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ADDENDUM 1

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Estimates of bias in numbers at age for a selected set of mackerel data

by

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Introduction

Brennan and Palmer (1976) calculated variances for quarterly estimates of number of mackerel caught at age for mackerel taken in the International Commission for the Northwest Atlantic Fisheries (ICNAF) Subarea 5 and Statistical Area 6 from 1972 to 1974, by the German Democratic Republic and Poland. In addition to variances, it is important to investigate the bias of these estimated numbers. The paper referenced above presents calculations of the precision of estimated numbers of mackerel caught at age: these numbers caught at age (N_a) were estimated for a quarter using individual age samples and monthly length distributions submitted to ICNAF. Two techniques were used to make these estimates of N_a: (1) applying a quarterly age-length key to monthly length samples, and (2) applying a monthly age-length key to monthly length samples. In each procedure, the number caught at age for a quarter were then estimated by summing N_a over all months in a particular quarter.

In this paper, biases of the estimated number of mackerel caught at age (Bias (N_a)) were calculated for each of the N_a as estimated by the two techniques described previously. With these biases and previously estimated N_a, a less biased estimate of N_a can be made.

Data base

In 1974, the International Commission for the Northwest Atlantic Fisheries Special Working Group on ICNAF Data Bases requested that individual age and length samples of cod, silver hake, and mackerel taken in ICNAF Subareas 3-6, 1972-1974, be submitted to ICNAF for analysis (ICNAF 1974). From these data, mackerel samples of Poland, 1972 and 1974, quarter 2, ICNAF Division 5Z, and of GDR, 1974, quarter 4, ICNAF Division 5Z plus SA 6 were selected for the analysis presented in this paper. These data sets were chosen because they were the only ones which had more than one age sample for each month for which a length-frequency sample was reported to ICNAF.

There were 12 mackerel age samples from Poland, 1972, quarter 2, ICNAF Division 5Z, representing April and May. There were no age samples for June. Twenty-nine mackerel age samples were submitted for Poland, 1974, quarter 2, ICNAF Division 5Z. All three months were represented. For GDR, 1974, quarter 4, ICNAF Division 5Z plus SA 6, six mackerel age samples were submitted for November and December. The monthly length samples used were those reported to ICNAF.

Bias estimations

The formula used to estimate numbers of mackerel caught at age for a quarter by Brennan and Palmer (1976) is:

$$N_a = \sum_{\ell=1}^{\ell^1} \sum_{h=1}^{k} M_h p_{\ell,h} p_{a/\ell,h}$$

where ℓ^1 = maximum length (cm) of fish sampled

k = number of months in which landings were reported in a quarter

M_h = numbers of fish landed in month h

$M_h = L_h/W_h$ = landings (kg) in month h divided by the mean weight of the fish sampled

W_h was estimated from the monthly length samples according to the expression: $W = (3.96 \times 10^6)\ell^{3.21}$, ℓ = length of fish (Brennan and Palmer 1976)

$p_{\ell,h}$ = percent of length ℓ fish in month h

$p_{a/\ell,h}$ = percent of length ℓ fish at age a in month h

When estimating N_a using a quarterly age-length key, h = 1 for the term $p_{a/\ell,h}$.

The bias of estimated numbers caught at age (Bias (N_a)) is estimated by the equation:

$$\text{Bias } (N_a) = \sum_{\ell=1}^{\ell^1} \sum_{h=1}^{k} (M_h p_{\ell,h} \text{ Bias } (p_{a/\ell,h}))$$

where (Cochran 1953)

$$\text{Bias } (p_{a/\ell,h}) = \frac{N-n}{Nn\bar{x}^2} (p_{a/\ell} S^2_{x\ell} - \rho S_{y\ell} S_{x\ell})$$

and $\frac{N-n}{N}$ = finite population correction and is assumed equal to one

$\bar{x}$ = mean numbers of fish aged at length

n = number of age samples in month h

$S^2_{x\ell}$ = variance of number of fish at length

$$= \sum_{i=1}^{n} \frac{(x_{i\ell} - \bar{x}_\ell)^2}{n-1}$$

$y_{i\ell}$ = numbers of fish of length ℓ and age a in sample i

thereby reducing to

$$\text{Bias } (p_{a/\ell,h}) = \frac{1}{n(n-1)\bar{x}^2} (p_{a/\ell} \sum_{i=1}^{n} (x_{i\ell} - \bar{x}_\ell)^2 - \Sigma(x_{i\ell} - \bar{x}_\ell)(y_{i\ell} - \bar{y}_\ell).$$

M_h, $p_{\ell h}$, k, and ℓ^1 are the same as defined previously.

When calculating Bias ($p_{a/\ell,h}$) for N_a estimated by applying a quarterly age-length key to monthly length samples, h = 1. Bias (N_a)

was estimated for all data sets. A less biased estimate of N_a was computed as $N_a = \hat{N}_a$ -Bias (N_a) for numbers at age estimated using a quarterly age-length key and for N_a estimated using a monthly age-length key (Tables 1-3).

Results and Conclusions

Biases calculated for N_a estimated from quarterly or monthly age-length keys indicate that, in most cases, N_a was overestimated or underestimated by no more than 12% (Tables 1-3). Ninety-two percent of N_aa estimated, using both techniques, were overestimated or underestimated by no more than 5%. N_a estimated using quarterly age-length keys were less biased than the estimates of N_a determined by applying monthly age-length keys to monthly length samples.

Brennan and Palmer (1976) concluded that there was a substantial increase in precision by using monthly age-length keys applied to monthly length samples as compared to quarterly age-length keys applied to monthly length samples. Calculations here show that this more precise estimate of N_a is also the more biased one. However, the difference in the biases of the monthly and quarterly estimates of N_a are considerably less than the difference between the variances.

In estimating the number of fish landed at a given age, a problem is presented by the use of a quarterly estimate of mean weight of a fish. The use of this constant tends to underestimate N_a for younger fish and overestimate N_a for older fish. As a result, the bias computed here is not a true bias since it does not take into account this problem.

Literature Cited

Brennan, J. A., J. E. Palmer. Estimates of variance of age composition of mackerel catches taken in Northwest Atlantic Ocean. Ninth Special Commission Meeting - December 1976, Res. Doc. No. 134, Serial No. 4025.

Cochran, W. G. 1959. Sampling Techniques. John Wiley & Sons, Inc., New York, 329 p.

Table 1. Estimates of numbers at age caught ($\hat{N}_a$) quarterly and associated biases calculated for data of Poland for 1972 Quarter 2, ICNAF Subarea 5Z, (1) when quarterly age-length keys were applied to monthly length samples and (2) when monthly age-length keys were applied to monthly length samples.

(1) Age (a)	($\hat{N}_a$)	Bias ($\hat{N}_a$)	(N_a)	Percent of Bias ($\hat{N}_a$) to $\hat{N}_a$
1	3,998,335	130,887	3,867,448	3.2
2	22,891,010	-123,345	23,014,355	0.5
3	12,236,400	-54,516	12,290,916	0.4
4	4,731,037	26,992	4,704,045	0.6
5	8,286,391	21,257	8,265,134	0.2
6	4,411,935	10,118	4,401,817	0.2
7	2,648,586	-40,380	2,688,966	1.5
8	479,276	19,507	459,769	4.1
9	134,656	1,650	133,006	NA
10	26,171	7,822	18,349	NA

(2) Age (a)	($\hat{N}_a$)	Bias ($\hat{N}_a$)	(N_a)	Percent of Bias ($\hat{N}_a$) to $\hat{N}_a$
1	2,221,090	24,576	2,196,514	1.1
2	23,848,478	-84,734	23,933,412	0.3
3	16,312,029	59,274	16,252,755	0.3
4	5,581,066	119,384	5,461,682	2.1
5	6,443,725	62,111	6,381,614	1.0
6	4,525,434	-123,625	4,649,059	2.7
7	1,611,070	-76,668	1,687,738	4.8
8	340,973	-39,893	380,866	11.7
9	17,244	NA	NA	NA
10	3,053	NA	NA	NA

Table 2. Estimates of numbers at age caught ($\hat{N}_a$) quarterly and associated biases calculated for data of Poland for 1974, Quarter 2, ICNAF Subarea 5Z, (1) when quarterly age-length keys were applied to monthly length samples and (2) when monthly age-length keys were applied to monthly length samples.

(1) Age (a)	($\hat{N}_a$)	Bias ($\hat{N}_a$)	N_a	Percent of Bias ($\hat{N}_a$) to $\hat{N}_a$
2	72,410	1,363	71,047	1.9
3	627,508	-3,315	630,823	0.5
4	660,759	155	660,604	<0.1
5	6,435,785	8,326	6,427,459	0.1
6	1,900,281	-10,141	1,910,422	0.5
7	733,467	-6,155	739,618	0.8
8	254,572	882	253,690	0.3
9	293,331	-5,033	298,364	1.7
10	510,531	13,917	496,614	2.7

(2) Age (a)	($\hat{N}_a$)	Bias ($\hat{N}_a$)	N_a	Percent of Bias (N_a) to $\hat{N}_a$
2	70,099	8,058	62,041	11.4
3	685,288	-10,208	695,496	1.5
4	660,209	6,024	654,185	0.9
5	6,788,543	-2,110	6,790,653	<0.1
6	1,670,230	-21,151	1,691,381	1.3
7	625,670	-9,300	634,970	1.5
8	281,982	2,319	279,663	0.8
9	309,669	20,908	288,761	6.8
10	394,793	19,299	375,495	4.9

Table 3. Estimates of numbers at age caught ($\hat{N}_a$) quarterly and associated biases calculated for data of GDR for 1974 Quarter 4, ICNAF areas 5 and statistical area 6, (1) when quarterly age-length keys were applied to monthly length samples and (2) when monthly age-length keys were applied to monthly length samples.

(1) Age (a)	$\hat{N}_a$	Bias ($\hat{N}_a$)	N_a	Percent of Bias ($\hat{N}_a$) to $\hat{N}_a$
1	21,891,380	91,407	21,799,973	0.4
2	6,449,451	93,423	6,542,874	1.4
3	2,744,519	7,257	2,737,189	0.3
4	5,272,915	13,274	5,286,189	0.3
5	4,037,654	33,800	4,003,884	0.8
6	3,563,452	-34,960	3,598,412	1.0
7	1,540,829	17,563	1,558,392	1.1
8	331,422	-7,730	339,152	2.3
9	71,888	3,081	68,807	4.3

(2) Age (a)	$\hat{N}_a$	Bias ($\hat{N}_a$)	N_a	Percent of Bias ($\hat{N}_a$) to $\hat{N}_a$
1	11,842,023	52,112	11,789,911	0.4
2	4,851,247	3,236	4,854,483	0.1
3	3,161,537	51,948	3,109,589	1.6
4	5,066,678	-142,093	5,208,771	2.8
5	4,242,691	89,846	4,152,845	2.1
6	3,439,271	-60,819	3,500,090	1.8
7	1,430,364	16,501	1,413,863	1.2
8	354,538	-660	355,198	0.2
9	62,382	-3,608	65,990	5.8

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Estimates of Variance of Age Composition of Mackerel Catches taken in Northwest Atlantic Ocean

by

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Abstract

A procedure for estimating the precision of quarterly estimates of numbers caught at age (N_a) was developed and applied to mackerel age samples of Poland (1972-1974) and the German Democratic Republic (GDR) (1974), for mackerel taken in ICNAF Division 5Z and Statistical Area (SA) 6. Age length keys were applied to monthly length samples as reported to ICNAF. Coefficients of variation (C.V.) of N_a were found to be under 50% for age classes 2 to 8, for the cases where there was more than one length sample/quarter, and at least one age sample for those months; most C.V.'s were under 20%. These C.V.'s must be considered underestimates of the true variation, since the absence of individual length samples corresponding to the age samples precluded the inclusion of a sample to sample component of variance for length samples in the formulae. Differences between the N_a estimated for a quarter and the N_a determined by applying monthly age samples to monthly length samples, were found in most cases to be less than 50% of the latter estimates; the variances of the former N_a were in most cases 1½ to 5 times greater than those calculated from monthly age samples applied to monthly length samples. In all cases, the C.V. of the total number landed each quarter was less than 8%.

Introduction

In recent years interest has risen in knowing the precision of routinely calculated statistics in fishery biology. This is due in part to the need of fishery managers to know precisely the consequences of setting catch quotas at various levels. Among the critical statistics

needed for the setting of these quotas in ensuing years, are estimates of numbers caught at age in current catch records.

The International Commission of the Northwest Atlantic Fisheries (ICNAF) has established guidelines on age and length sampling of catches taken in the Northwest Atlantic (ICNAF, 1976). The minimum sampling requirement is one sample per 1000 tons of each species caught in each ICNAF Division (Fig. 1), quarter of year and gear, where a sample consists of 200 fish measured for lengths, and at least one fish aged per centimeter interval. Adherence to these guidelines varied during 1973 and 1974 (Aikenhead, 1976) with less than 60% of the 59 stocks under quota regulation in 1974 adequately sampled in 1974, according to the guidelines. A shortcoming of the guidelines is that since only quarterly summaries of sampling activity are requested, catches from which individual samples are drawn are not specified. Although length samples have routinely been submitted each month, the absence of individual samples prevents estimation of within month vs. between month variability in length distributions. For age sampling, which is summarized by quarter, no estimates of sample to sample or month to month variability can be made.

In 1974, the Special Working Group on ICNAF Data Base requested individual age and length samples of cod, silver hake and mackerel taken in ICNAF Subarea 3-6, 1972-1974 (ICNAF, 1974). The availability of these samples permitted for the first time the estimation of variability of length samples routinely submitted on a monthly basis, and of age-length samples routinely submitted on a quarterly basis. Canada, Poland and the German Democratic Republic (GDR) submitted mackerel age samples, and USSR, Poland, FRG and USA submitted mackerel length samples. Doubleday (1976) examined a selected sample of the 1973 length samples of mackerel and concluded that since the important length groups in the samples from Divisions tended to separate themselves by months or seasons, that time of sampling was a more significant factor than location of sampling for these data. No firm conclusions concerning time vs. spatial sampling were made for age sampling.

The present paper presents a method for estimating the precision of the quarterly estimates of numbers caught at age, using these individual age samples and monthly length frequencies reported to ICNAF. The length samples

examined by Doubleday were not used here since there was practically no overlap by country and area, between length samples and age-length samples[1]. And, since Doubleday noted differences in mean length at age of samples taken within the same month for different countries, it did not seem wise to apply the length samples of one country to the age-length keys of another country. The data examined then included mackerel length and age samples of Poland for 1972-1974 and GDR for 1974, for mackerel taken in ICNAF Division 5Z and Statistical Area (SA) 6 (Figure 1). Canadian mackerel samples were not examined, since the associated catches are minimal, and since monthly length frequencies were not available for study. Estimates of numbers caught at age and variance of numbers caught at age were made on the Polish and GDR data by year, quarter and ICNAF area, according to the way (i.e. by Division, Subarea, etc.) the length samples were submitted to ICNAF. For the years, quarters and ICNAF Subareas where the sampling consisted of length samples from more than one month and at least one age sample for each of those months, estimates of monthly numbers caught at age and associated variances were calculated for comparison with the quarterly estimates.

Estimates of number caught at age

Standard formulae were used in calculating the necessary statistics from the available data. Gulland (1955) has outlined the routine procedure.

Estimates of numbers caught at age (N_a) during a quarter - year - gear - ICNAF Division (henceforth called a stratum) were calculated by the formula:

$$N_a = Np_a \qquad (1)$$

where N = number caught in a stratum, and

p_a = percent caught at age

The estimate of N used was

$$N = L/W \qquad (2)$$

where L = landings (in kilograms) reported in a stratum, and

W = average weight of the fish caught.

[1]Length samples of Poland consisted of the following: 2 samples April, 1972, Statistical Area (SA) 6; 1 sample September, 1973, Division 5Ze; 1 sample October, 1973, SA 6; 1 sample June, 1974, Division 5Ze.

The expression W was estimated from the monthly length samples according to the expression (Moores et al, 1975)

$$W = (3.96 \times 10^{-6})\ \ell^{3.21} \tag{3}$$

where ℓ = length of fish.

The correct procedure of calculating W for each fish and then averaging over all fish to estimate the average weight for each sample, was not used. For simplicity, the average length of the sample was used in (3) to provide an estimate of the average weight of the sample. Although this practice introduced a small bias into the estimates of N_a, the relative precision of the estimates are insensitive to the two procedures.

The percent caught at age (p_a) was estimated by the formula:

$$p_a = \sum_{\ell=1}^{\ell^1} p_{a/\ell}\ \ p_\ell \tag{4}$$

where $p_{a/\ell}$ = percent of age a fish among those of length ℓ,

p_ℓ = percent of length ℓ fish, and

ℓ^1 = maximum length of fish sampled.

Quarterly estimates of $p_{a/\ell}$ and p_ℓ were made from the formulas:

$$p_{a/\ell} = \sum_{i=1}^{n} y_{i\ell} \Big/ \sum_{i=1}^{n} x_{i\ell} \tag{5}$$

$$p_\ell = \sum_{i=1}^{k} M_i\ p_{i\ell} \Big/ \sum_{i=1}^{k} M_i \tag{6}$$

where $y_{i\ell}$ = number of fish of age a and length ℓ in sample i,

$x_{i\ell}$ = number of fish of length ℓ in sample i,

n = number of age samples reported in the stratum,

M_i = landings (kilograms) during month i in the stratum,

$p_{i\ell}$ = percent of length ℓ fish in sample of month i,

k = number of months in which landings were sampled in the stratum.

Combining (2), (5) and (6) then, expression (1) becomes

$$N_a = (L/W) \sum_{\ell=1}^{\ell^1} \left[\left(\sum_{i=1}^{k} M_i\ p_{i\ell} \Big/ \sum_{i=1}^{k} M_i \right) \left(\sum_{i=1}^{n} y_{i\ell} \Big/ \sum_{i=1}^{n} x_{i\ell} \right) \right] \tag{7}$$

For the monthly estimates of N_a also calculated, the same formulas were used. However, n would be the number of age samples reported during a month, and k = 1.

Variance estimates

The variance of N_a ($Var(N_a)$) in (7) was estimated by the following formulas:

$$Var(N_a) = Var(Np_a) \tag{8}$$
$$= p_a^2 Var(N) + N^2 Var(p_a)$$

$$Var(N) = L^2 Var(W) / W^4 \tag{9}$$

Estimates of N and p_a were assumed to be statistically independent, so the covariance term not included in (8) was assumed zero. The coefficient of variation of W was assumed to be 2%, so (9) reduces to

$$Var(N) = .0004 (L/W)^2 \tag{10}$$

The assumption of C.V.=2% seems reasonable; Wilk (pers. comm.) compiled statistics from a length-weight relationship for mackerel collected in the New York Bight area during June 1974 - June 1975, and found the C.V. of mackerel weights between 150-450 grams, to be between 3 and 1%, respectively.

Also,

$$Var(p_a) = \sum_{\ell=1}^{\ell^1} \left[p_{a/\ell}^2 \, Var(p_\ell) + p_\ell^2 \, Var(p_{a/\ell}) + 2 \, p_{a/\ell} \, p_\ell \, Cov(p_{a/\ell}, p_\ell) \right] \tag{11}$$

where $Cov(p_{a/\ell}, p_\ell)$ = covariance of $p_{a/\ell}$, p_ℓ, assumed to be zero here, since it was inestimable.

The variance of p_ℓ ($Var(p_\ell)$) was estimated by (Mendenhall et al., 1971)

$$Var(p_\ell) = \left(\frac{K - k}{K}\right)\left(k\bar{M}^2\right)^{-1} S_a^2 + \left(kK\bar{M}^2\right)^{-1} \sum_{i=1}^{k} M_i^2 p_{i\ell} q_{i\ell} / (m_i - 1) \tag{12}$$

$$S_a^2 = (k-1)^{-1} \sum_{i=1}^{k} M_i^2 (p_{i\ell} - p_\ell)^2 \tag{13}$$

$$\overline{M} = \sum_{i=1}^{k} M_i/k \qquad (14)$$

$$q_{i\ell} = (1.0 - p_{i\ell}) \qquad (15)$$

The finite population correction for the second term of (12) was assumed to be 1.00, since the number of fish measured during a month was negligible compared to the number of fish landed. The term K is the number of months in a quarter (stratum) in which catches were made, k of these having been sampled. All other notation is as defined earlier, in particular following equations (5) and (6).

In deriving the expression for the variance of $p_{a/\ell}$ (Var $(p_{a/\ell})$), it was assumed, as in accordance with the instructions of ICNAF outlined earlier, that each age sample was a subsample of a 200-fish length sample. $Var(p_{a/\ell})$ then was estimated from the formula (Cochran, 1959).

$$Var(p_{a/\ell}) = \frac{N'-n'}{N'} \left(n'(n'-1)\,\overline{x}_\ell^{\,2}\right)^{-1} \sum_{i=1}^{n'} \left(y_{i\ell}^{\,2} - 2\,p_{a/\ell} x_{i\ell} y_{i\ell} + p_{a/\ell}^{\,2} x^2_{i\ell}\right)$$

$$+ \left(n'N'\overline{M}^{\,2}\right)^{-1} \sum_{i=1}^{n'} \frac{(Z_i - m_i)}{Z_i\,(m_i-1)}\; M_i^{\,2} p_{a/\ell\ i}\, q_{a/\ell\ i} \qquad (16)$$

where N' = number of possible samples with length ℓ fish.

n' = number of samples taken with length ℓ fish,

Z_i = number of fish in the $i^{\underline{th}}$ length sample,

$p_{a/\ell i}$ = percent of fish of age a among those of length ℓ, for sample i,

$q_{a/\ell i} = 1.00 - p_{a/\ell i}$ and $\overline{x_\ell} = \sum_{i=1}^{n'} x_{i\ell} / n'$ (17)

Since n' is negligible compared to N', the expression (N'-n')/N' was assumed to be 1.00, and the second term of (15) was ignored.

As in the case for calculating the estimates of monthly numbers caught at age, when estimating the precision of N_a for months, K=k=1 in (12), and N' and n' of (16) are the appropriate entries for the number of possible length sample during the month and the number of those actually taken, respectively.

Using the expressions for N_a and $Var(N_a)$, (1) and (8) respectively, estimates of the "plus or minus" range of N_a were

calculated by the expression

$$\pm \text{ Range} = \pm 2.00 \sqrt{\text{Var}(N_a)} \tag{18}$$

from which 95% confidence intervals around N_a can be set as

$$(N_a - \text{Range},\ N_a + \text{Range}) \tag{19}$$

Also, coefficients of variation (C.V.) of Na were calculated by the standard formula

$$\text{C.V.} = \sqrt{\text{Var}(Na)}/Na \tag{20}$$

Relative precision-quarterly vs. monthly age sampling

The relative precisions of the estimates of N_a calculated from monthly age samples applied to monthly length samples, relative to the estimates of N_a calculated from quarterly age-length keys applied to monthly length samples, were calculated. The assumption was made that age samples were randomly drawn from the monthly landings. The relative precision of that technique, relative to a simple random sample for ages from a quarter's landings, was calculated as:

$$\text{Rel. prec.} = \text{Var } 1(N_a) \Big/ \text{Var } 2(N_a) \tag{21}$$

where Var $1(N_a)$ = Var (N_a) calculated using equation (8), and

$$\text{Var } 2(N_a) = \sum_{i=1}^{k} M_i^2 \text{ Var}(N_{a,i})/M^2,$$

$\text{Var}(N_{a,i})$ = $\text{Var}(N_a)$ calculated for month i, and

k, M_i and M are as explained earlier. The algebraic expression for $\text{Var}(N_{a,i})$ was outlined earlier.

Data Base

Estimates of percent caught at age (p_a) and number caught at age (N_a), and associated variances were made for the data sets cited in Table 1. The stratum breakdown was governed by the method of reporting length samples of Poland and GDR to ICNAF (ICNAF, 1976). Age samples of Poland typically consisted of 100 fish aged, and of GDR 50 fish aged. Also listed in Table 1 are the situations in which a length sample was reported but no associated age sample; a length sample with no reported landings, etc.

Results

Tables 2-19 list the quarterly - ICNAF - year estimates of p_a, N_a etc. using the formulae outlined earlier. The consistent decline during each year in average weight of the samples is a trend well-known to mackerel experts (E. Anderson, pers. comm.). This trend occurred even though the gears used in sampling for lengths were the same throughout each year (re: Table 1). The age distributions of the catches reflected this trend, with a switch to catching younger fish during the latter part of each year.

There was a general consistency in the age distributions of Division 5Z and SA 6 for each year and quarter, for data of Poland, where the length sampling occurred during the same months in each area. However, the lack of consistency between the distribution of the October, 1973 Polish length sample (Table 6), Division 5Z, and the December, 1973 sample from SA 6 (Table 13) illustrates the variability inherent in the length distributions in a quarter.

To examine this month to month variability more thoroughly, for the strata where there was more than one length sample and at least one age sample for each month (Tables 3, 5, 7, 8, 14 and 19), the age length key determined by the age samples drawn during a month, was applied to the monthly length sample, to assess the sensitivity of the quarterly estimates of p_a and N_a to the month to month differences in age samples. The results of these calculations are presented in Tables 20-25. The differences between the quarterly estimates of N_a and monthly N_a calculated by using age-length distributions and summing over months, ranged from 0-900% of the latter. Most of the differences were less than 20% of the latter. However, in view of the fact that not all monthly age samples covered the range of the length samples (cf. Tables 21, 24-25), solid conclusions about the differences, e.g. for which ages these differences are the most severe, cannot be drawn. It can be noted that the age distributions determined by the two methods did not agree; more importantly, the ordering of the age groups, from largest to smallest, did not agree. However, the differences tended to occur with the fourth largest and smaller age groups. Had the entire length range been sampled for ages each month, values of N_a calculated from montly age length keys applied to monthly lengths (Total 1) would increase more than values of N_a calculated from quarterly

age length keys (Total 2). Thus the ratios would decrease. Although this would certainly reduce the ratios to more moderate levels, the resultant differences between the N_a's of Total 1 and those of Total 2 could justify from a statistical point of view, applying monthly age-length keys to monthly distributions.

Calculations of variance of N_a

The variances of the estimated N_a are listed in Tables 2-19. The lower bound of each variance was dictated by the assumed coefficient of variation (2%) of W (equation (10)). Reducing the assumed coefficient to 1% does not substantially alter the results.

In interpreting the results of Tables 2-19 caution must be taken where there was no month to month variance estimated for length samples, and no sample to sample variance for age samples. Firm conclusions can be drawn from the results of Tables 3, 5, 7, 8, 14 and 19 for the ages 2 to 8. For the other ages, there was not always adequate sampling at each length, and case by case conclusions must be made.

For the strata (tables) and age classes noted, the C.V. of N_a was usually under 50%. Most C.V.'s were between 6 and 20%. There was no trend by year or quarter, although the percent at age was inversely related to the C.V., i.e. the larger p_a or N_a, the smaller C.V. For these age classes then, the estimates of N_a can be assumed to be within 100% of the true number caught at age, with most within 40%. Reduction in the confidence of the results (power) would allow smaller ranges to be attached to the N_a, but such calculations were not included here.

Relative precision of quarterly vs. monthly age length keys

Tables 20, 23 and 25 present the relative precision of the N_a calculated by applying monthly age length keys to monthly length samples and summing over months. Only these cases were considered since they were the only strata where there was more than one age sample each month. Although the age sampling was not in strict accordance with the assumptions of a stratified random sample, with stratification according to landings, the calculations suggest a considerable improvement in the precision of N_a. For data of Poland 1972, quarter 2, ICNAF Division 5Z (Table 20) and of GDR, 1974, quarter 4, ICNAF Division 5Z plus SA 6 (Table 25), the gain in precision (equation (21)) for each age averaged 200% of the variance of the N_a estimated from quarterly age-length keys. For the data

of Poland, 1974, quarter 2 (Table 23), the increases in precision were much more substantial, with the relative precision ranging from 2.00 to over 9.36, and an average gain in precision of 300%. Examination of the data generating these results indicated that for each age the p_a's of Table 23 varied more from month to month than did those of Table 20 or Table 25. In no case was the number of samples in each month strictly proportional to the landings each month in the quarter. However, there were more samples in the Polish, 1974, quarter 2 data than in the other data sets. Limitations of the data precluded further investigation of this aspect.

C.V. of numbers caught each quarter

Table 26 presents in summary form the results of Tables 2-19. Quarterly estimates of total numbers caught (N) and associated variances were calculated by year and ICNAF area by summing over ages the entries in the appropriate tables. All C.V.'s of the resultant estimates of numbers caught were less than 8%, a considerable reduction from the range of C.V.'s for individual age groups. This reduction follows algebracially for independent age classes since, for n constituent age classes where

$$\sum_{i=1}^{n} N_i = N, \tag{22}$$

letting $$C.V.(N_i) = a_i\, C, \tag{23}$$

where $$C = C.V.(N), \tag{24}$$

then $$C^2 = Var(N)/N^2 = \sum_{i=1}^{n} Var(N_i)/N^2 = \sum_{i=1}^{n} a_i^2\, C^2 N_i^2/N^2 \tag{25}$$

for at least one $a_i > 1.00$. Moreover, unless a_i is significantly greater than 1.00 for same i, all a_i must satisfy $a_i > 1.00$.

Discussion.

The variance estimates of numbers caught at age presented here have shortcomings. The covariance between $p_{a/\ell}$ and p_ℓ should not be overlooked. Moreover, the absence of individual length samples precludes inclusion of a true within month component of variance in the variance estimate of (12), and prevents the proper weighting of the individual samples. Further improvements in the estimates could

occur if the age samples covered the range of the length samples, since Tables 2-19 indicate that in some situations not all the range of the length distribution was accounted for (cf. Tables 4-6, 9, 13, 16-19).

The consistency between age distributions of catches in Divisions 5Z and SA 6, where the length sampling occurred during the same months, and the lack of consistency between the age distributions based on the October, 1973 Division 5 and the December, 1973 SA 6 age and length samples, suggests the importance of the time factor in sampling, noted by Doubleday. However, the differences between the estimates of numbers caught at age based on applying monthly age-length keys to monthly length samples (Tables 20-25) and those based on applying quarterly age-length keys to monthly lengths for same data (Tables 3, 5, 7, 8, 14, and 19), suggest the magnitude of the growth of mackerel during a quarter in an ICNAF Subarea, and the importance of age sampling where length sampling occurs, and of applying the samples to the catches from which they were drawn.

Despite the shortcomings of the samples used in the estimates of precision of numbers caught at age, the results can still be used to gain insight into the variability of other stock parameters. For example, Pope (1972) has developed a procedure of estimating the coefficients of variation of a year class size N_i, where the N_i are derived from a cohort analysis approach. The coefficient of variation of the fishing mortality on a year class can also be easily derived. Such information would be of immediate use to planners and managers.

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Table 1. Mackerel landings (MT) reported to ICNAF by area, month, sampling gear and country. Notation in parenthesis is as follows: (1) indicates length sample was reported to ICNAF for the associated landings; (2) indicates age samples were taken from the associated landings.

Country	Year	Sampling gear	Area	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Poland	1972	Otter trawl	5Z	5764	-	1805 (1,2)	1279 (1,2)	3789 (1,2)	4322	2156	3572	2180 (1,2)	3585	9878	23018
	1972	Otter trawl	6	21482	14838	19588 (1,2)	14912 (1,2)	7938	40	-	-	-	-	-	1715
	1973	Midwater trawl	5Z	9960	12747 (2)	14135 (2)	9847	13292 (1)	457	819 (1,2)	161 (1,2)	1385 (1,2)	7815 (1,2)	16936	13175
	1973	Midwater trawl	6	4646 (2)	1406 (2)	719 (2)	2832 (1,2)	99	-	-	-	-	-	-	6223 (1,2)
	1974	Pelagic trawl	5Z	7 (1,2)	-	6627 (1,2)	8001 (1,2)	4479 (1,2)	1114 (1,2)	- (2)	86 (2)	170* (1)	2134 (1,2)	9159	6765
	1974	Pelagic trawl	6A	20018 (1,2)	11501 (1,2)	354 (1,2)	959 (1,2)	125	-	-	-	-	-	-	9845
	1974	Pelagic trawl	6B	8689	4819 (1,2)	130	- (2)	-	-	-	-	-	-	-	796
GDR	1974	Midwater trawl	5Z	1640	-	1002	1186	186	-	-	8	26	5	2773	3269
			6	15361 (1,2)	13275	11586 (1)	4785 (1,2)	-	-	-	-	-	-	- (1,2)	4461 (1,2)

*Length sample did not cover range of age samples, and thus estimates of percent caught at age, etc. were not calculated.

Table 2. Data of Poland for 1972, quarter 1, ICNAF Div. 5Z. Length sample from March. Total landings = 1,805 tons. Average weight = .410 kg. 100% of length.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance [$Var(N_a)$]	± Range	Coef. of Var. (C.V.)
3	.152	671,813	1.45×10^{10}	240,850	.18
4	.088	388,155	6.82×10^{9}	165,222	.21
5	.508	2,234,971	2.44×10^{10}	312,896	.07
6	.157	691,185	5.35×10^{9}	146,306	.11
7	.036	158,803	2.94×10^{9}	108,365	.34
8	.011	49,705	2.32×10^{8}	30,480	.31
9	.029	129,431	4.98×10^{8}	44,651	.17
10	.018	79,849	7.60×10^{8}	55,159	.35

Table 3. Data of Poland for 1972, quarter 2, ICNAF Div. 5Z. Length samples from April and May. Total landings = 5,068 tons. Average weight = .441 kg. 100% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance [$Var(N_a)$]	± Range	Coef. of Var. (C.V.)
2	.066	72,410	1.87×10^{9}	86,620	.60
3	.055	627,508	7.27×10^{9}	171,660	.14
4	.058	660,759	9.89×10^{9}	198,908	.15
5	.560	6,435,785	7.37×10^{10}	543,048	.04
6	.165	1,900,281	2.81×10^{10}	335,512	.09
7	.064	733,467	1.15×10^{10}	214,022	.15
8	.022	254,572	2.31×10^{9}	96,121	.19
9	.026	293,331	3.31×10^{9}	115,053	.20
10	.044	510,531	6.31×10^{9}	158,881	.16

Table 4. Data of Poland for 1972, quarter 3, ICNAF Div. 5Z. Length sample from September; one age sample available. Landings = 2,180 tons. Average weight = .375 kg. 93% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance [$Var(N_a)$]	± Range	Coef. of Var. (C.V.)
2	.055	29,074	1.45×10^8	24,226	.41
3	.294	1,709,542	1.17×10^9	68,381	.02
4	.463	2,693,851	2.90×10^9	107,754	.02
5	.111	642,735	1.86×10^9	86,231	.07
6	.012	71,823	1.18×10^8	21,738	.15
7	.043	250,033	1.12×10^9	67,091	.13
8	.006	34,888	1.78×10^8	26,598	.38

Table 5. Data of Poland for 1973, quarter 3, ICNAF Division 5Z. Length samples from July - September. Total landings = 2,365 MT. Average weight = .301 kg. 99% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.349	2,740,106	1.38×10^{10}	234,990	.04
2	.083	648,995	1.10×10^{10}	209,975	.16
3	.032	252,260	5.80×10^9	152,374	.30
4	.175	1,374,454	1.70×10^{10}	260,957	.07
5	.098	774,010	5.88×10^9	153,323	.10
6	.142	1,114,214	1.18×10^{10}	217,621	.10
7	.072	566,587	7.11×10^9	168,615	.15
8	.031	241,007	1.13×10^9	67,239	.14
9	.008	60,391	3.41×10^8	36,908	.31
10	.001	9,632	1.50×10^8	24,550	.27

Table 6. Data of Poland for 1973, quarter 4, ICNAF Division 5Z. Length sample from October; one age sample available. Total landings = 7,815 MT. Average weight = .200 kg. 90% of length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.471	18,377,460	1.35×10^{11}	735,098	.02
2	.079	3,097,418	3.84×10^9	123,897	.02
3	.134	5,230,774	1.23×10^{11}	701,782	.07
4	.081	3,166,725	4.91×10^{10}	443,342	.07
5	.085	3,326,852	5.42×10^{10}	465,759	.07
6	.038	1,499,537	1.10×10^{10}	209,935	.07
7	.008	311,974	4.91×10^9	140,076	.22

Table 7. Data of Poland for 1974, quarter 1, ICNAF Division 5Z. Length samples from January and March. Total landings - 6,634 MT. Average weight = .192 kg. 100% of length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.015	526,061	7.62×10^{10}	552,000	.52
2	.504	17,455,570	2.20×10^{12}	2,964,658	.08
3	.215	7,435,068	1.64×10^{12}	2,527,686	.17
4	.128	4,429,677	5.89×10^{11}	1,535,321	.17
5	.076	2,625,458	2.84×10^{11}	1,065,783	.20
6	.030	1,037,005	8.36×10^{10}	578,112	.27
7	.028	960,479	9.53×10^{10}	617,299	.32
8	.004	151,630	6.55×10^9	161,877	.53

Table 8. Data of Poland for 1974, quarter 2, ICNAF Division 5Z. Length samples from April - June. Total landings = 13,590 MT. Average weight = .227 kg. 100% of length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.067	3,998,335	4.38×10^{11}	1,323,468	.17
2	.383	22,891,010	1.74×10^{12}	2,637,992	.06
3	.204	12,236,400	1.01×10^{12}	2,014,835	.08
4	.079	4,731,037	1.88×10^{11}	866,426	.09
5	.138	8,286,391	2.17×10^{11}	930,760	.06
6	.074	4,411,935	8.48×10^{10}	582,550	.07
7	.044	2,648,586	7.00×10^{10}	529,241	.10
8	.008	479,276	1.04×10^{10}	203,572	.21
9	.002	134,656	2.16×10^{9}	92,950	.35
10	.0004	26,171	1.63×10^{9}	80,700	1.54

Table 9. Data of Poland for 1974, quarter 4, ICNAF Division 5Z. Length sample from October; one age sample available. Total landings = 2,134 MT. Average weight = .130 kg. 86% of the reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.261	4,304,072	7.41×10^{9}	172,163	.02
2	.216	3,559,572	5.07×10^{9}	142,383	.02
3	.230	3,778,298	5.71×10^{9}	151,132	.02
4	.042	686,796	1.89×10^{8}	27,472	.02
5	.033	538,746	1.42×10^{9}	75,424	.07
6	.030	487,300	1.16×10^{9}	68,222	.07
7	.003	42,803	2.44×10^{7}	9,872	.12
8	.046	749,668	2.75×10^{9}	104,953	.07

Table 10. Data of Poland for 1972, quarter 1, ICNAF Area 6. Length sample from March. Total landings = 19,588 MT. Average weight = .417 kg. 100% of length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
2	.008	378,260	3.18×10^{10}	356,549	.45
3	.109	5,128,301	1.38×10^{12}	2,353,880	.23
4	.080	3,757,613	3.99×10^{11}	1,262,794	.17
5	.575	27,030,880	2.97×10^{12}	3,447,496	.06
6	.176	8,259,249	8.95×10^{11}	1,892,688	.11
7	.036	1,669,581	5.72×10^{11}	1,512,755	.45
8	.007	353,602	3.13×10^{10}	354,088	.50
9	.006	266,257	3.38×10^{10}	367,892	.69
10	.001	46,959	6.89×10^{8}	52,500	.56

Table 11. Data of Poland for 1972, quarter 2, ICNAF Area 6. Length sample from April. Total landings - 14,912 MT. Average weight - .355 kg. 100% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.064	2,678,260	9.09×10^{9}	190,725	.03
2	.024	1,022,824	2.24×10^{10}	299,019	.15
3	.124	5,212,419	8.71×10^{11}	1,866,428	.18
4	.099	4,148,004	1.04×10^{12}	2,038,564	.25
5	.373	15,674,670	1.19×10^{12}	2,181,746	.07
6	.207	8,717,272	1.16×10^{12}	2,157,880	.12
7	.071	2,965,717	9.53×10^{10}	617,397	.10
8	.016	677,867	1.03×10^{11}	641,841	.47
9	.009	366,471	6.67×10^{9}	163,300	.22
10	.013	543,971	7.01×10^{9}	167,451	.15

Table 12. Data of Poland for 1973, quarter 2, ICNAF Area 6. Length sample from April; one age sample available. Total landings = 2,832 MT. Average weight = .187 kg. 100% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.077	1,158,363	1.78×10^{10}	267,178	.12
2	.900	13,537,650	7.33×10^{10}	541,506	.02
3	.019	284,848	3.61×10^{9}	120,178	.21

Table 13. Data of Poland for 1973, quarter 4, ICNAF Area 6. Length sample from December. Total landings = 6,223 MT. Average weight = .381 kg. 99% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.133	2,173,194	1.15×10^{11}	679,106	.16
2	.113	1,840,668	2.51×10^{11}	1,001,151	.27
3	.086	1,408,045	2.34×10^{11}	967,875	.34
4	.200	3,267,235	1.21×10^{11}	788,213	.12
5	.131	2,134,152	2.75×10^{11}	1,049,696	.25
6	.250	4,072,322	1.56×10^{11}	789,606	.10
7	.054	873,849	9.35×10^{10}	873,849	.35
8	.024	399,398	5.13×10^{10}	453,150	.57
9	.000	-	-	-	-
10	.002	32,604	1.09×10^{9}	66,292	1.02

Table 14. Data of Poland for 1974, quarter 1, ICNAF Division 6A. Length samples from January - March. Total landings = 31,873 MT. Average weight = .262 kg. 99% of the reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.003	416,047	1.18×10^{10}	217,449	.26
2	.334	40,689,230	1.27×10^{13}	7,128,200	.09
3	.180	21,881,500	1.06×10^{13}	6,515,532	.15
4	.104	12,687,910	3.74×10^{12}	3,870,148	.15
5	.121	14,690,190	4.57×10^{12}	4,273,652	.15
6	.116	14,140,580	4.32×10^{12}	4,161,340	.15
7	.108	13,212,080	2.54×10^{12}	3,190,354	.12
8	.013	1,642,423	2.58×10^{11}	1,007,748	.31
9	.004	526,031	1.15×10^{11}	678,984	.63
10	.003	329,697	8.17×10^{10}	571,728	.86

Table 15. Data of Poland for 1974, quarter 2, ICNAF Division 6A. Length sample from April. Total landings = 959 MT. Average weight = .169 kg. 100% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.009	53,802	1.55×10^{8}	24,933	.23
2	.385	2,181,335	1.59×10^{10}	252,297	.06
3	.273	1,551,165	1.99×10^{10}	282,163	.09
4	.084	478,616	7.34×10^{9}	171,313	.18
5	.084	475,467	3.91×10^{9}	125,189	.13
6	.090	512,989	5.25×10^{9}	144,940	.14
7	.059	334,056	2.20×10^{9}	93,815	.14
8	.010	59,310	6.78×10^{8}	52,079	.44
9	.004	22,184	1.02×10^{8}	20,174	.45
10	.0004	2,269	2.74×10^{6}	3,311	.73

Table 16. Data of Poland for 1974, quarter 1, ICNAF Division 6B. Length sample from February; one age sample available. Total landings = 4,819 MT. Average weight = .194 kg. 99% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
2	.463	11,531,800	5.32×10^{10}	461,272	.02
3	.236	5,863,759	5.41×10^{10}	465,170	.02
4	.117	2,915,661	4.17×10^{10}	408,193	.07
5	.076	1,890,572	3.15×10^{10}	354,703	.09
6	.021	534,834	5.15×10^{9}	143,511	.13
7	.073	1,815,945	2.90×10^{10}	340,701	.09

Table 17. Data of GDR for 1974, quarter 1, ICNAF Areas 5 and 6. Length samples from January and March. Total landings = 29,589 MT. Average weight = .407 kg. 98% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.124	9,053,819	3.93×10^{11}	1,254,534	.07
2	.105	7,664,140	7.83×10^{11}	1,769,957	.12
3	.082	5,948,091	1.36×10^{12}	2,335,210	.20
4	.160	11,631,480	1.30×10^{12}	2,283,249	.10
5	.196	14,280,150	5.33×10^{12}	4,617,003	.16
6	.246	17,901,380	3.08×10^{12}	3,514,025	.10
7	.052	3,781,977	1.50×10^{12}	2,445,548	.32
8	.019	1,347,294	4.39×10^{10}	419,180	.16

Table 18. Data of GDR for 1974, quarter 2, ICNAF Areas 5 and 6. Length sample from April. Total landings = 5,971 MT. Average weight = .362 kg. 98% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (Na)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.180	2,979,139	3.99×10^{10}	399,693	.07
2	.200	3,309,667	1.23×10^{11}	702,606	.11
3	.095	1,572,970	7.16×10^{10}	535,074	.17
4	.073	1,204,716	2.83×10^{10}	336,951	.14
5	.097	1,599,329	7.39×10^{10}	544,040	.17
6	.176	2,913,914	5.87×10^{10}	484,597	.08
7	.092	1,526,920	3.78×10^{10}	388,690	.13
8	.030	495,298	2.65×10^{10}	325,737	.33
9	.010	161,036	7.97×10^{9}	178,603	.55
10	.009	144,520	6.28×10^{9}	158,533	.55
11	.008	136,262	4.17×10^{9}	129,111	.47
12	.005	90,841	8.32×10^{9}	182,425	1.00

Table 19. Data of GDR for 1974, quarter 4, ICNAF Areas 5 and 6. Length samples from November and December. Total landings = 10,503 MT. Average weight = .223 kg. 97% of reported length distribution accounted for.

Age (a)	Percent at age (p_a)	No. at age (N_a)	Variance ($Var(N_a)$)	± Range	Coef. of Var. (C.V.)
1	.464	21,891,380	4.21×10^{12}	4,105,122	.09
2	.137	6,449,451	5.29×10^{11}	1,454,361	.11
3	.058	2,744,519	2.10×10^{11}	916,963	.17
4	.111	5,272,915	5.02×10^{11}	1,417,579	.13
5	.086	4,037,684	3.05×10^{11}	1,104,281	.14
6	.076	3,563,452	3.37×10^{11}	1,160,979	.16
7	.033	1,540,829	7.92×10^{11}	562,717	.18
8	.007	331,422	2.31×10^{10}	303,808	.46
9	.002	71,888	4.25×10^{9}	130,366	.91

Table 20. Estimates of percent at age, number at age, etc. for Poland, 1972, quarter 2, ICNAF Division 5Z data. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2 = N_a from quarterly age-length keys, from Table 3. Rel. prec. = relative precision of equation (21).

Age (a)	P_a			N_a			Total 1	Total 2-Total 1 / Total 1	Rel. prec.
	April	May	June	April	May	June			
2	.007	.006		20803	49296		70099	.03	1.42
3	.062	.059		185654	499634		685288	.08	1.13
4	.046	.062		138657	521552		660209	.00	1.72
5	.489	.627		1473682	5314861		6788543	.05	2.59
6	.200	.126		603741	1066489		1670230	.14	2.90
7	.095	.040		287716	337954		625670	.17	2.31
8	.016	.028		48087	233895		281982	.10	1.18
9	.034	.024		101228	208441		309669	.05	1.69
10	.051	.028		153338	241455		394793	.29	3.29
Total landings reported (MT)	1.000	1.000							
	1,279	3,789							
# age samples	6(5Zw)	6(5Z)							

Table 21. Estimates of percent at age, number at age, etc. for Poland, 1973 quarter 3, ICNAF Division 5Z data. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2 = N_a based in quarterly age-length keys, from Table 5.

Age (a)	P_a			N_a			Total 1	Total 2-Total 1 / Total 1
	July	Aug.	Sept.	July	Aug.	Sept.		
1	-	.442	.301	-	281520	1351520	1633040	.68
2	.195	.083	.041	533305	52563	187697	773565	.15
3	.010	.067	.005	27349	43006	23741	94096	1.68
4	.256	.151	.179	699970	96294	804865	1601129	.14
5	.060	.109	.101	164515	69095	453679	687289	.13
6	.113	.112	.146	308387	70543	657366	1036295	.08
7	.054	.024	.096	147933	15085	431659	594677	.06
8	.007	.009	.039	19828	5957	177628	203413	.19
9	-	-	.007	-	-	31392	31392	.92
10	-	-	.002	-	-	9633	9633	.00
Total Reported landings (MT)	.695	.997	.917					
	819	161	1,385					
# age samples	1(5Ze)	3(5Ze)	3(5Ze)					

Table 22. Estimates of percent at age, number at age, etc. for Poland, 1974 quarter 1, ICNAF Division 5Z data. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2 = N_a from quarterly age-length keys, from Table 7.

Age (a)	p_a			N_a			Total 1	Total 2-Total 1 / Total 1
	Jan.	Feb.	Mar.	Jan.	Feb.	Mar.		
1	.001	-	.014	27	-	485702	485729	.08
2	.444	-	.439	12176	-	15199690	15211866	.15
3	.202	-	.251	5545	-	8685558	8691103	.14
4	.100	-	.136	2745	-	4724704	4727449	.06
5	.130	-	.075	3586	-	2606863	2610449	.01
6	.013	-	.042	365	-	1444653	1445018	.28
7	.038	-	.037	1055	-	1269930	1270985	.24
8	.053	-	.005	1466	-	172958	174424	.13
Total	.981		.994					
Reported landings (MT)	7		6,627					
#age samples	1(5Zw)		2(5Zw)					

Table 23. Estimates of percent at age, number at age, etc. for Poland, 1974, quarter 2, ICNAF Division 5Z data. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2= N_a from quarterly age-length keys, from Table 8. Rel. prec.= relative precision of equation (21).

Age (a)	p_a			N_a			Total 1	Total 2-Total 1 / Total 1	Rel. prec.
	April	May	June	April	May	June			
1	.021	.003	.260	856342	50544	1314204	2221090	.80	9.36
2	.509	.185	.079	20565600	2884052	398426	23848478	.04	4.42
3	.254	.306	.052	10273660	4756628	1281741	16312029	.25	3.12
4	.071	.133	.124	2882667	2069776	628623	5581066	.15	2.00
5	.050	.196	.275	2007139	3050519	1386067	6443725	.29	4.50
6	.046	.133	.121	1846963	2066881	611590	4525434	.03	2.60
7	.016	.037	.074	664131	574664	372275	1611070	.64	3.42
8	.006	.002	.011	254624	31135	55214	340973	.41	2.38
9			.003			17244	17244	6.88	N/A
10			.001			3053	3053	7.67	N/A
Total	.973	.995	1.000						
Reported landings (MT)	8,001	4,479	1,114						
#age samples	3(5Zw)	5(5Ze)	21(5Z)						

Table 24. Estimates of percent at age (p_a), number at age (N_a), etc. for Poland, 1974, quarter 1, ICNAF Division 6A. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2 = N_a from quarterly age-length keys, from Table 14.

Age (a)	p_a			N_a			Total 1	Total 2-Total 1 / Total 1
	Jan.	Feb.	Mar.	Jan.	Feb.	Mar.		
1	-	.002	-	-	146785	-	146785	
2	.274	.395	.067	17974500	23173820	87784	41236104	1.83
3	.235	.248	.298	15403760	14583140	392518	30379418	.01
4	.097	.237	.148	6376388	13940700	194884	20511972	.18
5	.116	.031	.182	7617891	1800055	239819	9657765	.38
6	.098	.007	.182	6406421	422741	239565	7068727	.52
7	.110	.056	.082	7199899	3296323	208370	10604592	1.00
8	.016		.011	1075814		14414	1090228	.25
9	.005		.005	327832		6258	334090	.51
10	.0005		.003	32783		3505	36288	.57
								8.09
Total	.952	.739	.978					
Reported landings (MT)	20,018	11,501	354					
#age sample	3(6A)	1(6A)	2(6A)					

Table 25. Estimates of percent at age (p_a), number at age (N_a), etc. for GDR, 1974, quarter 4, ICNAF Division 5Z+6 data. Calculations are the result of applying monthly age-length keys to monthly length samples. Total 2 = N_a from quarterly age-length keys, from Table 19. Rel. prec. = relative precision of equation (21).

Age (a)	p_a			N_a			Total 1	Total 2-Total 1 / Total 1	Rel. prec.
	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.			
1		.241	.250		2391978	9450045	11842023	.85	11.79
2		.179	.081		1780049	3071198	4851247	.33	1.96
3		.109	.055		1082272	2079265	3161537	.13	1.46
4		.159	.092		1574248	3492450	5066678	.04	2.54
5		.117	.082		1160652	3082039	4242691	.05	4.12
6		.116	.060		1154979	2284292	3439271	.04	2.41
7		.059	.022		581671	848693	1430364	.08	3.58
8			.009			354538	354538	.07	N/A
9		.005	.001		51740	10642	62382	.15	N/A
Total		.985	.650						
Reported landings (MT)		2,773	7,730						
# age samples		3(5Ze)	3(5Zw,6B)						

Table 26. Summary of statistics presented in Tables 2-19. Also, calculated coefficients of variation of estimated total number of fish caught by year, quarter, ICNAF area and country, for the months from which length samples were available for analysis (Table 1).

Country	Year	Quarter	Area	No. caught	Variance	Coef. of Var.
Poland	1972	I	5Z	4,403,412	5.55×10^{10}	.05
		II	5Z	11,488,644	14.44×10^{10}	.03
		III	5Z	5,431,946	$.75 \times 10^{10}$	.02*
	1973	III	5Z	7,781,656	7.40×10^{10}	.03
		IV	5Z	35,010,740	38.11×10^{10}	.02*
	1974	I	5Z	34,620,948	497.47×10^{10}	.06
		II	5Z	59,843,797	376.20×10^{10}	.03
		IV	5Z	14,147,255	2.37×10^{10}	.01*
	1972	I	6	46,890,702	631.36×10^{10}	.05
		II	6	42,007,475	450.45×10^{10}	.05
	1973	II	6	14,980,861	9.47×10^{10}	.02*
		IV	6	16,168,863	129.79×10^{10}	.07
	1974	I	6A	120,225,688	$3,893.65 \times 10^{10}$	.05
		II	6A	5,671,193	5.54×10^{10}	.04
		I	6B	24,552,571	21.47×10^{10}	.02*
GDR	1974	I	5+6	71,608,331	$1,344.63 \times 10^{10}$	.05
		II	5+6	16,134,612	48.97×10^{10}	.04
		IV	5+6	45,903,540	691.24×10^{10}	.06

*Only one age sample and one length sample available for analysis.

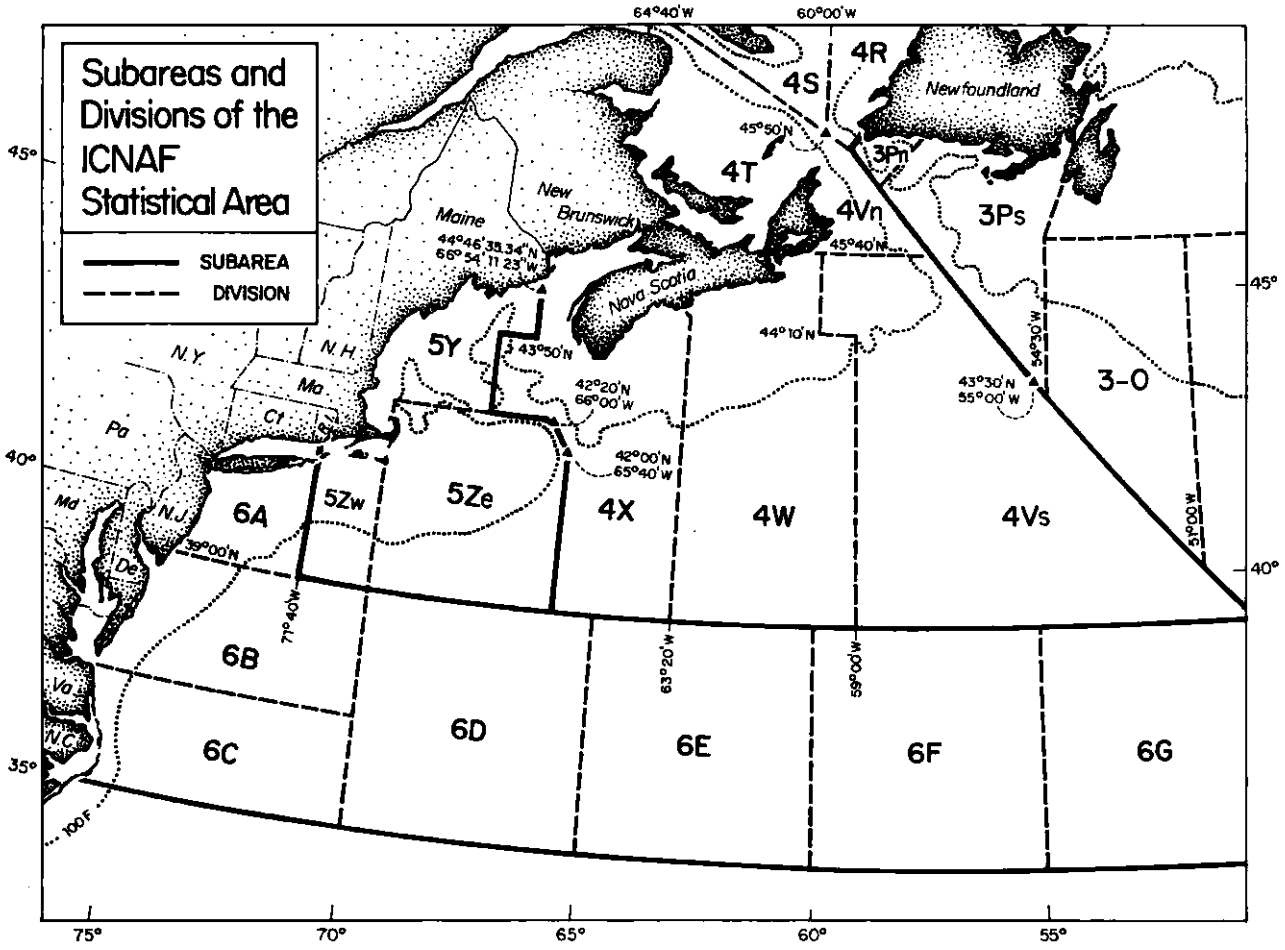

Fig. 1. Northwest Atlantic Ocean partitioned into ICNAF Subareas and Divisions.