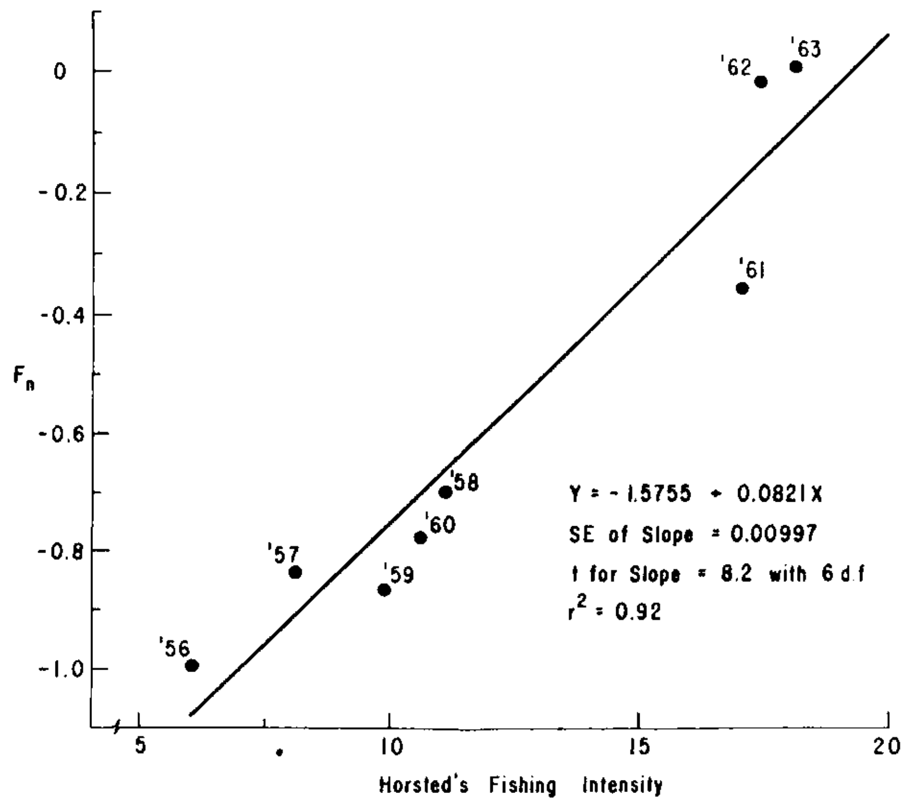

Fig. 6. Plot of estimated $\log_e$ fishery mortalitv vs Horsted's Fishing Intensity for Greenland cod.