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A simple iterative method to determine fishing mortality rates associated with varying catch levels

by

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Introduction

Doubleday (1975) described a simple iterative method for estimating the instantaneous rate of fishing mortality and stock size at age at the beginning of the year from catch at age data and population size at age at the end of the year. This method has proved useful in applying computer programming techniques to the virtual population analysis method as it provides a quick and precise method for solving the catch equation when catch in numbers and stock size at age at the end of the year are known. When calculating future stock sizes from those presently known, it is useful to associate a fishing mortality rate and a resulting stock size with a particular catch level. This paper describes a quick iterative method for estimating fishing mortality rate from catch and stock size at age at the beginning of the year by an approximate linearizing of the exploitation rate in relation to fishing mortality. This can be done by squaring the catch to stock size ratio for a particular age group.

Definitions

C_n - catch of fish in numbers from a year class at age n

F_n - instantaneous rate of fishing mortality on the year class in year n

M - instantaneous rate of natural mortality

P_n - stock size in numbers of a year class at the beginning of year n

exp - exponential function

Analysis

The catch equation of Beverton & Holt (1957):

$$C_n = \frac{F_n}{F_n+M} (1-\exp(-F_n-M))\ P_n$$

can be expressed as the exploitation rate:

$$\frac{F_n}{F_n+M} (1-\exp(-F_n-M)) = \frac{C_n}{P_n} \qquad 1)$$

Since the formula does not yield an analytical solution for F_n if C_n, P_n and M are known, it has to be solved numerically, either by an iterative procedure or by reference to tables.

Figure 1 shows $\left(\frac{C_n}{P_n}\right)^2$ as a function of F_n for values of M between 0 and 1. For $F_n > 0.1$, the curves are nearly linear. Since C_n, P_n and M are known, it is possible to solve equation 1 for F_n by squaring both sides and applying successive linear interpolations or extrapolations until F_n is estimated with a predetermined accuracy.

Once F_n is known, the resulting stock size after this fishing year (P_{n+1}) may be calculated by

$$P_{n+1} = \exp(-F_n-M)\ P_n$$

A program has been written on the Hewlitt packard 9821A calculator to test this method. Initial estimates of $F_1 = -\ln\ (1-\frac{C_n}{P_n})$ and $F_2 = F_1 + 0.5$ are used for the first iteration. In successive iterations, the latest estimate of F_n replaces either F_1 or F_2 depending which is farthest from the new estimate of F_n. The iterative procedure continues until the ratio of $\frac{C_n}{P_n}$ associated with the estimated F_n is within 10^{-4} of the true value. Negative estimates of F_n are replaced by 0.

Test runs indicate rapid convergence, with only 1 to 3 iterations usually required for M between 0 and 1.

Figure 2 shows a flow chart for a computer program employing this method.

Conclusion

This method provides a precise and quick iterative solution for F_n if P_n, C_n and M are known. This allows stock projection to be accomplished by specifying quota levels rather than fishing mortality rates.

References

Beverton, R. J. H., and S. J. Holt. 1957. On the dynamics of exploited fish populations. Fish. Invest. Series II, Vol. XIX.

Doubleday, W. G. 1975. A simple iterative solution to the catch equation. ICNAF Res. Doc. 75/42, Serial No. 3521.

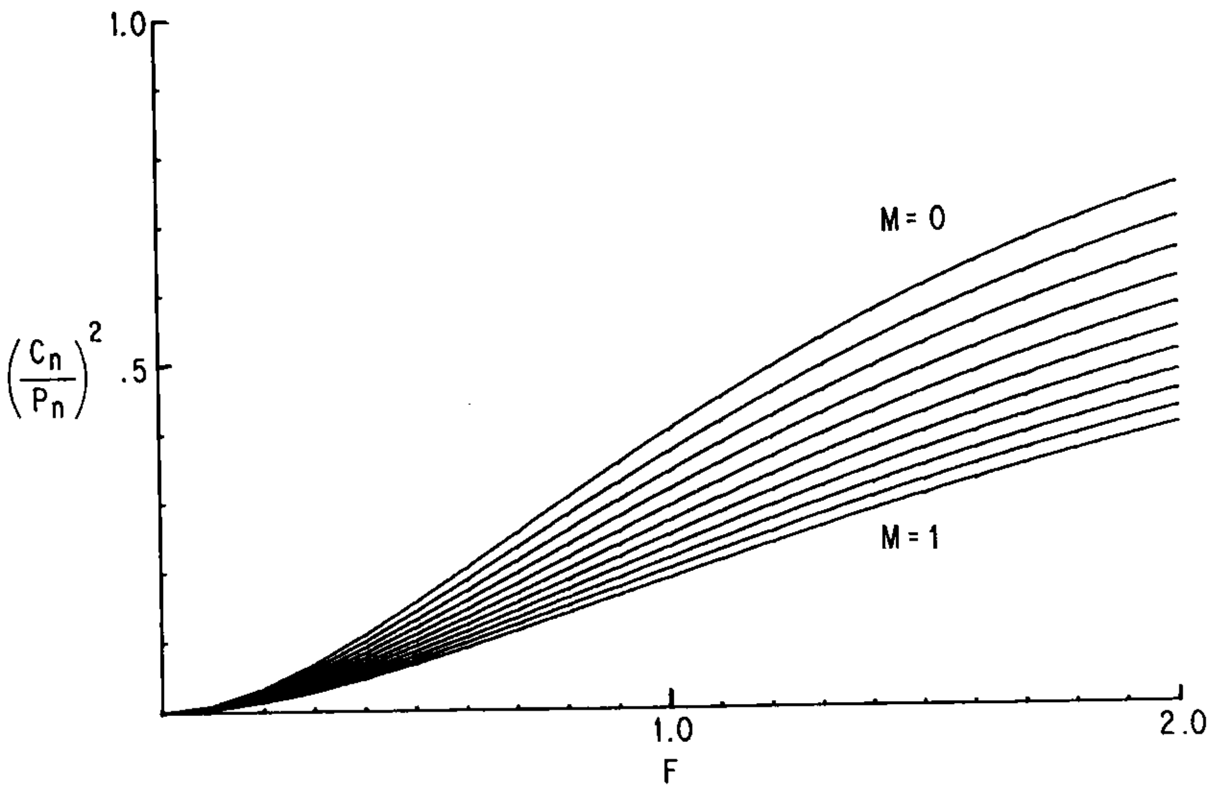

Fig 1 $\left(\frac{C_n}{P_n}\right)^2$ as a function of F for M between 0 and 1

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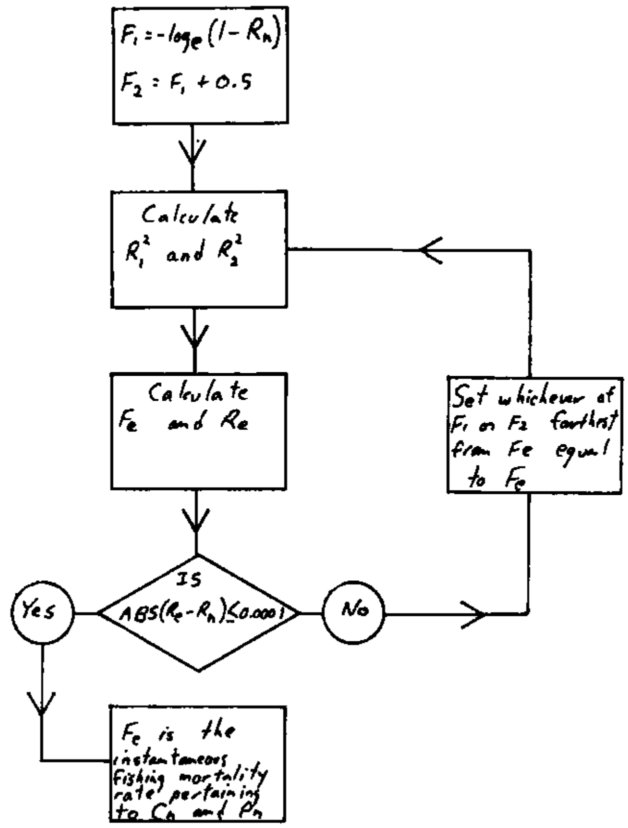

Fig. 2. Flow chart for estimating F_n if P_n, C_n, and M are known.

F_1, F_2 - estimates of F used in each iteration

$$R_1 = \frac{F1}{F_1+M} \left(1-\exp(-F_1-M)\right)$$

$$R_2 = \frac{F2}{F_2+M} \left(1-\exp(F_2+M)\right)$$

F_e - estimated instantaneous fishing mortality rate

$$R_e = \frac{F_e}{F_e+M} \left(1-\exp(-F_e-M)\right)$$

$$R_n = \frac{C_n}{P_n}$$