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Graphical Methods for Estimating Parameters in Simple Models of Fisheries

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Abstract

A graphical method for calculating the coefficients for a Schaefer model of a fishery is introduced. It involves plotting catch per effort vs. effort data and then correcting the values for disequilibrium of the fishery. A hypothetical and a realistic example are presented.

1. Introduction.

The "simple" mathematical models of fisheries studied by Schaefer (1968), Pella and Tomlinson (1969), Fox (1970), and Walter (1973) have an advantage over the more detailed models in that they require only catch and effort data for their use. One of the greatest sources of difficulty with their use lies in the determination of the parameters which appear in each of them. The usual procedures, after the calculation of effort and catch per effort data, involve using these data to derive certain equations in the parameters and then solving these equations. Such procedures, as well as search procedures such as that of Pella and Tomlinson, require the use of a computer. However, with any of them, the results must be tempered by common sense and a knowledge of the fishery.

Another approach in which this tempering is easier involves using graphical methods to determine the parameters. These involve first plotting the catch per effort (CPE) versus the effort over the history of the fishery. In the Schaefer model, a straight line with negative slope is fitted in some way to these points. In others it is a curve of predetermined type. This line or curve represents the points of equilibrium between fishing effort and CPE (or stock biomass which is assumed proportional). If the fishery is actually in equilibrium for some portion of its history it may be fitted by least squares (or by eye) to these points of equilibrium. Unfortunately, few exploited fisheries are in equilibrium for any extended period of time and methods other than fitting the plotted points must be devised. Schaefer's method is to take the average of the CPE over a period of low exploitation and over a period of high exploitation and draw the line through the two points determined thus. The method of Gulland (1968) is to plot CPE versus the effort averaged over the number of years classes in the fishery and fit the best line or curve to these plotted points.

In this work we shall describe a graphical method which involves correcting the CPE by an amount corresponding to disequilibrium of the fishery, plotting the resulting equilibrium values of CPE versus effort and fitting the best straight line to them. The coefficient of catchability is determined graphically as part of this procedure. Extensions are made to non-linear models and to delay models. Hypothetical examples as well as realistic examples are presented.

2. A Hypothetical Example.

We construct in this section an example of a fishery which exactly follows the Schaefer model, i.e.,

$$\frac{1}{p}\frac{dp}{dt} = b - ap - qf \qquad (1)$$

where p is the stock biomass and f is the fishing effort. We shall then try to work backwards from the generated data to determine the equilibrium line and investigate how well different methods recover the line

$$0 = b - ap - qf.$$

Let us take $q = 0.1$, $a = 0.01$, and $b = 1$, and assume first that the fishery is an expanding one over the course of 10 years with f increasing sporadically from 1 to 10. The values of U (i.e., CPE) and of f are plotted in Figure 1. Here $U = qp = 0.1p$. We start with $p(0) = 100$ and use the difference quotient $\frac{\Delta p}{\Delta t}$ to approximate $\frac{dp}{dt}$. It should be observed that the equilibrium line is generally below the plotted points.

If the Schaefer method of determining the equilibrium line is used, a line above the original one is obtained. If the Gulland method in which a running average of effort over three years is used, the resulting line again lies above the equilibrium (see Figure 1). In the method we shall introduce the resulting line is much closer to the line of equilibrium (see Figure 2). The method involves first using an initial approximation to the equilibrium to determine the parameters needed in the correction. Then the data points are corrected to the values they would have if the fishery were in equilibrium and a line is fitted to these corrected values.

3. Mathematical Formulae.

We shall calculate the amount of the correction needed for equilibrium in a number of different ways and find approximations to this correction. One formula is based on the difference between the equations for equilibrium and non-equilibrium. If p_e represents the biomass at equilibrium, then for a given level of f

$$0 = b - ap_e - qf. \qquad (2)$$

This may be subtracted from equation (1) to obtain

$$\frac{1}{p}\frac{dp}{dt} = -a(p - p_e) \qquad (3)$$

or

$$p_e = p + \frac{1}{ap}\frac{dp}{dt}. \qquad (4)$$

This is given approximately by replacing $\frac{dp}{dt}$ by $\frac{\Delta p}{\Delta t}$ and is given in terms of CPE by replacing p by U/q:

$$U_e = U + \frac{q}{aU}\frac{\Delta U}{\Delta t}. \qquad (5)$$

Thus the correction is $\frac{q}{aU}\frac{\Delta U}{\Delta t}$.

Another expression for p_e and hence for the correction may be obtained from the integrated form of the equation. Let p_i equal the biomass at the beginning of the i-th year and assume that the effort is constant at a level f. Then equation (1) may be solved to find that during the year

$$p(t) = [\frac{a}{b-qf} - (\frac{a}{b-qf} - \frac{1}{p_i})e^{-(b-qf)(t-i)}]^{-1} \qquad (6)$$

The equilibrium level p_e during this i-th year is given by (2). Hence the biomass at the end of the i-th year, p_{i+1}, may be expressed in terms of p_i and p_e by letting

$t = i+1$ and $b - qf = ap_e$ in (6):

$$p_{i+1} = [\frac{1}{p_e} - (\frac{1}{p_e} - \frac{1}{p_i})e^{-ap_e}]^{-1} . \qquad (7)$$

If the biomass is above equilibrium at the start of the year at that particular level of f, then it is also above at the end of the year at the same level of f. This is plausible since we would expect the biomass to decline toward but never attain equilibrium. This can also be shown analytically by solving (7) for p_e:

$$p_e = p_{i+1}[1 - e^{-ap_e}(1 - \frac{p_e}{p_i})]. \qquad (8)$$

This shows that $p_i > p_e$ implies that $p_{i+1} > p_e$. Furthermore (8) may be expressed as

$$p_{i+1} = p_i[\frac{p_i}{p_e} - (\frac{p_i}{p_e} - 1)e^{-ap_e}]^{-1}$$

which may be used to deduce that

$$p_{i+1} < p_i.$$

Thus we obtain the relation

$$p_e < p_{i+1} < p_i$$

for fixed f. In other words, the point (f, p_{i+1}) is closer to (f, p_e) than is (f, p_i). In fact in certain cases it is a very good approximation.

If ap_e is large with respect to 1, then e^{-ap_e} is approximately zero and (8) reduces to

$$p_e \approx p_{i+1}. \qquad (9)$$

The value ap_e as a term in the expansion $b - ap_e$ is an instantaneous growth rate. It will be large for rapidly growing species, though rarely much larger than 1. For slowly growing species for which ap_e is small, we use the approximation

$$e^{-ap} \approx 1 - ap.$$

In this case equation (8) reduces to

$$p_{i+1} \approx [\frac{1}{p_e} - (\frac{1}{p_e} - \frac{1}{p_i})^{(1-ap_e)}]^{-1}$$

$$= [\frac{1}{p_i} + ap_e(\frac{1}{p_e} - \frac{1}{p_i})]^{-1}$$

$$= [\frac{1}{p_i}(1 - ap_e) + a]^{-1},$$

which may be solved for p_e to give

$$p_e \approx p_i + (\frac{p_{i+1} - p_i}{ap_{i+1}}) \tag{10}$$

This, in turn, may be expressed in terms of U to obtain another expression for the correction. It should be compared to (5) with which it agrees approximately.

We shall use the CPE values corresponding to the biomass values in (9) as a first approximation to the true equilibrium points. They are plotted against effort and the best straight line drawn through them. The equation of this line, say $U = \beta - \gamma f$, may be used to obtain an initial estimate for the parameters in the equation

$$\frac{1}{U}\frac{dU}{dt} = b - \frac{a}{q}U - qf. \tag{11}$$

The number β will approximate $\frac{bq}{a}$ and γ will approximate $\frac{q^2}{a}$. Since there are three parameters in (11), another equation is needed to obtain them all. This may be done in a number of ways, of which we mention threc.

A graphical method consists of plotting $\frac{\Delta U}{U}$ versus $\beta - U - \gamma f$ and fitting the best straight line through the origin with position slope to these points. Then we would obtain

$$\frac{\Delta U}{U} \approx \alpha(\beta - U - \gamma f)$$

from which we could calculate

$$b = \alpha\beta, \quad q = \alpha\gamma, \quad \text{and} \quad a = \alpha^2\gamma.$$

An analytic method consists of integrating both sides of (11) and then calculating α as the quotient

$$\alpha = (\sum_{i=1}^{n} \beta - U_i - \gamma f_i)^{-1} \ln \frac{U_n}{U_o}. \qquad (12)$$

If the value of q can be determined independently, say from estimates of fishing mortality, b and a can be determined from it and γ and β.

Once the estimates for a, b, and q are all made, U_i could be corrected again by using (5). This correction, however, is very sensitive to error in the values of q/a since it appears in the denominator. It would be better to devise an approximate correction which is less sensitive to such errors. Accordingly, we solve (8) for p_e in the form

$$\begin{aligned} p_e &= p_{i+1} \frac{(1-e^{-ap_e})}{(1 - \frac{p_{i+1}}{p_i} e^{-ap_e})} \\ &= p_{i+1} - \left(\frac{p_i - p_{i+1}}{e^{ap_e} p_i - p_{i+1}}\right) p_{i+1} \end{aligned} \qquad (13)$$

The second term in the last line is the exact error term but contains the expression to be estimated (p_e). However, if the initial approximation to p_e by p_{i+1} is used, this equation will give us a new approximation.

The expression (13) may be written in terms of the CPE U as

$$U_e = U_{i+1}\left(1 + \frac{\Delta U_i}{e^{\frac{a}{q}U_e}U_i - U_{i+1}}\right)$$

$$\approx U_{i+1}\left(1 + \frac{\Delta U_i}{e^{\frac{a}{q}U_{i+1}}U_i - U_{i+1}}\right) \qquad (14)$$

This has two advantages over (5), one that it is not as sensitive to error in a/q, and the other that it can be iterated to achieve any degree of accuracy desired provided a/q is known.

4. Back to the Hypothetical Example.

Let us now use the procedure of section 3 on the hypothetical example introduced earlier in section 2. The first approximation is obtained by plotting the points (f_i, U_{i+1}) and fitting a straight line through them. This is done in Figure 2. The equation of this line is approximately $U = 10 - 0.9f$.

We now follow the graphical procedure of plotting $\frac{\Delta U}{U}$ versus $10 - U - 0.9f$ and fitting the best straight line (Figure 3). Here we have used $(U_{i+1} - U_i)/U_{i+1}$ as the value of $\frac{\Delta U}{U}$. We find the slope of this line to be .17.

This is our estimate for a/q. It differs considerably with the true value of 0.1 but if we use (14) to correct our first approximation, this shouldn't make much difference. The corrections are shown again on Figure 2. The straight line fitting these points is $U = 10 - .96f$ which should be compared to the line we started with $(U = 10 - f)$.

5. A realistic Example - Mackeral in ICNAF Statistical Areas 5 and 6. (Data from Anderson)

The fishing effort is plotted against CPE in Figure 4, for this fishery. The initial corrections are made as in the hypothetical example and a straight line fitted to the corrected values. Its equation is $U = 2 - \frac{1}{300}f$. The values of $2 - U - \frac{1}{300}f$ were plotted vs. $\frac{\Delta U}{U}$ and the straight line fitted through points was found to have a slope of about 1, the estimate for a/q in this case. The values shown in Figure 4 are again corrected by the amount calculated from (14) and a new straight line drawn. Its equation is

$$U = 2.1 - .0042f.$$

If the maximum sustained yield is calculated from this, it is found to equal 263,000 MT.

6. Other Models.

The graphical method outlined in section 3 applies to the Schaefer model. If there is some valid biological reason for using a value of $m \neq 2$ in the model

$$\frac{1}{p}\frac{dp}{dt} = b - ap^{m-1} - qf \qquad (15)$$

then the procedure may be modified by first plotting U^{m-1} vs. f and fitting the best straight line with the equation $U^{m-1} = \beta - \alpha f$. Then as before $\frac{\Delta U}{U}$ is plotted against $U^{m-1} + \alpha f$ and the slope, say γ, calculated.

Now, however, the correction is not the same. U^{m-1} rather than U is corrected to obtain

$$U_e = (U^{m-1} + \frac{q^{m-1}}{aU}\frac{\Delta U}{\Delta t})^{\frac{1}{m-1}}. \qquad (16)$$

In the case of a model with a delay term of the form

$$\frac{1}{p}\frac{dp}{dt} = b - a_1 p_t + a_2 p_{t-\omega} - qf \qquad (17)$$

it is necessary to procede in three stages. The first is to ignore the delay term and use either a graphical or analytic procedure to estimate b, a, and q for the Schaefer model. The delay model assumes that natural growth is composed of two components, one due to individual growth and the other due to recruitment. It further assumes that recruitment is proportional to the spawning population from whence it came. Thus the coefficient b of the Schaefer model must be split into two terms $b' + a_2 p(t - \omega)$. The proper value of these new coefficients may be obtained by plotting

$$\frac{1}{U}\Delta U + qf$$

versus $U(t - \omega)$ where the latter is the dependent variable. The y intercept is then b' and the slope is $\frac{a_2}{q}$.

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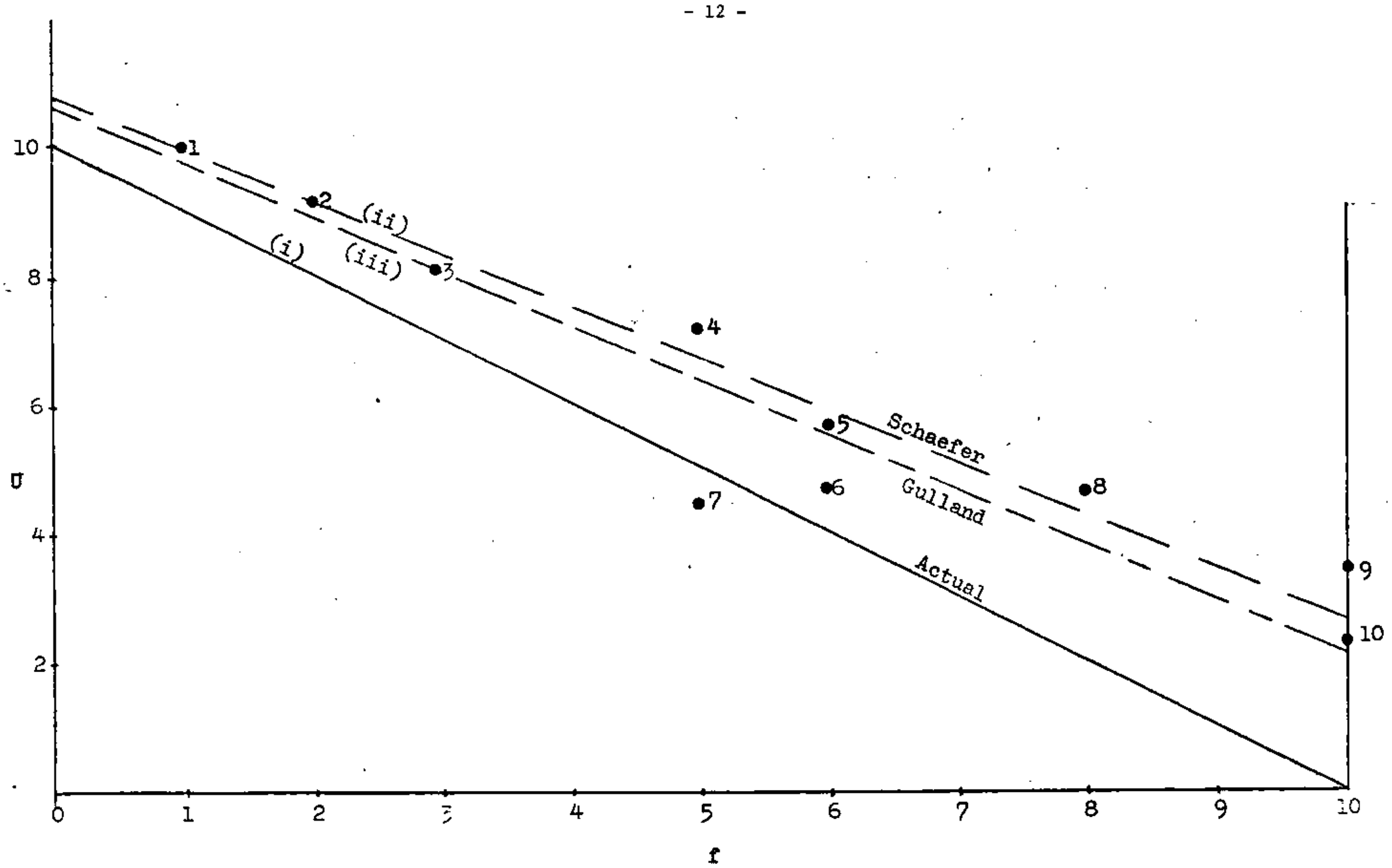

Fig. 1. Plot of catch per effort U vs. effort f for a hypothetical fishery governed exactly by the equation $\frac{1}{p}\frac{dp}{dt} = 1 - 0.01p - 0.1f$. The actual equilibrium line is given by (i). The lines that would result from the Schaefer and Gulland methods respectively are given by (ii) and (iii).

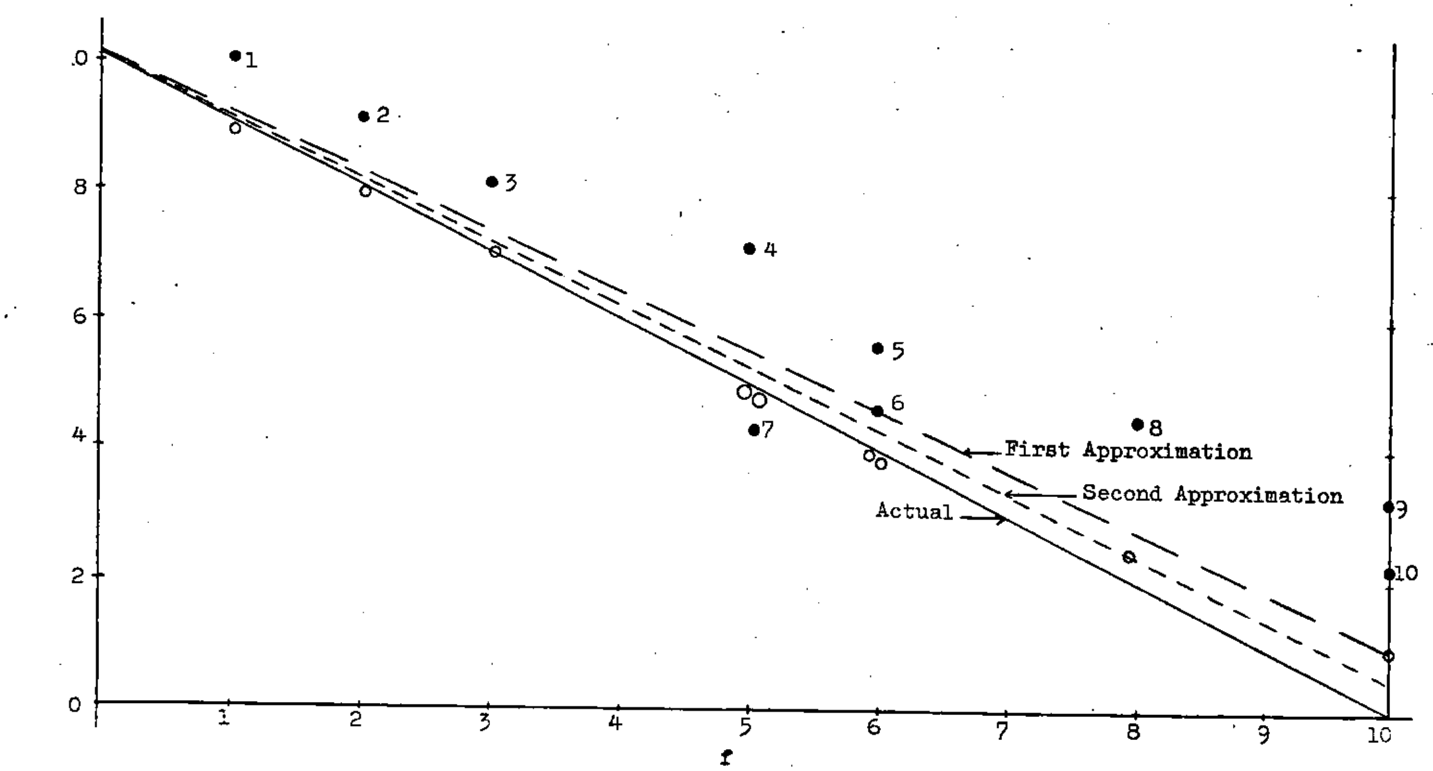

Fig. 2. Plot of catch per effort U vs. effort f uncorrected • and corrected ο for disequilibrium for the hypothetical fishery of Fig. 1. The first and second approximations to the equilibrium line given by the new method are shown.

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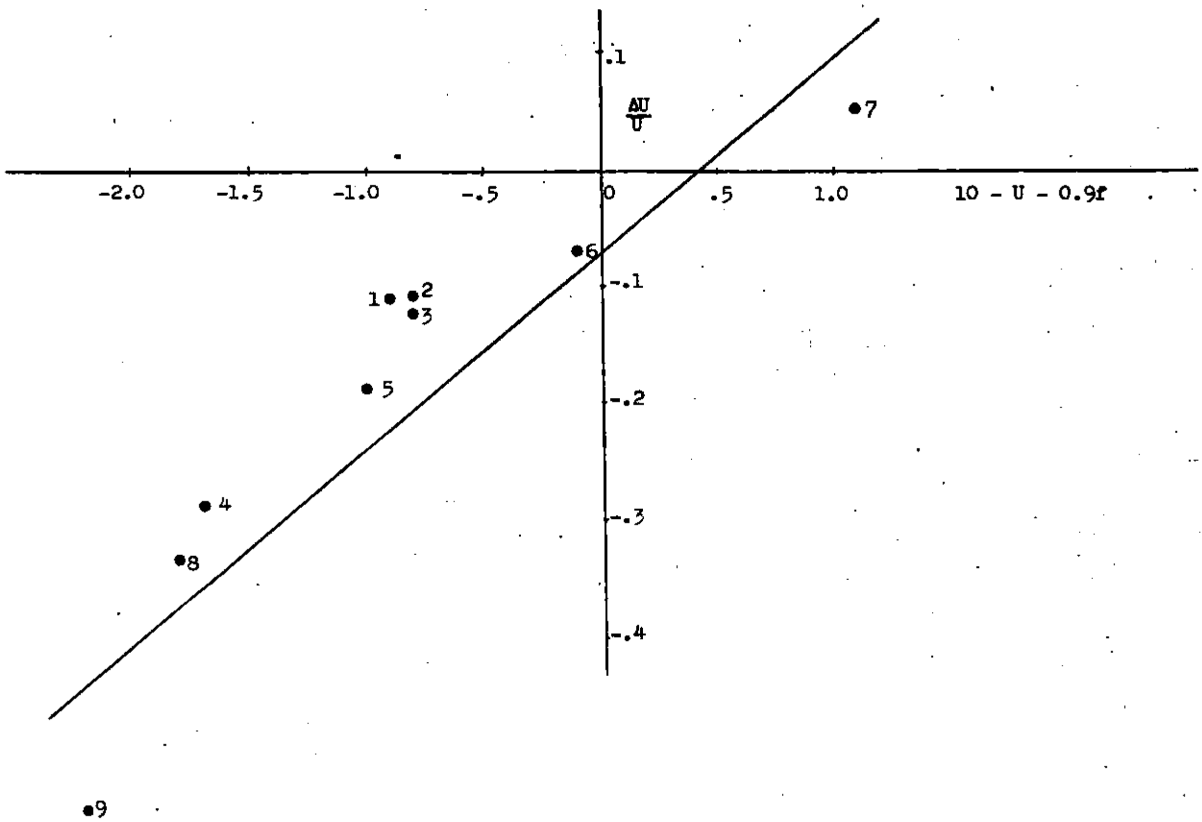

Fig. 3. Plot of $\frac{\nabla U}{U}$ vs. 10 - U - 0.9f for the hypothetical example of Fig. 1. The line represents the best fit (by eye) to the points.

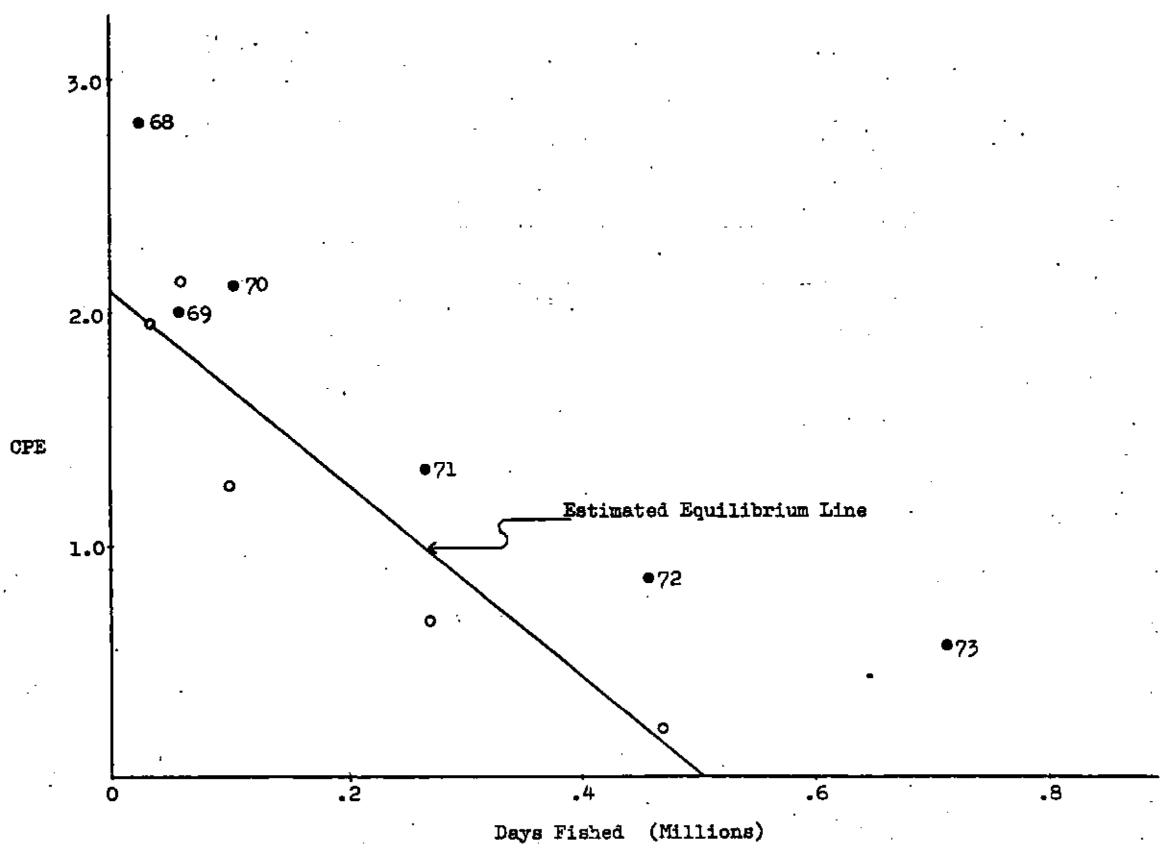

Fig. 4. CPE vs. f for mackerel in ICNAF Statistical Areas 5 and 6. Data from Anderson. Both original values and corrected values o are shown.