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Enhancing age length keys

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Abstract:

Age length keys are subject to two sources of error - sampling fluctuations in length frequencies and incorrect ageing. In this paper, a method of analysis of length frequency samples proposed by Buchanon-Wollaston (1929) is adapted for use on age length keys. When applicable, the method smooths sampling fluctuations and considerably reduces the effects of incorrect ageing. The method applied to an age-length key for cod.

Introduction:

In the absence of effective ageing procedures, length frequency samples have been treated as mixtures of normal distributions of length at age with one component per year class (Tanaka 1962, Cassie 1963, Bhattacharya 1967, Macdonald 1969, Mathews 1974). Several methods of separating the components have been proposed, but none is satisfactory when adjacent components overlap heavily (Macdonald 1969, Mathews 1974).

If the assumption of a normal distribution of length at age is applied to an age length key, the problem of overlap is reduced to the problem of incorrect ageing which is more manageable.

Theory:

Let Y(x) represent the number of fish at length x and a given age in a random sample of 1000 fish. If the distribution of length at age is normal, then

$$Y(x) = \frac{K}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where K is the number of fish in the age class per 1000 fish, μ is the mean length, and σ is the standard deviation of the distribution of lengths.

Then $$l_n Y(x) = l_n\left(\frac{K}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$$

$$= a_o + a_1 x + a_2 x^2$$

Thus $$\mu = -\frac{a_1}{2a_2}$$

$$\sigma = \sqrt{\frac{-1}{2a_2}}$$

$$K = \sqrt{\frac{-\pi}{a_2}} \exp\left(a_0 - \frac{1}{4}\frac{a_1^2}{a_2}\right)$$

If the natural logarithm of the estimated number of fish at a given age and length per 1000 fish is plotted against length, a series of parabolas, one for each age, results. Buchanon-Wollaston (1929) observed this and used the method of least squares to estimate the relevant parameters for the comonent parabolas of length frequency samples.

Fig. 1 shows a logarithmic plot of an age length key with three ages. The spacing and shape of the parabolas was motivated by the cod example considered below.

Fig. 2 illustrates how the method of least squares can be used to estimate the curves. The ordinates of the parabolas of Fig. 1 at each cm were randomized by adding normally distributed random deviates with mean 0 and variance 0.04 and parabolas were estimated by least squares.

Figs. 3, 4, 5 show the effects of ageing errors on the curves of Fig. 1. In Fig. 3, ten percent of all fish are incorrectly aged, in Fig. 4, the rate is twenty percent. Fig. 5 shows the effects of the error rate increasing with age. There are two observations which can be made from these figures. Firstly, the region near the peak of a parabola is depressed but not distorted. Secondly, the tails of the parabolas are distorted giving rise to skewed tails and even false modes beneath the modes of adjacent age classes. Even if the overlap of adjacent parabolas is considerable, errors in ageing are easily detected.

Methods:

The main practical difficulty in fitting parabolas to age length keys is overcoming the ageing errors. Fortunately the error rate increases with age so that it is possible to work from the youngest age to the oldest with all parabolas "to the left" previously defined.

The first step in analysis is to estimate the number at age and length in a random sample of 1000 fish for the initial key. If the key was derived from a random sample, all numbers are multiplied by a single raising factor. If the key was derived from a stratified sample, one raising factor per stratum is required.

The logarithms of the numbers at length and age are then calculated and plotted against length.

The plot is examined for false modes and distorted tails. Every fish in a false mode should be added to the mode above it.

A parabola is fitted to the youngest age group. In the author's experience, the variance of the l_n numbers is fairly constant with the exception of observations consisting of one fish which should not be used in the fitting procedure. Points are chosen from the left until the area of distortion (if any) on the right tail is reached. The remaining points are not used for fitting the parabola.

The estimated parabola is plotted and the estimates of numbers at length for the next age group are corrected by subtracting the estimates due to the parabola just fitted from the total number of fish at length.

The next parabola is then fitted by the same means.

The procedure is repeated until all ages are exhausted or the method breaks down. If there is a very high error rate in ageing old fish and also few fish at the corresponding lengths, it is best to stop and leave the remainder unresolved.

From the parabolas, a new age length key can be constructed straight - forwardly.

Example:

Table 1 contains an age length key for cod in ICNAF Subarea 1D for 1971 (sampling yearbook vol. 16, 1972, p. 34). The key is based on five samples collected by the Federal Republic of Germany in December.

Fig. 6 shows the estimated logarithms of numbers at age and length up to age length using the original table corrected for stratification. Observe that distortion begins at 45 cm for age 3 and 72 cm for age 5. These are false modes at 60, 69, and 78 cm.

Fig. 7 shows the estimated parabolas for ages 3 to 7. The dotted lines may represent ages 8 and 9, but there was insufficient data to fit parabolas beyond age 7. Observe the orderly progressions of mean length at age. The estimated standard deviations increased from age 3 to age 4 and then decreased steadily as would be expected as the growth rate slows down.

Table 2 shows the enhanced age length key. Numbers are given to 1/10 of a fish to avoid rounding errors.

Extension:

Fig. 8 shows the effect of one month's growth on the hypothetical key of Fig. 1. If samples widely spaced through time are pooled, the parabolas are distorted. If the growth rate is constant, however, the model can easily be extended to make μ and a_0 linear functions of time (σ changes very slowly).

Design of Sampling:

Ideally, sampling should aim for a constant coefficient of variation of numbers at age and length in the initial key. If large samples (thousands) of fish are to be taken, fish should be measured to the nearest cm so that the number of points in the fitting process is increased. If stratified samples are taken, the areas of overlap of successive age groups should have larger samples as should the larger lengths.

Conclusion:

If length at age can be assumed to have a normal distribution, an age length key can be considerably enhanced by the above method. A certain amount of judgement is required in detecting distortion although most examples seem to be clear cut. The subjective decisions enter the method only qualitatively: the estimation procedure is automatic once false modes have been labeled. It is possible that biases may result from use of this method but these biases will normally be much smaller than the effects of errors in ageing.

The normality assumption may break down if two stocks are present in the catch or if spring and fall spawners (for example) are represented. In this case, individuals would have to be classified by stock as well as age and the estimation procedure would become more complicated.

Where applicable, the method promises to improve considerably the reliability of age length keys.

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TABLE 1

ACE/LENGTH TABLE - COD - 1D - 1971 (4th QUARTER)

GEAR	Otter Trawl (Research)
CATCHES OR LANDINGS	Catches
COUNTRY	Germany, Fed. Rep.
TONNAGE	>1800
STRUCTURES USED IN AGE DETERMINATION	Otoliths

Length Group (3 cm) \ Age	3	4	5	6	7	8	9	10	11	12	13	Total No. of Ages Read	LENGTH COMPOSITION PER MILLE Months Within Quarter: Oct	Nov	Dec
24	1	-	-	-	-	-	-	-	-	-	-	1	-	-	1
27	4	-	-	-	-	-	-	-	-	-	-	4	-	-	6
30	8	-	-	-	-	-	-	-	-	-	-	8	-	-	27
33	12	-	-	-	-	-	-	-	-	-	-	12	-	-	53
36	20	1	-	-	-	-	-	-	-	-	-	21	-	-	78
39	20	2	-	-	-	-	-	-	-	-	-	22	-	-	74
42	16	5	1	-	-	-	-	-	-	-	-	22	-	-	63
45	14	8	-	-	-	-	-	-	-	-	-	22	-	-	70
48	6	12	1	-	-	-	-	-	-	-	-	19	-	-	46
51	-	16	3	-	-	-	-	-	-	-	-	19	-	-	47
54	-	17	12	1	-	-	-	-	-	-	-	30	-	-	67
57	-	6	28	1	-	-	-	-	-	-	-	35	-	-	69
60	-	1	45	2	-	-	-	-	-	-	-	48	-	-	75
63	-	-	34	1	-	-	-	-	-	-	-	35	-	-	66
66	-	-	17	6	1	1	-	-	-	-	-	25	-	-	48
69	-	-	16	20	5	2	-	-	-	-	-	43	-	-	60
72	-	-	5	9	1	5	1	-	-	-	-	21	-	-	40
75	-	-	2	7	2	7	3	-	-	-	-	21	-	-	31
78	-	-	-	5	1	2	1	-	-	-	-	9	-	-	30
81	-	-	-	6	1	4	1	-	-	-	-	12	-	-	18
84	-	-	-	1	1	1	-	-	-	-	-	3	-	-	7
87	-	-	-	1	-	1	-	-	-	-	1	3	-	-	8
90	-	-	-	-	-	-	-	2	2	1	-	5	-	-	9
93	-	-	-	1	-	-	-	-	-	-	-	1	-	-	2
96	-	-	-	-	-	1	-	-	1	-	1	3	-	-	2
99	-	-	-	-	-	-	-	1	1	-	-	2	-	-	2
102	-	-	-	-	-	-	-	-	1	-	1	2	-	-	1
													-	-	1000
Total No. of Ages Read	101	68	164	61	12	24	6	3	5	1	3	448			

Serial Numbers — No. of Age Samples 5

	Oct	Nov	Dec
Number of fish measured each month	-	-	2127
Mean length of fish (cm) each month	-	-	55.6
Mean weight of fish (kg) each month	-	-	-
Total weight caught or landed (tons) each month	-	-	-
Estimated number caught or landed (whole numbers) each month	-	-	-
Range of depths (m) each month	-	-	170-310
Mesh size (mm)	-	-	130

AGE COMPOSITIONS OF LANDINGS (PER MILLE)

Month	3	4	5	6	7	8	9	10	11	12	13	Total
Oct	-	-	-	-	-	-	-	-	-	-	-	-
Nov	-	-	-	-	-	-	-	-	-	-	-	-
Dec	334	170	297	109	21	43	11	5	7	1	2	1000

TABLE 2

Length \ Age	3	4	5	6	7	8+	Total
24	1.8						1.8
27	7.3						7.3
30	21.7						21.7
33	46.1	0.4					46.5
36	70.6	1.9					72.5
39	77.8	6.2					84.0
42	61.9	14.9					76.8
45	35.2	26.8	0.2				62.2
48	14.5	36.0	1.4				51.9
51	4.3	35.9	7.5				47.7
54	0.9	26.7	25.8				53.4
57		14.8	55.1				69.9
60		6.1	73.9				80.0
63		1.9	62.1	1.9			65.9
66			32.7	15.9			48.6
69			10.8	43.7	0.5		55.0
72			2.2	40.3	4.3		46.8
75			0.3	12.5	15.8		28.6
78				1.3	26.5		27.8
81					19.9		19.9
84					6.8	1.2	8.0
87					1.0	8	9.0
90						2	2.0
93						2	2.0
96						2	2.0
99						2	2.0
102						1	1.0
Total	342.1	171.6	272.00	115.6	74.8	18.2	994.3

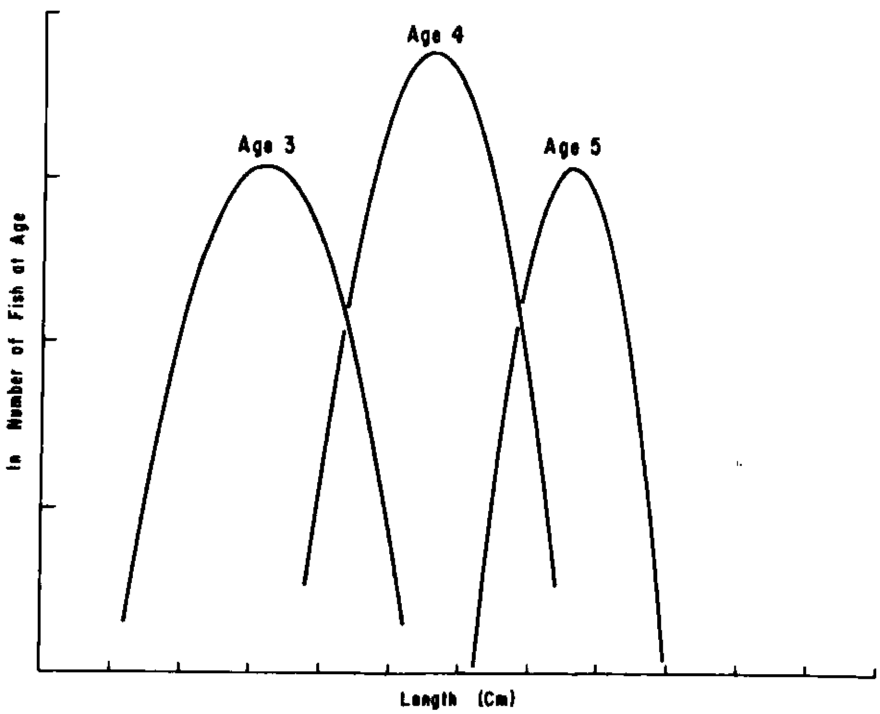

Fig. 1. Hypothetical age length key.

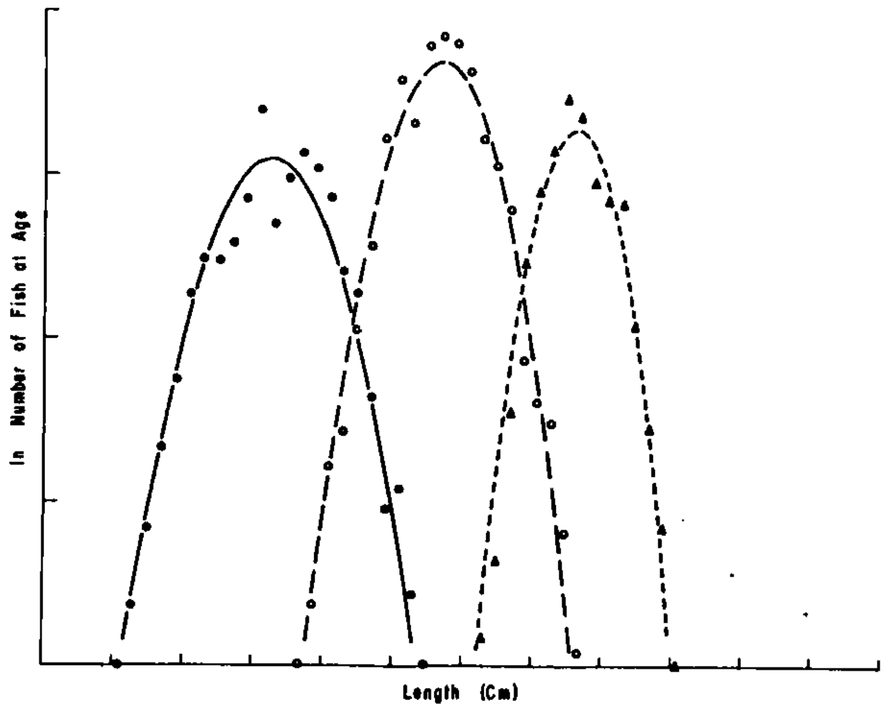

Fig. 2. Hypothetical estimated age length key.

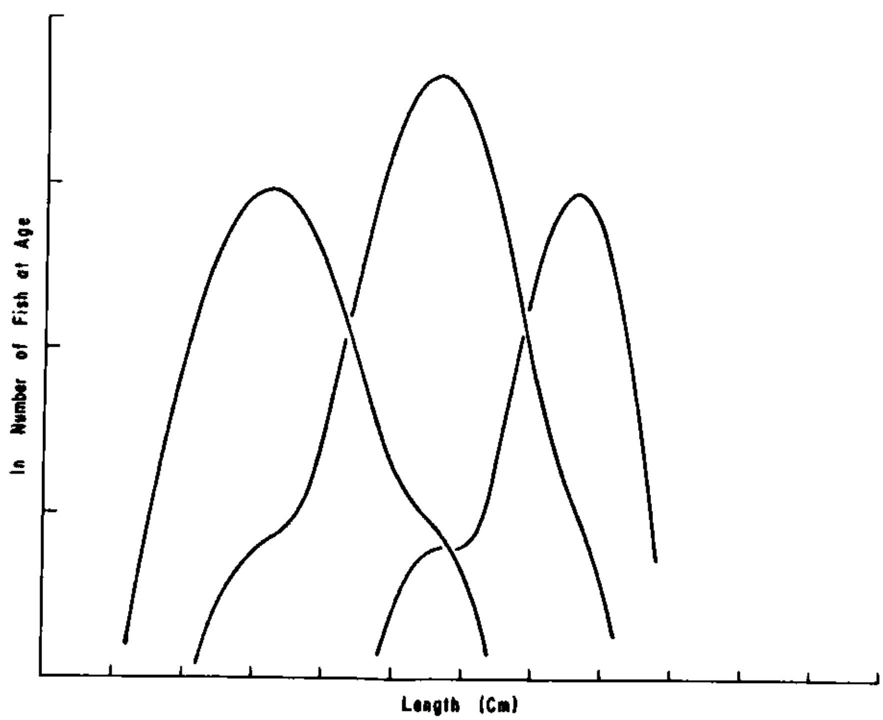

Fig. 3. Key with 10% ageing error rate.

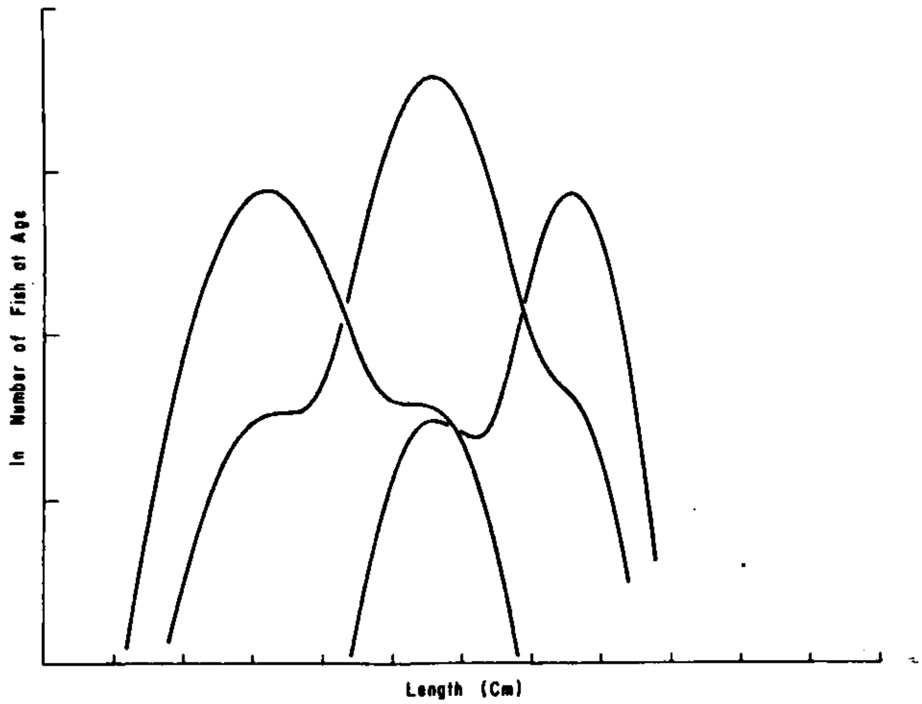

Fig. 4. Key with 20% ageing error rate.

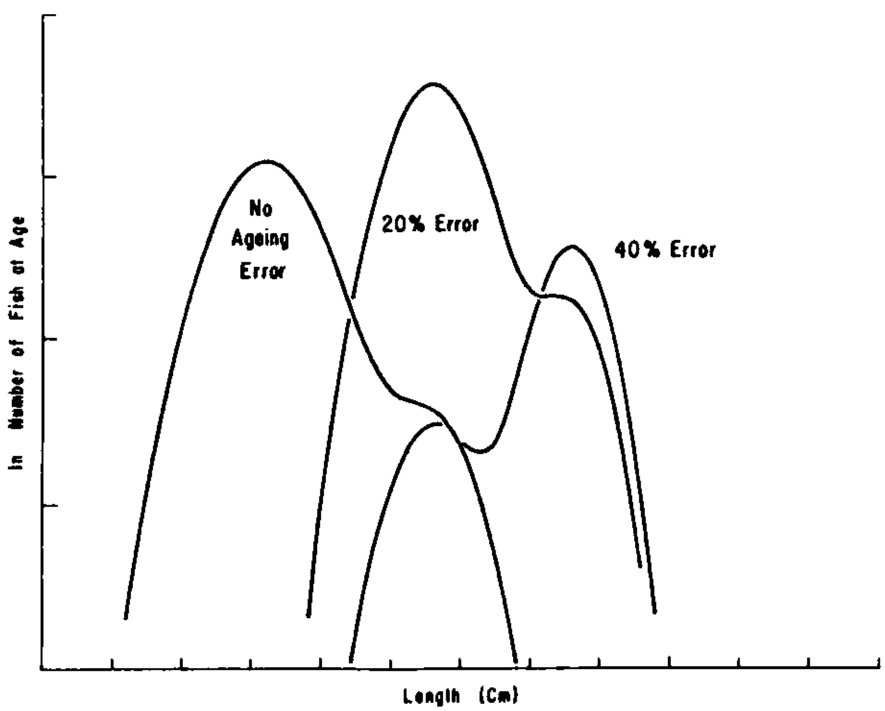

Fig. 5. Key with ageing error rate increasing with age.

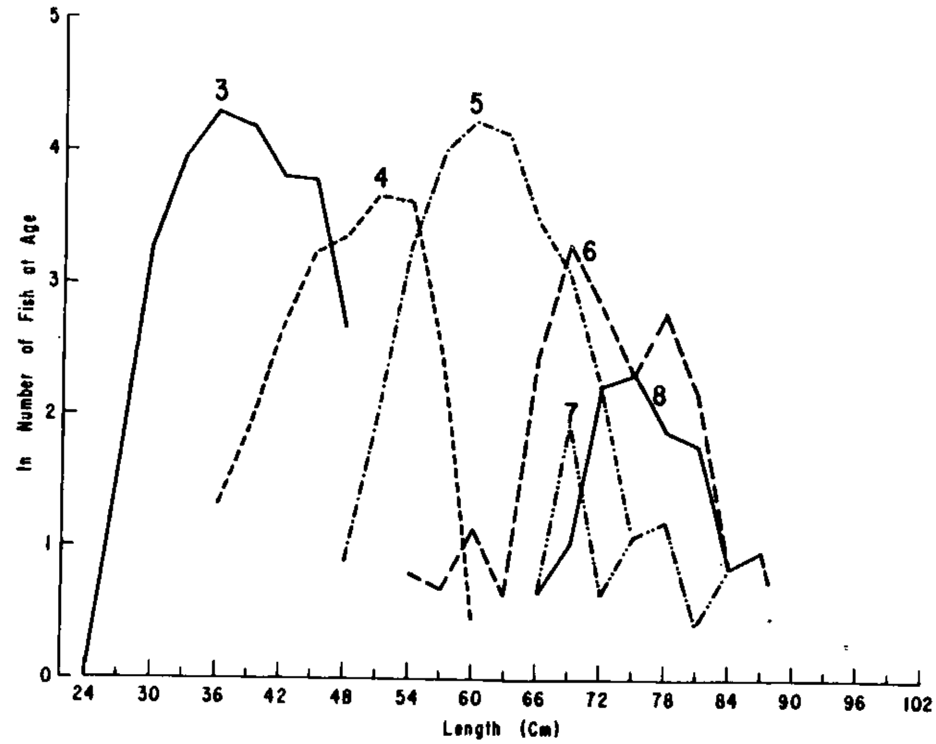

Fig. 6. Key for Div. 1D cod.

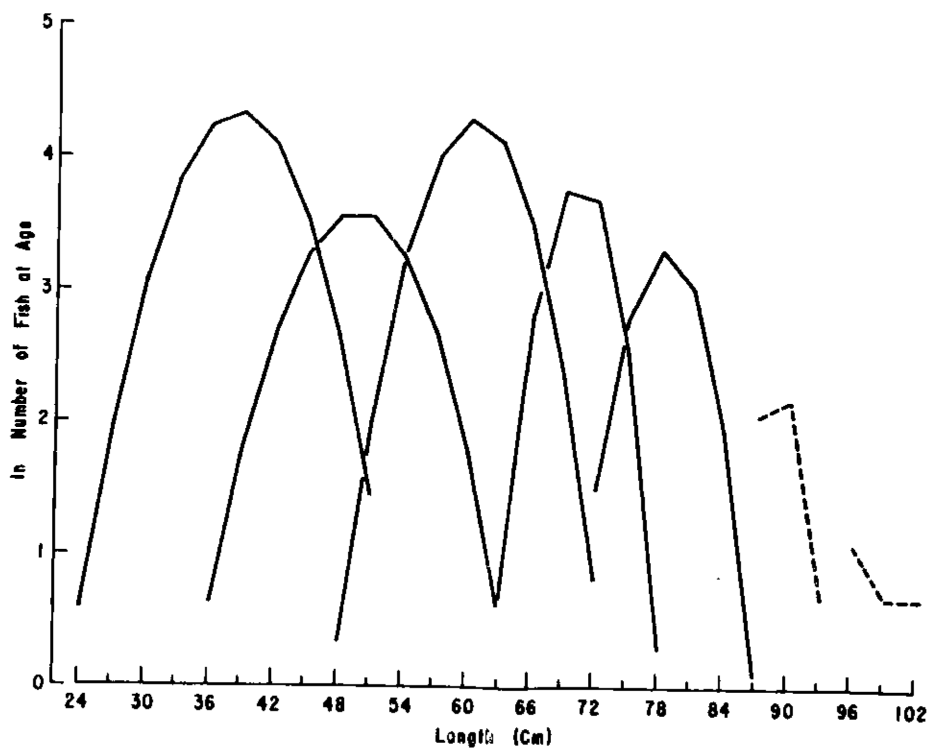

Fig. 7. Enhanced key for Div. 1D cod.

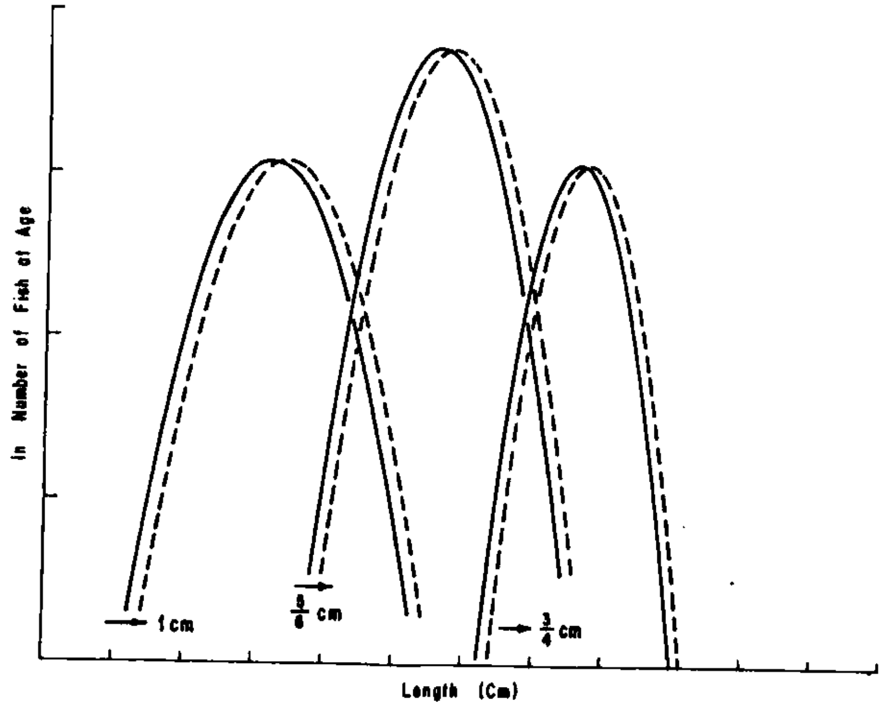

Fig. 8. Effect of growth on the hypothetical key.