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The Schaefer model and the disequilibrium of the fisheries

One formula - two examples

by

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Abstract.

A formulation of the relation between effort and catch per effort data is presented, as a consequence of the simple mathematical model studied by Schaefer.

This formula takes into account the natural disequilibrium of the fishery and allows the determination of the parameters which appear in the Schaefer model.

Introduction.

Many technics have been proposed for the determination of the parameters enclosed in the Schaefer model. Recently WALTER (1975) presented graphical methods with correction of the catch per effort to take into account the disequilibrium of the fisheries.

Indeed this disequilibrium between effort and stock is a general case whatever the level of exploitation of the fisheries may be, the balanced production being the result of a tendency, an ideal state, reached more or less quickly in relation with time.

We suggest a formulation between effort and the mean catch per unit of effort, deducted from the basic equation of the model and giving a simple analytical expression of the divergence between balanced state and real condition of the fisheries.

I - Mathematical statement -

The simple mathematical Schaefer model has for main advantage that it requires only effort and catch data for the determination of equilibrium state.

The basic equation of the model involves that at every moment the growth of the fished population is determined by the natural increase minus an amount equal to the rate of catching of the fish :

(1) $dS/dt = K\,S\,(S_M - S) - Sqf$ where

- k is a constant,
- S the stock biomass,
- q the instantaneous rate of fishing mortality per unit of fishing effort,
- f the number of units of fishing effort, and
- S_M the maximum population which the living space can support.

Let $u = Sq$ be the catch per unit of effort, integration of the equation (1) gives, for a constant effort f, the value of u as a dependent variable of time t.

$$(2) \quad U_{(t)} = Uo\,(U_M - \frac{q^2}{K} f) \Big/ \left[Uo + (U_M - \frac{q^2}{K} f - Uo) \exp\left(-\frac{K}{q}(U_M - \frac{q^2}{K} f)t\right)\right]$$

with $U_M = qS_M$ and

$U = Uo$ when $t = o$

$(U_M - \frac{q^2}{K} f)$ is the analytical expression of a straight line, termed "the line of equilibrium conditions" by Schaefer.

Let us set $\hat{U} = U_M - \frac{q^2}{K} f$ and

$a = q/k$

Then equation (2) becomes

$$(3) \quad U_{(t)} = Uo\,\hat{U} \Big/ \left[Uo + (\hat{U} - Uo) \exp(-\hat{U}\,t/a)\right]$$

and when $t = 1$

$$(4) \quad U_1 = Uo\,\hat{U} \Big/ \left[Uo + (\hat{U} - Uo) \exp(-\hat{U}/a)\right]$$

Integrating over the year, the equation (3) we obtain $\overline{U}$, the mean catch per unit of effort during the year 1.

$$(5) \quad \overline{U}_1 = \int_o^1 Uo\,\hat{U}\,dt \Big/ \left[Uo + (\hat{U} - Uo) \exp(-\hat{U}t/a)\right]$$

and

$$(6) \quad \overline{U}_1 = a \text{ Log } \left[(Uo \exp(\hat{U}/a) + \hat{U} - Uo) / \hat{U}\right]$$

equivalent to

$$(7) \quad \overline{U}_1 = a \text{ Log } \left[\exp(\hat{U}/a)\left\{(Uo + (\hat{U} - Uo) \exp(-\hat{U}/a)\right\} / \hat{U}\right]$$

From (4) and (7) it follow that

$$(8) \quad \overline{U}_1 = a \text{ Log } \left[\exp(\hat{U}/a)\,(Uo/U_1)\right]$$

or

$$(9) \quad \overline{U}_1 = \hat{U} + a \text{ Log } (Uo/U_1)$$ or more generally for any year, I,

$$(10) \quad \underline{\overline{U}_i = \hat{U}_i + a \text{ Log } (U_{i-1}/U_i)},$$ where U_{i-1} is the catch per unit effort at the beginning of the year i, U_i the catch per unit effort at the end of the same year, and

$\hat{U}_i = U_M - aqfi$

The expression "$a\ L\ (U_{i-1}/U_i)$" gives a measure of the difference between the observed mean CPUE and the equilibrium value, for a given effort f.

If for any year, i, the real effort equals the equilibrium effort, then,

$$U_{i-1} = U_i = \hat{U}_i = \overline{U}_i$$

In the other cases the difference will be positive or negative according as the real effort is greater or lower than the balanced effort.

So, one may expect the differences with regard to the "equilibrium line" be rather positive for great levels of the biomass, one effort, even relatively small, tending to lower it in these cases :

$$U_i < U_{i-1}$$

On the other hand if the stock at the beginning of the year is low, the observed effort has more chance to be smaller than the equilibrium effort and the stock biomass tends to get higher :

$$U_i > U_{i-1} \quad \text{with correlatively } (\overline{U} - \hat{U}_i) < o$$

So, with regard to the equilibrium line, the deviation has a systematic complexion, as it appears clearly on the second example that we give.

The relationship (6) between $\overline{U}$ and f (or $\hat{U}$) depends on the parameter $Uo = qSo$, that is to say from the level of the stock biomass at the beginning of the year. For each value of this parameter we can draw a curve which intersects the equilibrium straight line for a value f such as $Uo = U_M - aqf$.

So, we obtain a family of curves on which the observed values $\overline{U}$ move, going from one to the other, attracted towards the equilibrium state.

II - Two examples -

An estimation of the parameters of the equilibrium line may be obtained from the equation (10) in a method of successive approximations.

U_{i-1} and U_i may be estimated as was done by Schaefer (1954) by

$$U_{i-1} = (\overline{U}_{i-1} + \overline{U}_i)/2 \qquad U_i = (\overline{U}_i + \overline{U}_{i+1})/2$$

First the fishing effort is plotted against CPUE, the term of correction, a L (U_{i-1}/U_i), being neglected. A straight line with negative slope is fitted to these points ; so we obtain a first approximation for U_M and aq and consequently for $\hat{U}_i$.

The equation $\overline{U}_i - \hat{U}_i = a \, L \, (U_{i-1}/U_i)$ may be used to obtain an initial estimate for the parameter a.

Substituting this value of a into (10) we now compute a second approximation for U_M and aq.

Continuing this procedure a series of successive approximations are obtained.

a) We have applied this method to the Halibut fishery of the north Pacific, data reported by Schaefer.

The approximations converge very rapidly and three iterations are enough. We obtain the following values :

U_M	aq	a	
156,1	0,216	72,6560	initial values
140,8	0,182	68,6946	final values

From these estimates we have $1/q = 377 \times 10^3$ near the value given by Thomson (Schàefer, 1954).

b) Our second example deals with data from Berthome (1975) on the redfish in ICNAF Statistical Area 4.

In figure (1) the data for the years 1962 through 1974 have been plotted.

In this case the convergence was slower and needed ten iterations

U_M	aq	a	
4,613	0,0000310	0,7526	initial values
5,540	0,0000704	2,581545	final values.

Therefore, with observations so scattered, the speed of the convergence may depend on the choice, more or less well-advised, of the first estimated straight line.

With these values we obtain $1/q = 368 \times 10^3$.

Conclusion.

From the basic equation of the Schaefer model we obtain an equation between the mean CPUE $\overline{U}$ and the effort which is not linear.

This equation comprises a correcting term we must apply on $\overline{U}$ to obtain a linear relation and compute the parameters of the equilibrium straight line.

Nowadays "it is necessary that the changes in populations associated with changes in fishing effort be sufficiently great in relation to the variations due to other cause to permit a carefully reliable determination of the parameters" (Schaefer 1957).

But this is true whatever may be the method of estimation which is used.

References

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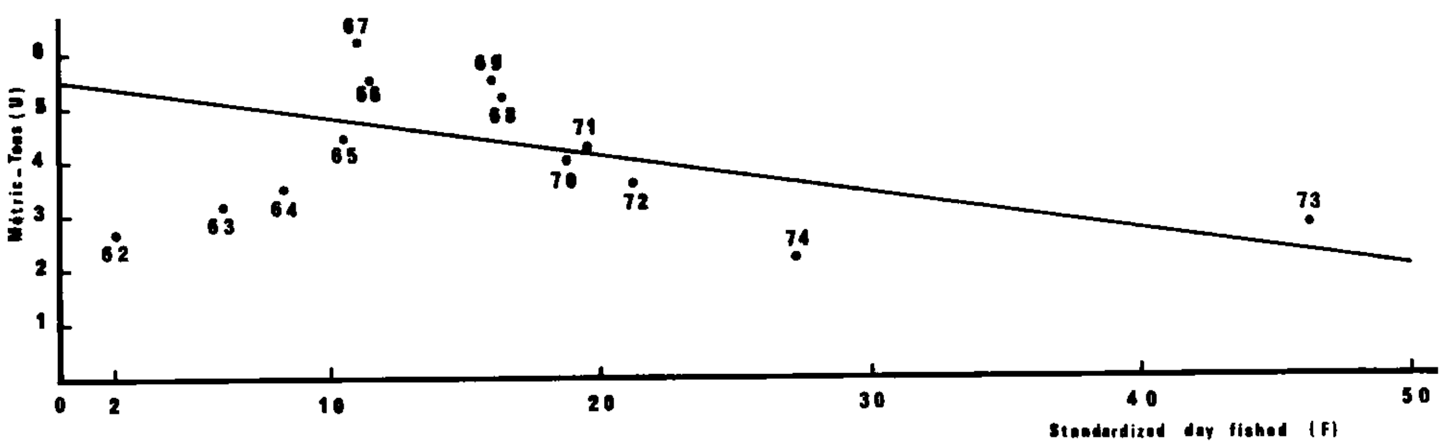

Fig. 1 - Mean catch per unit of effort set plotted against standardized effort. The estimated equilibrium line.

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ADDENDUM

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Theorical considerations were presented by CHEVALIER (1976) on the Schaefer model. A mathematical formulation between effort and the mean catch per unit of effort was proposed, deduced from the basic equation of the model and giving a simple analytical expression of the divergence between equilibrium state and real condition of the fisheries.

Two examples were presented. In order to estimate the parameters of the model for these observed data, the least squares regression of $\overline{U}$ on f was employed. This procedure may be appropriate for a probleme of predicting, but it is not necessarily appropriate for finding the structural relationship.

So, for estimating the parameters of the functional relationship we propose here a method mentionned by LEBART and FENELON (1971), the correcting term $L(U_{i-1}/U_i)$ being integrated as a supplementary variable.

1. Results previously established.

For any year i the equation relating the mean catch per unit of effort $\overline{U}$ to the fishing effort f is :

$$(1) \quad \overline{U}_i = \hat{U}_i + a\,L\,(U_{i-1}/U_i),$$

U_{i-1} is the catch per unit of effort at the beginning of the year i,

U_i, the catch per unit of effort at the end of the same year and

$\hat{U}_i = U_M - aqf_i$ is the line of equilibrium conditions.

The position of the equilibrium line is liable to two types of variation : some are caused by errors of measurement and others are the result of natural variability. In these circumstances the mean square regression of one variable on the others is not accurate for estimating the structural relationship : the observed regression coefficients are biased estimates of the structural regression coefficients.

2. Functional regression.

Principles.

Principal component analysis gives, together with the first factor, the linear combination of variables which explains the greatest part of the total variance. Actually, the variance of this combination, when the variables are standardized is the greatest eigenvalue of the correlation matrix.

In the same way the smallest eigenvalue and the associated eigenvector give the linear combination with the smallest variance.

The relation

$$(2)\quad b_1\bar{U} + b_2 f + b_3 z = 0$$

where (b_1, b_2, b_3) are the components of this eigenvector and

$z = L(U_{i-1}/U_i)$ is the equation of the functional regression plane :

the sum of the squares of the distances between this plane and the observed points, measured orthogonally to the plane, are minimized.

So, we can obtain estimates of the structural regression coefficients from equation (2).

Let us set R the correlation matrix of the variables $\bar{U}_i$, f and z . The eigenvalues can be extracted from the characteristic equation :

$$(3)\quad l^3 - 3l^2 + l(3 - \sum r^2_{ij}) + \sum r^2_{ij} - 2 \prod r_{ij-1} = 0$$

by usual methods.

The components of the eigenvector associated to the smallest latent root l are determined from the homogeneous system :

$R\,V = l\,V$, where V is the column matrix of the eigenvector (b_1, b_2, b_3) with $\sum b_i^2 = 1$.

Examples.

We have applied this method to the two sets of data given in our first paper mentioned above.

a) Halibut fishery of the north pacific (Schaefer, 1954)

$l = 0{,}0743$ $\qquad b_1 = -0{,}6901$

$b_2 = -0{,}6744$

$b_3 = +0{,}2626$

The regression plane equation becomes

$0{,}6901\ \bar{U} + 0{,}6744\ f - 0{,}2626\ z$ with standardized variables, or with original data

$$\bar{U} = 146{,}8 - 0{,}196\ f + 72{,}99\ L\ (U_{i-1}/U_i)$$

The estimated functional relationship of the equilibrium line

$\hat{U} = 146{,}8 - 0{,}196\ f$ slightly differs from the least square regression

$$\hat{U} = 140{,}8 - 0{,}182\ f.$$

b) Redfish fishery in ICNAF Statistical Area 4 (BERTHOME J.P., FOREST A., 1976)

$l = 0{,}1931$ $\qquad b_1 = 0{,}2478$

$b_2 = 0{,}7274$

$b_3 = -0{,}6399$

For the standardized variables the equation of the regression plane is :

$$0{,}2478\ \bar{U} + 0{,}7274\ f - 0{,}6399\ z = 0$$

or with original data :

$$\bar{U} = 9{,}18 - 0{,}000299\ f + 17{,}64\ L\ (U_{i-1}/U_i)$$

In this case the equilibrium line

$\hat{U} = 9{,}18 - 0{,}000299\ f$ differs clearly from the estimate obtained by the least squares regression

$$\hat{U} = 5{,}54 - 0{,}000070\ f.$$

Nevertheless it may be seen from figure 2 that the curve based on the functional regression line (A) appears to correspond much more adequately to the data than when calculated from the predictive regression (B).

Conclusion.

As emphasized by SCHAEFER (1957), if we wish to obtain estimates of the parameters of the structural relationship between the variables, the mean square regression of one variable on the others is not necessarily suitable.

Besides according to RICKER (1973), since the range of efforts available is often incomplete at both ends "the functional regression is also best for predicting". So it is usually preferable to use the functional regression in all uncertain situations.

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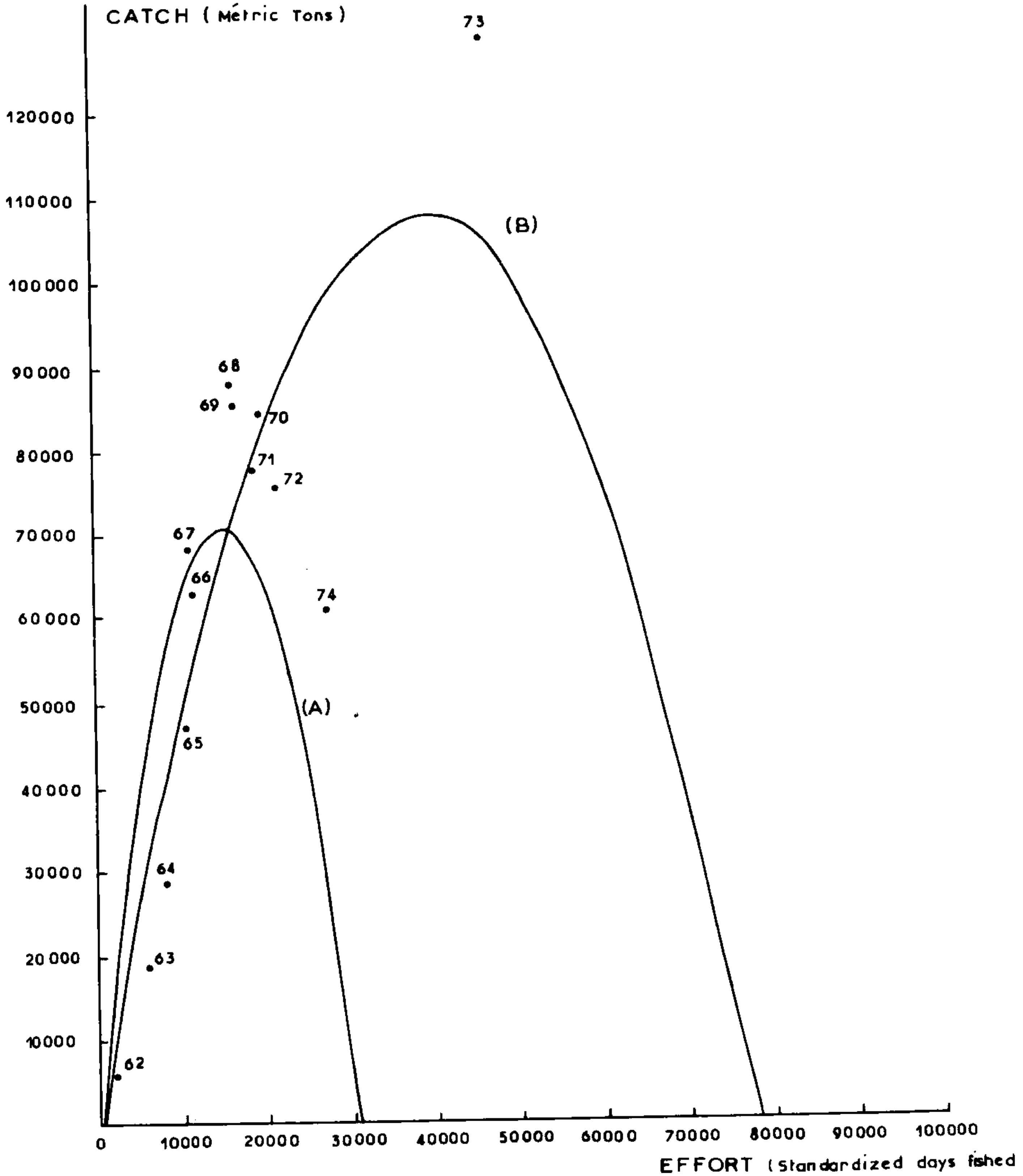

Fig. 2. Redfish. The line of equilibrium conditions obtained from - functional regression analysis (A) - predictive regression of $\bar{U}$ on f (B).