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International Commission for the Northwest Atlantic Fisheries

Serial No. 5358

ICNAF Res. Doc. 79/II/32
Revised

SPECIAL MEETING OF STACRES - FEBRUARY 1979

Biological Charactersitcs and Biomass Estimates of Capelin in ICNAF Div. 2J+3K Using a Sequential Capelin Abundance Model

by

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Introduction

Since 1972, substantial catches of capelin have been made in ICNAF Div. 2J3K by an offshore trawler fleet operating in the fall of the year. At this time capelin are schooling and feeding in preparation for the overwintering period.

Estimates of capelin biomass by acoustic and photogrammetric methods have been produced since 1974 (Serebrov et al 1975, Bakenev et al 1976, Klochkov et al 1977, Bakanev and Seliverstov 1978, Miller et al 1978).

This paper discusses the biological characteristics of capelin occurring in Div. 2J3K in 1978. In addition, a sequential capelin abundance model for this stock is presented and discussed.

Catch and Catch per Unit Effort

Catches in ICNAF Subarea 2 and Div. 3K showed an increase from 1972 to 1976 but have declined in 1977 and 1978 (Fig. 1). Estimates of C/E reveal a decline in 1977 and 1978 for 2J and a decline from 1976 to 1978 for Div. 3K. Combined estimates for C/E for 2J3K show a peak in 1974 and a general decline through to 1978. The estimates of C/E for 1978 are less than half of the estimates for 1974 (Table 1).

Table 1. Estimates of catch per unit effort (fishing day) for USSR trawlers >2000 GRT fishing capelin in ICNAF Div. 2J and 3K.

Year	Div. 2J	Div. 3K	Div. 2J3K
1972	24.2	32.6	28.7
1973	35.0	39.2	37.2
1974	64.4	51.5	59.8
1975	40.0	53.3	43.3
1976	43.3	42.1	42.6
1977	38.4	38.1	38.3
1978*	22.4	22.1	22.2

* From Canadian FLASH

Biological Characteristics

All data in this section unless otherwise specified were collected from commercial catches by Canadian Foreign Fisheries Observer Program.

Catches of capelin in Div. 2J3K in 1978 showed variation in age-composition by month. Most of the catches in 2J occurred in August and September and older fish predominated in these catches (Fig. 2). For males, the 1975 year-class and for females, the 1973 year-class was strongest.

In Div. 3K, two year-old fish were common during August and September; the proportions of 2 year-olds declined but one year-olds appeared in the catches (Fig. 3). In this respect one year-old capelin were unusually strong; although one year-olds have been recorded in other years (Bakanev and Seliverstov 1978) they were more apparent during the 1978 fishing season. The 1975, 1974 and 1973 year-classes were about of equal strength in females but in males the 1975 year-class predominated.

Capelin from Div. 2J were larger at age than capelin from Div. 3K (Table 2). The higher proportions of immature fish from samples in Div. 3K may account for the smaller mean length-at-age.

Large proportions of the one and two year-olds were maturing (Table 3). It is difficult to evaluate the significance of this finding. It may indicate that the 1977 year-class is faster growing than normal and is maturing at an earlier age. This would be unusual since Winters (1974) reported that capelin from the northern stock tended to grow more slowly and mature at a later age.

Table 2. Mean lengths of capelin from commercial catches in ICNAF Div. 2J and 3K, 1978.

	Age							
Males	1	2	3	4	5	6	7	8
Div. 2J		152	182	186	193	188		
Div. 3K	126	151	166	174	180	174	233	
Females								
Div. 2J		147	168	178	186	192	201	202
Div. 3K	123	143	158	172	181	187	195	

Table 3. Proportions of mature capelin by month, sex and age in samples from commercial capelin fishery in ICNAF Div. 2J and 3K, 1978.

Males	Age					
Month	1	2	3	4	5	6
Aug.		100	100	75	66	100
Sept.	100	100	100	100	100	
Oct.	43	94	99	100	100	100
Nov.	67	86	100	98	85	58
Females						
Aug.		100	100	100	100	100
Sept.	100	100	100	100	100	100
Oct.	34	83	100	100	100	100
Nov.	39	78	99	99	98	98

On the other hand, high proportions of mature fish in the younger age-groups may simply reflect a behavioral pattern of matures and immatures forming separate schools. In our research vessel samples taken between Oct. 8-30, 1978, we found that the proportions of immatures in a set varied between 0% and 95% (Table 4).

Table 4. Maturity composition of capelin and set details from Gadus Atlantica 14, Oct. 8-30, 1978.

Set No.	Date	Position	No. Males	% Immature	No. Females	% Immature	Total of Immature
1	Oct. 8	50-00N, 54-06W	114	82	86	94	87
2	Oct. 9	50-11N, 56-04W	21	29	179	0	3
3	Oct. 9	50-13N, 54-25W	102	0	98	0	0
6	Oct. 10	50-29N, 54-48W	116	0	84	0	0
7	Oct. 11	50-32N, 55-48W	54	7	146	5	6
22	Oct. 22	50-52N, 54-21W	120	3	80	5	4
23	Oct. 23	50-22N, 54-36W	107	0	93	0	0
26	Oct. 26	50-06N, 54-40W	99	77	101	64	70
27	Oct. 26	50-04N, 55-36W	99	96	101	95	95
29	Oct. 27	50-48N, 55-19W	92	27	108	12	19
30	Oct. 28	50-36N, 54-52W	88	13	112	13	13
33	Oct. 29	50-28N, 55-02W	98	33	102	21	27
34	Oct. 30	50-02N, 54-43W	97	9	103	8	9
35	Oct. 30	50-33N, 54-58W	110	73	90	69	71
		Mean		31		25	28

Sequential Capelin Abundance Model 2J3K (SCAM 2J3K)

(1) Seasonal Aspects of the Model:

Because the fishery occurs during only a portion of the year and not during spawning, the model was designed to account for the timing of these events.

The year is assumed to be a calendar year that is, 1 January-31 December. The period from 1 January to 1 July is considered to be a pre-spawning period with only natural mortality occurring. Spawning is assumed to occur on 1 July of each year and is treated as an instantaneous event. Between 1 July and 1 September only natural mortality occurs on the spawning survivors, immature fish and maturing fish. It is assumed that by 1 September fish that were immature and did not spawn during the preceding spawning period have now matured and will spawn the following year. It is assumed that the fishery occurs on both mature and immature fish from 1 September to 31 December.

(2) Natural Mortality and Spawning Mortality:

Natural mortality is 0.3 throughout. Spawning mortalities are from Carscadden and Miller (1979). Thus, for ages 3-5 spawning mortality is 1.94.

(3) Proportions Mature at Age:

The values for p = proportions mature at age are from Carscadden and Miller (1979) and are assumed to be that proportion of fish mature after the fishery and at the start of the next year. For instance, after the fishery on age 2, the value for p on January 1 of the following year will be 0.6 (age 3).

Age	p
2	0
3	.6
4	.9
5	1.0

(4) Ages used in the Model:

Ages 2 to 5, inclusive, are used in the model.

(5) Numbers-at-age:

Numbers-at-age for 1972-1977 were calculated separately for 2J and 3K and then combined. Mean weights for individual fish were taken from Fig. 2 of Bakanev and Seliverstov (1978). Percent age compositions in Div. 2J and 3K for 1972, 1974, 1975 and 2J-1973, 3K-1976 were from Fig. 2 of Bakanev and Seliverstov (1978). Percent age compositions for 2J in 1976 was from Kostantinov and Noskov (1977) and for 2J3K in 1977 was from Konstantinov and Noskov (1978). Estimates of percent age-composition for Div. 3K in 1973 was from Canadian sampling data.

Numbers-at-age for 1978 were also calculated separately from data collected by the Canadian Foreign Fisheries Observer Program. Weights of individual capelin by sex by month were estimated using length-weight regressions constructed from Canadian research data.

Numbers-at-age and mean-weights-at-age are found in Table 5.

Table 5. Numbers-at-age (ooo's) removed by Div. 2J+3K capelin fishery and mean weight (gm) at each age used in the model.

Age	1972	1973	1974	1975	1976	1977	1978	Mean wt. (gm)
2	100,675	1,109,438	641,645	5,170,024	1,378,590	147,388	285,037	10.0
3	1,511,343	1,012,872	2,389,121	2,838,028	7,622,039	1,594,565	689,601	15.8
4	440,865	2,517,772	601,896	833,162	478,471	3,361,226	599,373	24.3
5	37,950	329,464	437,466	294,155	54,901	391,655	545,529	30.4

(6) Partial Recruitment and Fishing Mortality:

It was assumed that fishing takes both matures and immatures. Estimates of partial recruitment and starting fishing mortality were estimated in 2 ways.

a. Examination of the distribution of F with age for the years 1972-1975 from initial runs of the model was made. The values of F for each age in these 4 years were averaged and expressed as a proportion of the largest F (i.e. from the oldest age group, age 5) to give estimates of partial recruitment (Table 6).

Values of starting F for the oldest age group in each year were selected as in Carscadden and Miller (1979). A number of runs of the model using values of starting F from 0.01 to 1.00 were made. Estimates of fishing effort for 1972-1978 were made by dividing the total catches by the C/E of USSR trawlers >2000 GRT. Using values of F on fully recruited age groups and effort for the years 1972-1977 functional regressions were calculated. Values for r and predicted F did not vary greatly over a wide range of starting F's from 0.1 to 1.0. Therefore, the predicted value of F = 0.213 was chosen since it fell in approximately the middle of the range of predicted F's (Table 7).

In calculating the functional regressions the value for effort in 1976 was considered to be abnormal and was not used. Estimates of C/E were constant throughout the 1976 fishing season and lower than 1974 and 1975. This was unusual considering that the 1973 year-class was abunaant as 3 year-olds and the total catch was good. Klochkov et al (1977) reported that stationary overwintering concentrations of capelin were formed one month later than usual. This suggests that capelin were more dispersed earlier in the season thus reducing the catch per unit effort.

b. For each age group in each year, catch per unit effort was calculated by dividing the removals at age by the total effort as calculated above. Then functional regressions were calculated for each age group with catch per effort for each age-class as the x-variable and numbers-at-age at the start of the year predicted from the model (F = .213) as the y-variable (Table 8). Thus, there was a regression for each age-class and using the C/E for 1978, an estimate of the numbers at each age was predicted. Partial recruitment and values of F were then estimated. This method gave a starting F of 0.200.

Table 6. Estimates of partial recruitment used with F = 0.213.

Age	Partial recruitment
2	0.050
3	0.300
4	0.970
5	1.000

Table 7. Coefficients and r^2 values and predicted values of F for functional regressions at different values of starting F.

Starting F	0.1	0.2	0.5	1.0
r^2	.86	.92	.93	.93
a	-0.16024	-0.169560	-0.17985	-0.174000
b	0.000152	0.000163	0.000175	0.000175
Predicted F	0.196	0.213	0.231	0.237

Table 8. Coefficients and r^2 values for functional regressions for each age, predicted fishing mortality F and calculated partial recruitment.

	Age			
	2	3*	4	5
r^2	.88	.98	.83	.89
a ($\times 10^6$)	31.42	16.97	31.05	2.94
b ($\times 10^6$)	.19848	.11028	.00568	.02692
Predicted F	.007	.037	.118	.200
Partial Recruit	.035	.185	.590	1.00

* Values for 1972 and 1974 not used.

(7) The Model:

A schematic representation of the model is given in Appendix A.

(A) The exploitation rate μ in year N is

$$\mu = \frac{F}{F + MDF} \quad \text{(for oldest age-class)}$$

or

$$\mu = \frac{F}{F + MDF}\left(1 - e^{-F-MDF}\right) \quad \text{(for younger age-classes)}$$

where natural mortality during fishing, MDF = 0.1.

The total population before fishing TP_{BFN} is calculated by $TP_{BFN} = \frac{C}{\mu}$.

This estimate of TP_{BFN} includes both matures and immatures and is an estimate of the size of the year-class before the fishery.

The size of the population after spawning TP_{ASN} is calculated by

$$TP_{ASN} = TP_{BFN}\, e^{MAS}$$

where the natural mortality before fishing and after spawning MAS = 0.05. The population after spawning is composed of spawning survivors (PM_{ASN}) and immature fish (PIM_{ASN}).

Thus, $TP_{ASN} = PM_{ASN} + PIM_{ASN}$

Although we do not know the proportions mature (pM) and proportions immatures (pIM) after spawning, we do know the value of p, the proportions of capelin mature at the start of the year as well as spawning mortality (SM) and natural mortality before spawning (MBS).

Thus, $TP_{ASN} \times pM = PM_{ASN}$

$TP_{ASN} \times pIM = PIM_{ASN}$

(B) and $PM_{ASN}\, e^{SM + MBS}$ gives PM_{BYN}

$PIM_{ASN}\, e^{MBS}$ gives PIM_{BYN}

(C) The total population at the beginning of the year TP_{BYN} is composed of PM_{BYN}, the population of matures plus PIM_{BYN}, the population of immatures. The proportion mature $p = PM_{BYN} / TP_{BYN}$.

Although the numbers of matures and immatures after spawning are not known, there is a unique algebraic solution (see Appendix B) to the problem and TP_{BYN} can be calculated.

(D) We know the value of TB_{BYN} which is the population size after the fishery in year N-1. Since we have an estimate of catch in year N-1 and MDF, a value for F can be calculated.

Then, $T_{BYN}\, e^{F + MDF + MAS}$ gives TP_{ASN-1}

The model is continued for each year-class in the population.

Results from the SCAM 2J3K Model

Population and total biomass estimates, calculated fishing mortality and total population and mature biomass at the beginning of 1979 (starting F = 0.213) are found in Tables 9, 10 and 11, respectively. Similar results using a starting F of 0.200 given in Table 12, 13, and 14.

Although the values of starting F are similar the results differ especially for the years 1976-1978. In these years, total calculated biomass is over 1 million tons higher in each year with a starting value of F = 0.200. However, both runs of the model indicate similar trends in total biomass levels especially for the first four years of the analysis. The calculated biomass in 1972 was moderate at about 5 million tons, then dropped in 1973 and 1974 to about 3.5 million tons and subsequently rose to about 6.5 million tons in 1975. In 1976, the model run with F = 0.200 shows the highest biomass at about 7.3 million tons while the run at F = 0.213 indicates the total biomass remained at about 6.5 million tons. The high biomass levels in 1975 and 1976 are probably the result of the presence of the strong 1973 year-class.

Both model runs exhibited a decline in biomass in 1977 and 1978. The model run with F = 0.200 indicated that the 1978 biomass was about the same as 1973 and 1974, but the other run with F = 0.213 showed the biomass in 1978 was lower than any of the previous years.

Table 9. Total population (000's), total biomass (000 tons) and biomass (000 tons) of mature fish at beginning of year in Div 2J+3K. (Starting F = 0.213.)

Age	1972	1973	1974	1975	1976	1977	1978
2	80,038,262	46,469,417	113,523,713	254,325,038	74,677,770	39,864,096	34,531,573
3	132,821,064	59,198,088	33,370,783	83,490,027	183,493,968	54,012,145	29,391,749
4	23,515,905	46,405,806	20,360,269	9,752,410	27,376,877	58,856,173	17,939,766
5	613,940	3,576,566	5,496,610	2,887,499	868,907	4,196,622	6,813,540
Total biomass	3,489	2,636	2,324	4,187	4,338	2,310	1,453
Mature Biomass	1,792	1,685	929	1,093	2,365	1,927	878

Table 10. Calculated values of fishing mortality on total population in Div. 2J+3K. (Starting F = 0.213.)

Age	1972	1973	1974	1975	1976	1977	1978
2	0.002	0.031	0.007	0.026	0.024	0.005	0.011
3	0.030	0.046	0.209	0.094	0.116	0.081	0.064
4	0.111	0.361	0.181	0.645	0.103	0.384	0.207
5	0.111	0.361	0.209	0.645	0.116	0.384	0.213

Table 11. Total population (000's) and mature biomass (tons) at the beginning of 1979. (Starting F = 0.213.)

Age	Jan. 1, 1979
2	73,000,000
3	25,310,620
4	9,931,631
5	2,478,943
Mature biomass	533,000

The estimates of mature biomass at the start of 1979, 0.72 and 1.19 million metric tons, are similar to the acoustic estimate of .75 million metric tons (Miller and Carscadden 1979).

Both runs of the model showed that the 1969 and 1973 year-classes were strong with the 1973 year-class being about 1.5 times more abundant than the 1969 year-class. The run of the model with starting F = 0.200 suggested that the 1975 and 1976 year-classes were of moderate strength but much weaker than both the 1969 and 1973 year-classes. With the other model run (F = 0.213), the 1975 and 1976 year-classes appear to be less abundant than previous year-classes.

The sensitivity of the model with different levels of p, proportions mature at age 4, was tested. When the value of p was decreased by 1%, the maximum change in total population was an increase of 2.8% and the maximum change in F on the oldest age group was a decrease of 3.7%. These values were identical for runs of the model at F = 0.200 and F = 0.213.

Table 12. Total population (000's) total biomass (000 tons) and biomass (000 tons) of mature fish at beginning of year in Div. 2J+3K. (Starting F = 0.200.)

Age	1972	1973	1974	1975	1976	1977	1978
2	80,077,856	46,586,915	113,852,984	257,449,329	120,280,980	67,816,350	52,443,267
3	132,905,173	59,227,420	33,457,827	83,733,955	185,808,492	87,795,510	50,099,077
4	23,519,492	46,436,102	20,370,834	9,783,748	27,464,732	59,690,019	30,108,206
5	613,987	3,577,176	5,501,755	2,889,293	874,296	4,211,547	6,955,035
Total Biomass	3,491	2,639	2,329	4,223	4,833	3,644	2,259
Mature Biomass	1,793	1,686	930	1,096	2,535	2,266	1,345

Table 13. Calculated values of fishing mortality on total population in Div. 2J+3K. (Starting F = 0.200.)

Age	1972	1973	1974	1975	1976	1977	1978
2	0.002	0.031	0.007	0.026	0.015	0.003	0.007
3	0.030	0.046	0.208	0.094	0.114	0.049	0.037
4	0.111	0.360	0.181	0.642	0.102	0.377	0.118
5	0.111	0.360	0.208	0.642	0.114	0.377	0.200

Table 14. Total population (000's) and mature biomass (tons) at the beginning of 1979. (Starting F = 0.200.)

Age	Jan. 1, 1979
2	89,700,000
3	38,579,921
4	17,390,313
5	4,545,874
Mature biomass	884,000

Discussion

The results of SCAM 2J3K illustrate the impact of strong year-classes on the estimates of capelin biomass. Biomass levels were high in 1972 because of the strong 1969 year-class and also in 1975 and 1976 because of the strong 1973 year-class. In years when strong year-classes were absent, average total biomass levels were in the order of 3.5 million metric tons while the average mature biomass was 1.2-2.5 million metric tons.

Winters and Carscadden (1978) estimated that the annual consumption of capelin by cod in 2J3KL during 1947-1951 was approximately 3.97 million metric tons. However, during approximately the next two decades this cod stock declined and annual consumption of capelin by cod was estimated to be approximately 3.0 million metric tons. In the same paper, it was estimated that 300,000 metric tons of capelin were eaten by the present stock of harp seals and that approximately 250,000 tons of capelin were eaten by fin whales. In total, these predators consumed approximately 3.5 million metric tons of capelin annually during the years that their stocks were low. Minet and Perodou (1978) estimated that cod in 2J3KL consumed 2.0-3.4 million metric tons of capelin annually during the period 1965-1969.

These estimates of annual consumption of capelin are not directly comparable to the population estimates of capelin for Div. 2J3K. The cod feeding estimates were derived for the 2J3KL cod stock; the seal and whale feeding estimates were derived for the Newfoundland and Labrador area as a whole. It is reasonable to assume then that feeding by predators in the 2J3K area only would be lower than 3.5 million metric tons. Thus, under conditions of average recruitment in the 2J3K capelin stock, the supply of capelin would be in excess of the needs of the three major capelin predators.

The stock size of capelin in 1978 was below average. The presence of the 1977 year-class as one year-olds (not included in our calculations) and the reduced stock of mature fish in 1978 probably contributed to the decrease in catch and catch per effort in 1978.

The data from the commercial fishery indicated that the 1977 year-class was relatively abundant. Comparisons of this year-class with the strong 1969 and 1973 year-classes are not possible at this time. However, caution should be exercised in analyzing these data because the relatively poor abundance of the 2 and 3 year-olds in 1978 may lead to a conclusion that the 1977 year-class is stronger than it actually is. Thus, we would recommend that for purposes of estimating recruitment, the 1977 year-class should be considered to be of average strength.

Acknowledgements

We thank Dr. G.H. Winters and Dr. W. Doubleday for advice and comments during preparation of this paper.

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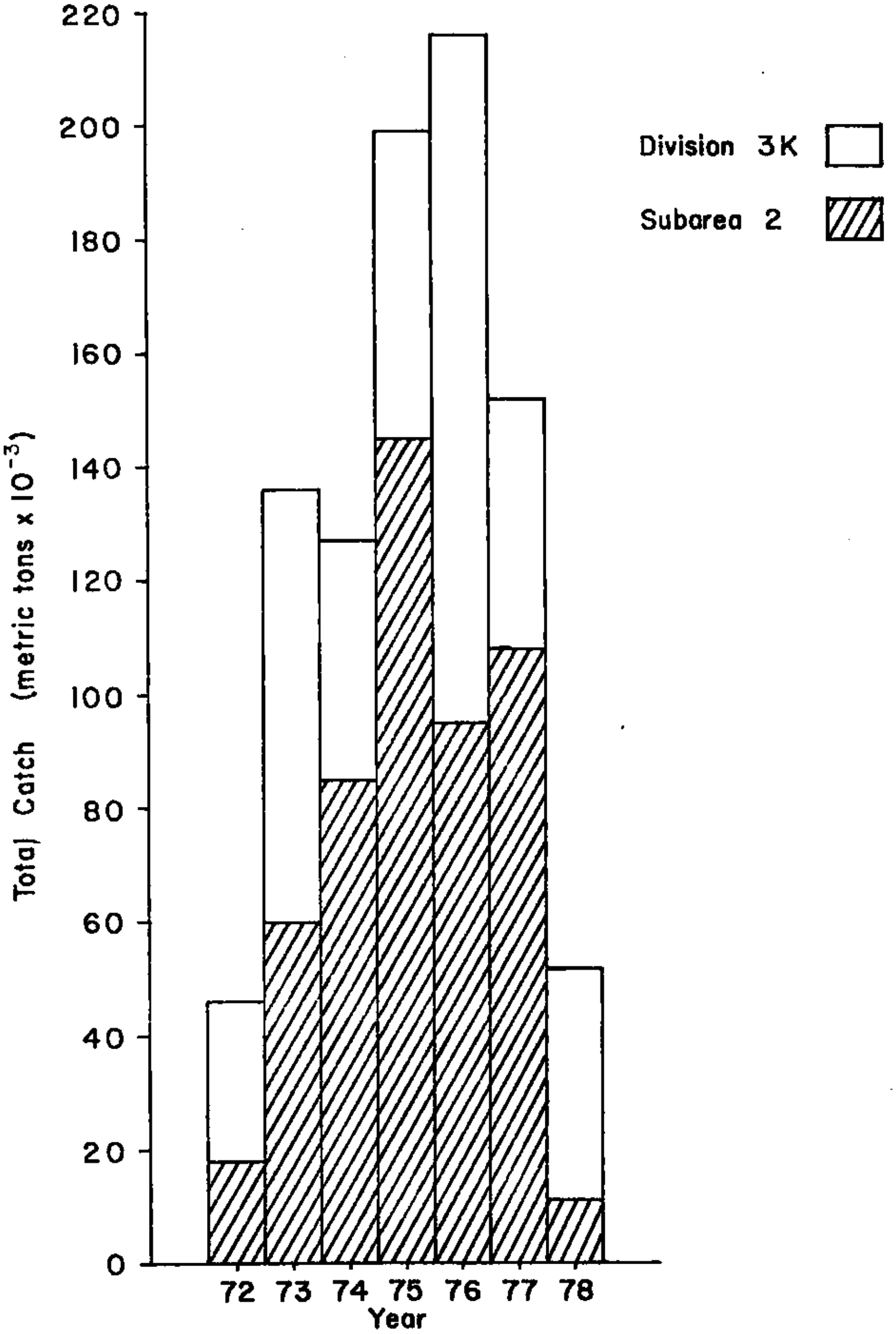

Fig. 1. Catches of capelin in ICNAF Subarea 2 and Div. 3K from 1972-1978.

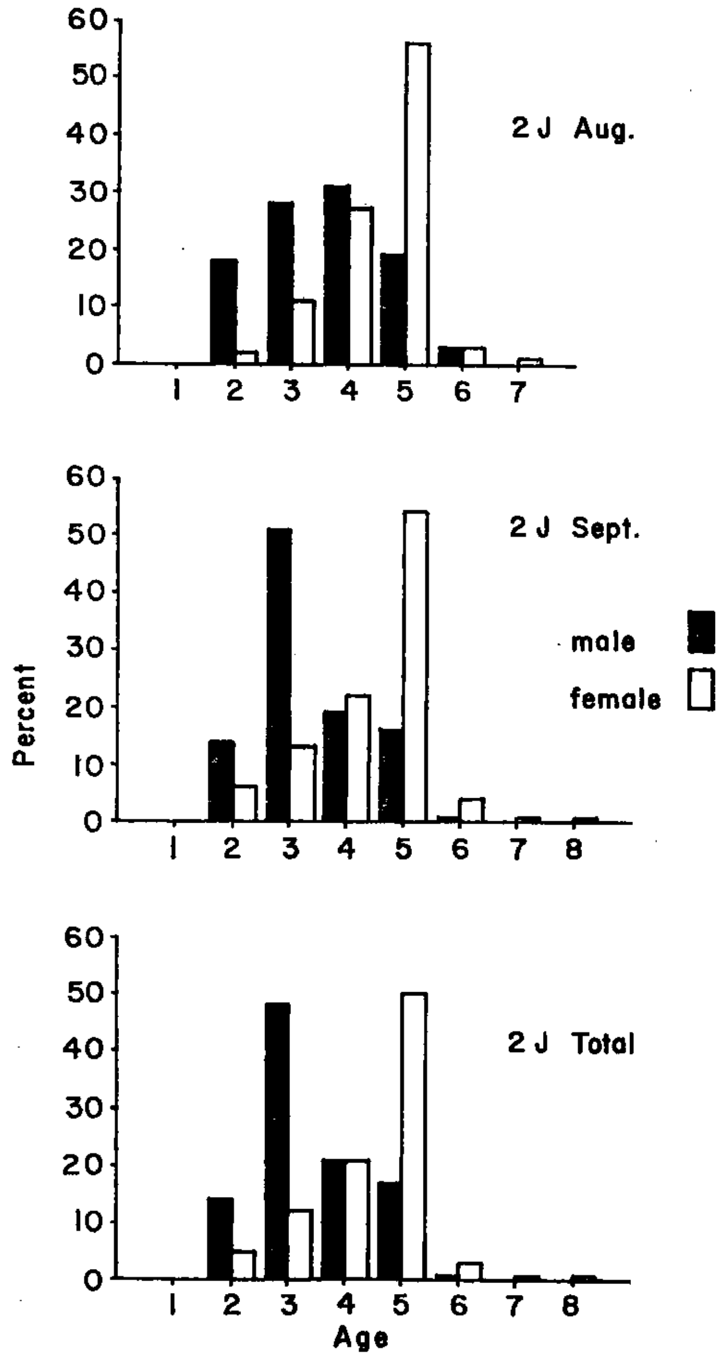

Fig. 2. Age composition of capelin in commercial catches from ICNAF Div. 2J in 1978.

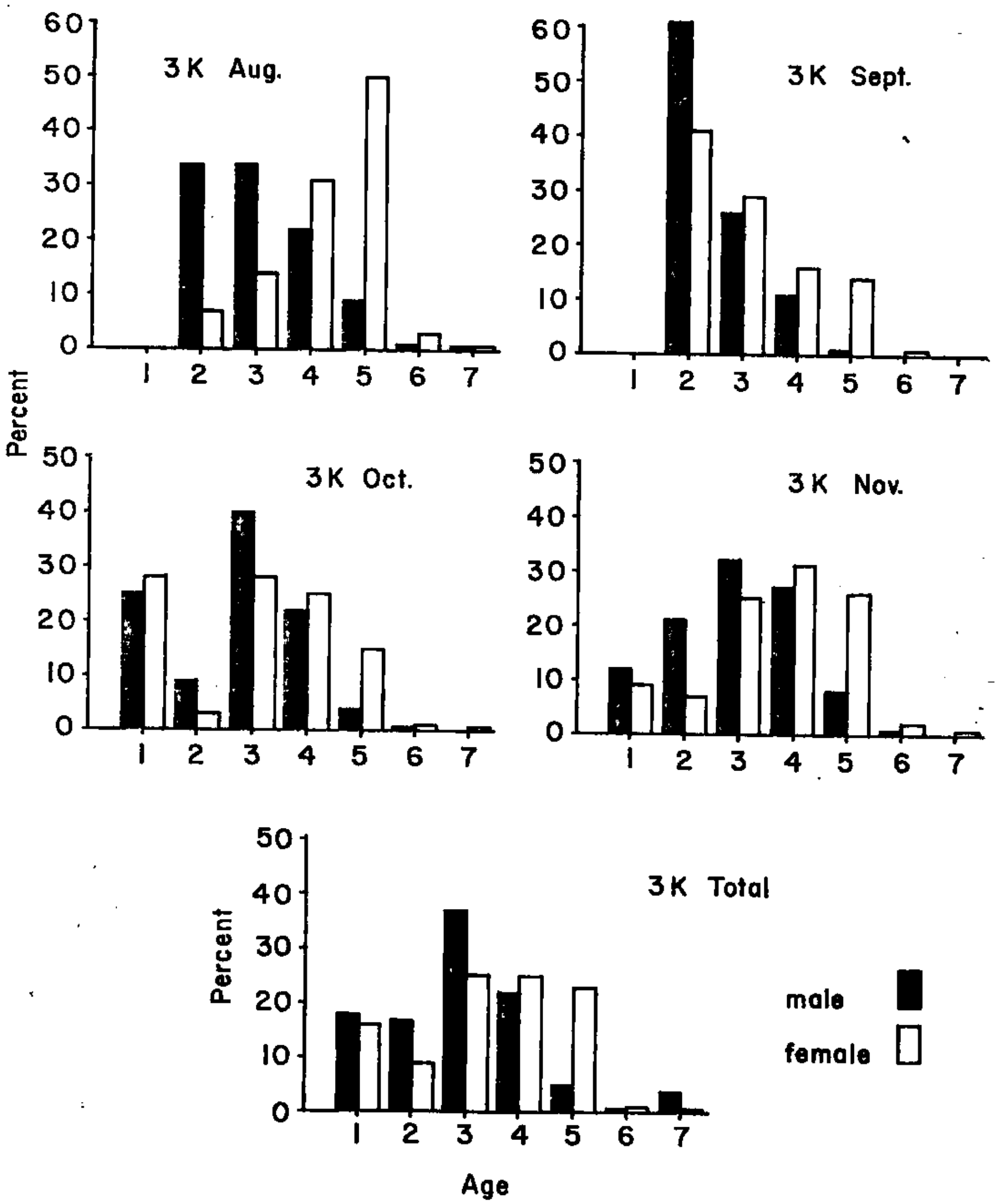

Fig. 3. Age composition of capelin in commercial catches from ICNAF Div. 3K in 1978.

Schematic Representation of SCAM 2J3K

SM = spawning mortality
MDF = natural mortality during fishery (1 Sept.-31 Dec.)
MAS = natural mortality after spawning (1 July -1 Sept.)
MBS = natural mortality before spawning (1 Jan. - 1 July)

(A) $\mu = \frac{F}{F + MDF}$ (for oldest age class)

or

$\mu = \frac{F}{F + MDF}(1 - e^{-F-MDF})$ (for younger age-classes)

$$\frac{CN}{\mu} \longrightarrow TP_{BFN}\, e^{MAS} \longrightarrow TP_{ASN} \left\} \begin{array}{l} \times\ pM \longrightarrow PM_{ASN} \\ \times\ pIM \longrightarrow PIM_{ASN} \end{array} \right.$$

(B) $PM_{ASN}\, e^{SM + MBS} \longrightarrow PM_{BYN}$

$PIM_{ASN}\, e^{MBS} \longrightarrow PIM_{BYN}$

(C) $TP_{BYN} = PM_{BYN} + PIM_{BYN}$

$pN = PM_{BYN} / TP_{BYN}$

(D) In year N-1

F calculated based on

$\frac{C}{TP_{BYN}}$ knowing MDF

Then, $TP_{BYN}\, e^{F + MDF + MAS} \longrightarrow TP_{ASN-1} \left\} \begin{array}{l} \times\ pM \longrightarrow PM_{ASN-1} \\ \times\ pIM \longrightarrow PIM_{ASN} \end{array} \right.$

$PM_{ASN-1}\, e^{SM + MBS} \longrightarrow PM_{BYN-1}$

$PIM_{ASN-1}\, e^{MBS} \longrightarrow PIM_{BYN-1}$

$TP_{BYN-1} = PM_{BYN-1} + PIM_{BYN-1}$

$pN\text{-}1 = PM_{BYN} / TP_{BYN-1}$

Algebraic solution for determining the number of mature fish after spawning in SCAM 2J3K.

(1) $PIM_{AS} = TP_{AS} - PM_{AS}$

(2) $p = \dfrac{PM_{AS}\, e^{SM}}{PM_{AS}e^{SM} + PIM}$

$= \dfrac{PM_{AS}\, e^{SM}}{PM_{AS}e^{SM} + TP_{AS} - PM_{AS}}$ (from Eq. 1)

(3) $PM_{AS}e^{SM} + TP_{AS} - PM_{AS} = \dfrac{PM_{AS}e^{SM}}{p}$

(4) $\dfrac{PM_{AS}e^{SM}}{p} + PM_{AS} - PM_{AS}e^{SM} = TP_{AS}$

(5) $PM_{AS}\left(\dfrac{e^{SM}}{p} - e^{SM} + 1\right) = TP_{AS}$

(6) $PM_{AS} = TP_{AS} \Big/ \left(\dfrac{e^{SM}}{p} - e^{SM} + 1\right)$

All values on right of equation are known - solve for PM_{AS}.