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ANNUAL MEETING - JUNE 1976The effect of random fluctuations on a hypothetical fishery

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Abstract

The Schaefer (1954) stock-production model was stochastically simulated numerous times for several combinations of the ratio of maximum equilibrium catch to maximum population size, the ratio of initial population size to maximum population size, the variance of deviations from the model and the autocorrelation of the deviations. The rate of fishing mortality is set equal to the level that will produce the maximum equilibrium catch ( $CE_{max}$ ) for the deterministic version of the Schaefer model. The results indicate that the long-term average catch is likely to be below  $CE_{max}$  for realistic values of the variance of the deviations from the model. The probability of a population declining to extremely low levels increases as the autocorrelation of the deviation increases.

Introduction

Most fisheries models do not explicitly incorporate random variations in population parameters resulting from fluctuations of the environment. Researchers generally assume that the time-span over which data were collected to estimate model parameters is long enough so that these estimates are adequate for representing the fishery at average environmental conditions. If the fishery is conducted so as to maintain fishing mortality at the level corresponding to the maximum equilibrium catch,  $CE_{max}$ , then it is usually assumed that the long-term average catch will be about  $CE_{max}$ . The validity of this assumption is the topic considered here.

Method

Assume the following equation describes the population dynamics of an exploited species:

$$\frac{dP}{dt} = (a - bP) \cdot P \cdot A - F \cdot P \quad (1)$$

where  $P$  is a measure of population size,  $F$  is the instantaneous fishing mortality rate,  $a$  and  $b$  are species specific model parameters, and  $A$  is a random variable with an expected value of 1. Equation (1) is identical to the Schaefer (1954) model for  $A=1$ , where  $CE$  is the catch that results in  $dP/dt = 0$  and  $CE_{max}$  and  $P_{max}$  are the maximum levels of  $CE$  and  $P$ . For the Schaefer model,  $CE_{max}$  occurs when  $P$  is half its maximum level ( $P_{msy}$ ) and  $F_{msy}$  is the fishing mortality rate that will maintain the population at this level.

These terms describing the hypothetical fishery are related in the following manner:

$$a = 4 CE_{\max} / P_{\max} \quad (2)$$

$$b = a / P_{\max} \quad (3)$$

$$P_{\text{msy}} = P_{\max} / 2 \quad (4)$$

$$F_{\text{msy}} = a / 2 \quad (5)$$

If A and F are assumed constant over an interval of time (from i to i+1), then

$$P_{i+1} = \frac{\alpha}{\beta} \left[ 1 + e^{-\alpha} \frac{(\alpha/\beta - P_i)}{P_i} \right]^{-1} \quad (6)$$

$$C \approx F (P_{i+1} + P_i)/2 \quad (7)$$

where  $\alpha = a/A - F$ ,  $\beta = bA$  and C is catch during the interval. The length of the interval can be reduced until Equation (7) reaches an acceptable level of accuracy.

The random variable A may be assumed to represent a biotic response to the fluctuation of an environmental factor, W. According to Ricker (1958, p. 232), the relationship between A and W should be multiplicative since for a specific environmental anomaly each individual in the population (not a fixed number) may be affected. This being the case, then the relationship between  $\log_e A$  and W should be additive (or linear). It is reasonable to assume that W is normally distributed, particularly if it is the average state of an environmental factor over some time (for example, annual average temperature or annual average wind velocity). Therefore, it is appropriate to assume that A is log normally distributed with mean 1. The range of the log normal distribution is from 0 to  $\infty$  which is the logical range of A.

Let  $X = \log_e A$  (a linear function of W) be normally distributed with mean  $\mu$  and variance  $\sigma^2 (X \sim N(\mu, \sigma^2))$ . If X is autocorrelated then

$$X_{i+1} \sim N(\mu + \rho(X_i - \mu), \sigma^2 (1 - \rho^2)) \quad (8)$$

where  $\rho$  is the first order autocorrelation coefficient of X (correlation between  $X_{i+1}$  and  $X_i$  where the subscript refers to time). Equation (8) is based on the conditional probability density function of a bivariate normal distribution (Hogg and Craig, 1970). The mean and variance of A ( $E(A)$  and  $V(A)$ ) where  $\rho = 0$  or where the previous value of A is unknown are as follows:

$$E(A) = e^{\mu + \sigma^2/2} \quad (9)$$

$$V(A) = e^{\sigma^2 + 2\mu} (e^{\sigma^2} - 1) \quad (10)$$

(Brownlee, 1965). Since  $E(A) = 1$ , then  $\mu = -\sigma^2/2$  and  $V(A) = e^{\sigma^2} - 1$ . Therefore it is possible to generate an autocorrelated log normal random variable with mean 1 for use in simulating a fishery described by Equation (1) by the following transformation:

$$A_{i+1} = \exp \left[ Z_{i+1} (\sigma \sqrt{1 - \rho^2}) - \sigma^2/2 + \rho(X_i + \sigma^2/2) \right] \quad (11)$$

where  $Z_{i+1}$  is the standard normally distributed ( $N(0,1)$ ) random variable used to calculate A for time period i+1 and  $X_i$  is the natural log of A from the previous time period.

Pseudo standard normal random numbers can be generated by a computer algorithm described by IBM (1970). It is possible to generate random numbers because of the modular arithmetic used by digital computers. If m is the largest positive integer number that can be accommodated by a single computer register, then the product of any multiplication exceeding m results in some residual being stored in another register. For certain multipliers the residuals have the property of uniform random numbers. The average of several of these converges to a normal distribution.

Using computer generated random numbers and Equations (6), (7) and (11), a hypothetical fishery described by Equation (1) was simulated. Since each simulation was stochastic in nature, it was repeated (with different random numbers) numerous times to estimate the probability of particular outcomes. Several values of the ratio of  $CE_{max}$  to  $P_{max}$  ( $R$ ),  $\sigma^2$ ,  $\rho$  and the initial condition of  $P$  relative to  $P_{max}$  ( $P_0$ ) were considered (Table 1). Note that the shape of  $dP/dt$  as a function of  $P$  is uniquely determined by  $R$  and that  $\mu$  is determined by  $\sigma^2$ . The probability of  $A > 2$  for each value of  $\sigma^2$  is also given. For each combination of  $R$ ,  $\sigma^2$ ,  $\rho$  and  $P_0$ , Equation (1) was simulated 100 times for 25 time intervals (years, for example).  $F$  was always set equal to  $F_{msy}$ . In order to calculate  $A$  for the first interval simulated, a seed value of  $X(X_0)$  was needed for use in Equation (11).  $X_0$  was selected randomly for  $P_0 = 0.50$ . When  $P_0 = 0.25$ ,  $X_0 = \log_e A_0$  was set so that  $dP/dt = 0$  for  $F_{msy}$ ,  $P_0$  and  $A_0$ . This implies that the stock had declined as a result of unfavorable environmental factors and that for  $\rho \neq 0$ , conditions will tend to remain unfavorable at the onset of the simulations. Clearly, for  $A=1$  ( $\sigma^2=0$ ) catch converges to  $CE_{max}$  for any value of  $P_0$  (between 0 and  $P_{max}$ ). The results of these simulations were intended to demonstrate the likelihood of this outcome when  $A$  is a random variable.

#### Results and Discussion

The catch relative to  $CE_{max}$  ( $C_r$ ) during each time interval was used to construct a catch frequency table for all 36 combinations of  $R$ ,  $P_0$ ,  $\rho$  and  $\sigma^2$  (Tables 2.1-2.9). The catch frequency tables on any single page are for the same value of  $\sigma^2$  and  $\rho$ . Therefore, the sensitivity of the results to  $R$  and  $P_0$  is revealed by comparing frequency tables on the same page. The value of  $\sigma^2$  increases for each group of 3 pages of tables (2.1-2.3, 2.4-2.6, 2.7-2.9) and  $\rho$  increases as the table number increases within each group.

$C_r$  is more variable for  $R=1.0$  than for  $R=0.25$ . This is more obvious for  $\sigma^2=0.09$  than for larger values of  $\sigma^2$ . For larger values of  $\sigma^2$ , it is clear that the probability of complete collapse of the fishery is higher for  $R=1.0$  than for  $R=0.25$ . It also appears that the mean of  $C_r$  is lower for  $R=1.0$ .

There does not seem to be any long-term effect of a low initial value of  $P_0$  (0.25 vs. 0.50= $P_{msy}$ ). Note that the fishery recovers to  $C_r=1.0$  at times 3 and 7 for  $R=1.0$  and 0.25, respectively, when  $\sigma^2=0.0$  ( $A=1$ ) and  $P_0 = 0.25$ .

There is little question that the outlook is for lower values of  $C_r$  as  $\sigma^2$  increases. This is most clear when the upper left-hand corners of Tables 2.1 and 2.7 are compared. For autocorrelated random variations ( $\rho \neq 0.0$ ),  $C_r$  appears more likely to reach extreme levels (high and particularly low).  $C_r$  seldom declines to 0 (actually <0.05) for  $\sigma^2=0.64$ ,  $R=1.0$  and  $\rho = 0.0$ , but reaches this low level for about 25% of the simulations when  $\sigma^2=0.64$ ,  $R=1.0$  and  $\rho = 0.8$ .

The mean value of  $C_r$  was calculated (Table 3) for each combination of  $R$ ,  $P_0$ ,  $\sigma^2$  and  $\rho$  for all years after the required recovery period (from  $P_0 = 0.25$  if  $A=1$ ). These mean values of  $C_r$  support the discussion above. The mean value of  $C_r$  decreases as  $\sigma^2$  and  $R$  increase. The mean of  $C_r$  after the recovery period is not greatly influenced by  $P_0$  nor by  $\rho$ , although as already noted the probability of extinction appears to increase as  $\rho$  increases.

Now that the response of Equation (1) to specific values of  $R$ ,  $P_0$ ,  $\sigma^2$  and  $\rho$  has been discussed, it is important to consider the likelihood of these values being applicable to fisheries of interest. Since  $P_{msy} = 1/2 P_{max}$  (Equation (4)), then  $CE_{max} = 1/2 F_{msy} P_{max}$  (Gulland, 1971) which implies that  $R = 1/2 F_{msy}$ . For  $R=1.0$ ,  $F_{msy}$  would be 2.0 which could only occur for species having a very high production to standing crop ratio. For  $R=0.25$ ,  $F_{msy}$  would be 0.5 which may be realistic for many species in the Northwest Atlantic.

The autocorrelation of the environmental factor influencing  $A$  is  $\rho$ . The effect of temperature on fisheries has been discussed by numerous authors (ICNAF, 1965). The autocorrelation of the annual average surface water temperature at Boothbay Harbor, Maine, is 0.70 for the period 1905-1975. Therefore,  $\rho$  as large as 0.8 seems realistic.

The variances of  $\log_e A$  considered in this work are equivalent to a .5, 5 and 10% probability of  $A > 2$ . Sissenwine (1974) estimated CE for the Southern New England yellowtail flounder fishery for 1944-1965. For these 22 years, CE exceeded its mean twice or about 9% of the time. Curves representing the relationship between CE and P and between recruitment and spawning stock have been fit for several populations. Table 4 gives the number of data points below and above each curve and the percent of points at least double their expected value ( $A=2.0$ ) for several populations. This summary (Table 4) indicates that  $\sigma^2=0.25$  (5% probability of  $A > 2$ ) is probably commonplace and that  $\sigma^2=0.64$  (10% probability of  $A > 2$ ) may occur. Table 4 also indicates that the distribution of A is likely to be asymmetric as was assumed in this work. In only 3 of 18 cases are there as many deviations above the curve as below.

Thus any combination of R,  $P_0$ ,  $\sigma^2$  and  $\rho$  considered here could occur, although  $R=1.0$  is unlikely for commercially exploited species in the Northwest Atlantic. This work indicates that the longer term average catch at  $F_{MSY}$  may be well below  $CE_{max}$  even for a fishery perfectly described by Equation (1) with the parameters known. As  $\sigma^2$  increases, the average catch declines; and as  $\rho$  increases, the probability of the population declining to extremely low levels increases. One explanation of this result is that the population declines below  $P_{MSY}$  more rapidly when  $A < 1$  than it grows above  $P_{MSY}$  when  $A > 1$ . When  $A > 1$ ,  $F_{MSY}$  is too low, but as P increases  $dP/dt$  decreases thus slowing population growth. But when  $A < 1$ ,  $F_{MSY}$  is too high so P decreases which in turn decreases  $dP/dt$  thus speeding the decline. When  $\rho > 0$ , the probability of the population growing or declining over several consecutive time intervals is increased; thus extremely high or low levels of catch become more likely.

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Table 1. Values of  $R$ ,  $\sigma^2$ ,  $\rho$  and  $P_0$  used to simulate a fishery described by Equation (1). The probability of  $A > 2$  ( $\text{Prob}(A > 2)$ ) is also given for each value of  $\sigma^2$ .

$R$	$P_0$	$\rho$	$\sigma^2$	$\text{Prob}(A > 2)$
1.00	0.50	0.0	0.09	.005
.25	0.25	0.5	0.25	.05
		0.8	0.64	.10

Table 2.1. Relative catch frequency tables for  $\sigma^2 = 0.09$  and  $\rho = 0.0$ .

RELATIVE CATCH

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00  
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50  
AUTOCORRELATION OF LOG OF A IS .98 .00  
VARIANCE OF LOG OF A IS .0050

TIME

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .83  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50  
 AUTOCORRELATION OF LOG OF A IS .15 .00  
 VARIANCE OF LOG OF A IS .050

AUTOCORRELATION OF LOG OF A 18 .00  
 VARIANCE OF LOG OF A 18 .000  
 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25  
 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

RATIO OF MAXIMUM EGYPTIAN CATCH TO MAXIMUM POPULATION SIZE IS .024  
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .024  
AUTOCORRELATION OF LOG OF A IS .00

Table 2.2. Relative catch frequency tables for  $\sigma^2 = 0.09$  and  $\rho = 0.50$ .

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION IS 1.00	
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50	
AUTOCORRELATION OF LOG OF A IS .50	
VARIANCE OF LOG OF A IS .090	
RELATIVE CATCH	TIME
2.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.9	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.8	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.7	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.6	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.5	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.4	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.3	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.2	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.9	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.8	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.7	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.6	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.5	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.4	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.3	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.2	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION IS 1.00	
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50	
AUTOCORRELATION OF LOG OF A IS .50	
VARIANCE OF LOG OF A IS .090	
RELATIVE CATCH	TIME
2.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.9	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.8	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.7	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.6	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.5	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.4	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.3	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.2	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.9	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.8	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.7	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.6	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.5	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.4	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.3	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.2	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Table 2.3. Relative catch frequency tables for  $\sigma^2 = 0.09$  and  $\rho = 0.80$ .

**RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00**

**RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50**

**AUTOCORRELATION OF LOG OF A IS .80**

**VARIANCE OF LOG OF A IS .050**

**RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00**

**RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .25**

**AUTOCORRELATION OF LOG OF A IS .80**

**VARIANCE OF LOG OF A IS .050**

**RELATIVE CATCH**

**TIME**

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .25	
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50	
AUTOCORRELATION OF LOG OF A IS .40	
VARIANCE OF LOG OF A IS .050	
RELATIVE CATCH	TIME
.05	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.10	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.25	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.30	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.35	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.40	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.45	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.50	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.55	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.60	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.65	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.70	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.75	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.80	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.85	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.90	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
.95	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
1.00	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Table 2.4. Relative catch frequency tables for  $\sigma^2 = 0.25$  and  $\rho = 0.0$ .

Detailed description: This is a scatter plot with 'RELATIVE CATCH' on the vertical y-axis and 'RATIO OF MAXIMUM EQUILIBRIUM CATCH TO INITIAL POPULATION' on the horizontal x-axis. Both axes range from 0.0 to 1.00. The data points, represented by small squares, show a clear negative correlation, starting near (0.0, 1.0) and ending near (1.0, 0.0). A horizontal dashed line is drawn at y = 0.50, and a vertical dashed line is drawn at x = 0.50.

**TABLE 1** Ratio of maximum equilibrium catch to maximum population size is 1.00  
 Ratio of initial population to maximum population is .50  
 Autocorrelation of log of a is .00  
 Variance of log of a is .00

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION IS .18 .95  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .95  
 AUTOREGULATION OF A LOG OF A 12,000  
 VARIANCE OF LOG OF A IS .950

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .88  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .85  
 AUTOCORRELATION OF LBG OF A 13-900  
 VARIANCE OF LBG OF A 13-900

Table 2.5. Relative catch frequency table for  $\sigma^2 = 0.25$  and  $\rho = 0.50$ .

TIME	RELATIVE CATCH		VARIANCE OF LOG OF A IS -220		RELATIVE CATCH	
	1	2	1	2	1	2
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0

Table 2-6. Relative catch frequency tables for  $\sigma^2 = 0.25$  and  $\rho = 0.80$ .

The figure consists of two vertically stacked line graphs sharing a common x-axis labeled "TIME".

**Top Graph:** The y-axis is labeled "RELATIVE CATCH" with values from 0.0 to 1.0. The legend indicates "RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.000". The data series starts at approximately 0.95 and rises steadily to about 0.99 over 25 time units.

**Bottom Graph:** The y-axis is labeled "RELATIVE CATCH" with values from 0.0 to 1.0. The legend indicates "RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .950". The data series starts at approximately 0.95 and rises more rapidly than the top graph, reaching about 0.99 over 25 time units.

The figure consists of two vertically stacked line graphs sharing a common x-axis labeled "TIME".

**Top Graph:**

- Y-axis: "RELATIVE CATCH" ranging from 0 to 1.
- X-axis: "TIME" ranging from 1 to 25.
- Legend: "RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION = .90" (solid line)
- Legend: "RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION = .90" (dashed line)
- Description: The solid line starts at (1, 0.9) and remains nearly constant. The dashed line starts at (1, 0.9), peaks at approximately 0.95 around time 10, and then gradually declines towards 0.9 by time 25.

**Bottom Graph:**

- Y-axis: "RELATIVE CATCH" ranging from 0 to 1.
- X-axis: "TIME" ranging from 1 to 25.
- Legend: "RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION = .95" (solid line)
- Legend: "RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION = .95" (dashed line)
- Description: Both lines are nearly identical, starting at (1, 0.95) and remaining constant throughout the 25-time period.

Table 2.7. Relative catch frequency tables for  $\sigma^2 = 0.64$  and  $\rho = 0.0$ .

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50  
 AUTOCORRELATION AT LAG OF 16 IS .16 .00  
 VARIANCE OF LOG OF A IS .0001

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .25  
 AUTOCORRELATION OF LOG OF A IS .00  
 VARIANCE OF LOG OF A IS .000

The figure consists of two vertically stacked line graphs sharing a common x-axis labeled "TIME". The top graph is titled "RELATIVE CATCH" and the bottom graph is titled "VARIANCE OF LIS". Both graphs have y-axes ranging from 0 to 10.

**RELATIVE CATCH:**

- Group A (solid line): Starts at ~8.5, peaks at ~9.5 around day 10, then fluctuates between 8.5 and 9.5.
- Group B (dashed line): Starts at ~7.5, peaks at ~8.5 around day 10, then fluctuates between 7.5 and 8.5.

**VARIANCE OF LIS:**

- Group A (solid line): Starts at ~8.5, peaks at ~9.5 around day 10, then fluctuates between 8.5 and 9.5.
- Group B (dashed line): Starts at ~7.5, peaks at ~8.5 around day 10, then fluctuates between 7.5 and 8.5.

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .25  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .50  
 AUTOCORRELATION OF LOG OF A IS .00  
 VARIANCE OF LOG OF A IS .00

RATIO OF MAXIMUM EQUILIBRIUM LATCH TO MAXIMUM POPULATION IS .007  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .25  
 AUTOCORRELATION OF LOG OF A IS .00  
 VARIANCE OF LOG OF A IS .600

RATIO OF MAXIMUM EQUILIBRIUM LATENT TO MAXIMUM POPULATION IS .001  
 RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .025  
 AUTOCORRELATION OF LOG OF A IS .00  
 VARIANCE OF LOG OF A IS .001

Table 2.8. Relative catch frequency tables for  $\sigma^2 = 0.64$  and  $\rho = 0.50$ .

**RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.00**

**RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .95**

**AUTOCORRELATION OF LOG OF A IS .50**

**VARIANCE OF LOG OF A IS .000**

**RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.05**

**RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .95**

**AUTOCORRELATION OF LOG OF A IS .50**

**VARIANCE OF LOG OF A IS .040**

RELATIVE CATCH

TIME

Table 2.8. Relative catch frequency tables for  $\sigma^2 = 0.64$  and  $\rho = 0.80$ .

RELATIVE CATCH

TIME

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.000  
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .90  
AUTOCORRELATION OF LOG OF A IS .80  
VARIANCE OF LOG OF A IS .000

RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS 1.000  
RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .90  
AUTOCORRELATION OF LOG OF A IS .80  
VARIANCE OF LOG OF A IS .000

The figure consists of two vertically stacked line graphs sharing a common x-axis representing time from 1 to 25.

**Top Graph:** The y-axis is labeled "RELATIVE CATCH". It shows four curves starting at different points on the y-axis (approximately 0.05, 0.15, 0.35, and 0.65) and all converging towards a value of 1.0 as time increases. The curves are labeled with their respective initial ratios: 0.05, 0.15, 0.35, and 0.65.

**Bottom Graph:** The y-axis is labeled "LIVING CATCH". It shows four curves starting at different points (approximately 0.05, 0.15, 0.35, and 0.65) and all converging towards a value of 1.0 as time increases. The curves are labeled with their respective initial ratios: 0.05, 0.15, 0.35, and 0.65.

**Text Labels:**

- RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .050
- RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .950
- AUTOCORRELATION OF LOG OF A IS .800
- VARIANCE OF LOG OF A IS .000
- RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .150
- RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .850
- AUTOCORRELATION OF LOG OF A IS .800
- VARIANCE OF LOG OF A IS .000
- RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .350
- RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .650
- AUTOCORRELATION OF LOG OF A IS .800
- VARIANCE OF LOG OF A IS .000
- RATIO OF MAXIMUM EQUILIBRIUM CATCH TO MAXIMUM POPULATION SIZE IS .650
- RATIO OF INITIAL POPULATION TO MAXIMUM POPULATION IS .550
- AUTOCORRELATION OF LOG OF A IS .800
- VARIANCE OF LOG OF A IS .000

Table 3. Mean value of relative catch ( $C_r$ ) after the period required to recover ( $A=1.0$ ) from  $P_0 = 0.25$ .

$\sigma^2$	R	0.09		0.25		0.64	
		.25	1.0	.25	1.0	.25	1.0
$P_0$							
0.0	.50	.98	.95	.95	.87	.87	.72
	.25	.98	.95	.94	.87	.86	.72
0.5	.50	.97	.93	.91	.84	.80	.67
	.25	.96	.93	.90	.83	.78	.66
0.8	.50	.96	.93	.88	.83	.76	.67
	.25	.93	.91	.85	.80	.72	.64

Table 4. The number of data points above and below curves representing Stock-Production or Stock-Recruitment relationships. The percent of points at least double the expected value (according to the curves) is also given.

Stock	Type of curve	Above	Below	Double
Pacific Halibut <sup>1</sup>	Stock-Production	16	32	4%
Antarctic Blue Whale <sup>2</sup>	"	8	13	5%
Buchen Atlantic Herring	Stock-Recruitment <sup>3</sup>	10	14	8%
Dogger Atlantic Herring	"	11	18	10%
Kodiak Is. Pacific Herring	"	5	6	9%
Sakhalin Pacific Herring	"	17	26	16%
British Columbian N. Pacific Herring	"	10	14	13%
British Columbian Lower E. Pacific Herring	"	13	10	4%
Halibut Area 2	"	12	21	3%
Halibut Area 3	"	14	15	0%
Petrale Sole	"	12	8	15%
Plaice	"	11	13	8%
Arctic Cod	"	9	12	0%
St. Lawrence Cod	"	3	7	0%
North Sea Haddock	"	9	7	6%
Georges Bank Haddock	"	11	20	10%

<sup>1</sup>Based on Figure 13.2A from Ricker (1975).

<sup>2</sup>Based on Figure 8B from Gulland (1972).

<sup>3</sup>All Stock-Recruitment curves based on Figures 42-47 from Cushing (1973).

